

# Predicting Future Observations of Functional and Structural Measurements in Glaucoma Using a Two-Dimensional State-based Progression Model

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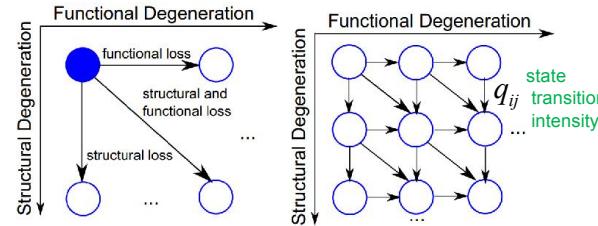
## Purpose: Future observation prediction based on 2-D continuous-time hidden Markov model (2D CT-HMM)

- Glaucoma progression:** structural (retinal nerve fiber loss) and functional (visual field loss) degeneration processes often occur asynchronously over the disease course.

### The proposed 2-D state-based CT-HMM model:

- \* Define disease states based on joint structural and functional measures, and model their transition intensities to capture their intricate dynamic relationship.
- \* The learned state transition intensities, and state dwelling time distribution, can be intuitively visualized for progression understanding.

\* Covariate (such as age, treatments, etc.) effects can also be learned and incorporated into the model for individual-specific disease state decoding and future state path prediction.



## Methods: Learn the state transition intensities from the longitudinal data for state-based future path prediction

- 2-D disease state definition:** visual field index (VFI) and global mean circumapillary retinal nerve fiber layer (RNFL) thickness from OCT.

### The likelihood function for one individual with unknown parameters $q_{ij}$ (Q matrix):

$$p(O, S^* | \lambda) = \max_{S^* = s_1, \dots, s_n} \{ p(o_1 | s_1) p(s_1) \prod_{k=2}^n p(o_k | s_k) P_{s_{k-1}, s_k}(t_k - t_{k-1}) \}$$

state data emission prob. state transition prob. with time interval ( $t_k - t_{k-1}$ )

where  $P(d) = e^{Qd}$  is the state transition probability matrix with duration  $d$ , computed from the matrix exponential of intensity matrix  $Qd$ . The  $P_{ij}(d)$  entry represents the probability that if the current state is  $s_i$ , then after duration  $d$ , the state will be  $s_j$  (there can be many state jumps in the time interval).

O: noisy observation sequence  
S\*: best hidden state sequence  
(ok, tk): one visit's data (observations, time)  
 $q_{ij}$ : state transition intensity between  $s_i, s_j$   
Q: state transition intensity matrix composed by  $q_{ij}$   
P(d): state transition prob. matrix with duration d  
 $\lambda$ : model parameters

- Maximize the overall likelihood from all individuals to estimate the parameters:**  
Expectation-Maximization (EM)-based method to find the instantaneous state transition rates  $q_{ij}$  for each link, which defines the transition intensity matrix Q.

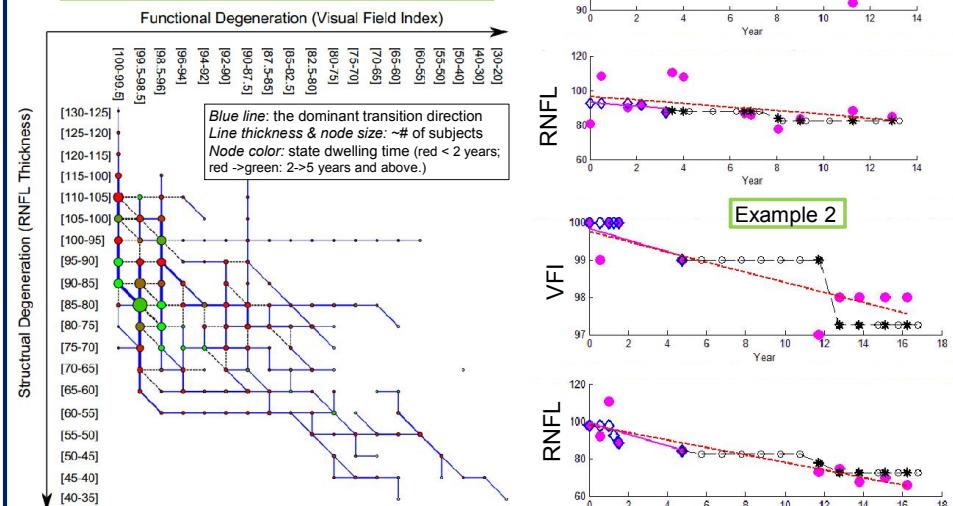
- Future state prediction:** decode the hidden disease state path from the noisy history data using Viterbi algorithm, then predict the future state given any future time  $t$  by  $j = \max_i P_{ij}(t)$ , where  $i$  denotes the current state.

## Results: 2D CT-HMM method outperforms linear regression (LR) prediction

- Dataset:** 81 glaucomatous eyes from 46 patients followed for 12.4+-4.3 years; each eye has at least 6 visits (average 8.5 +- 2.9 visits).
- Testing:** 10-fold cross validation; for a testing eye, the first 5 visits were used as history data to decode the hidden states, then used for future observation prediction.
- Performance assessment:** mean absolute error (MAE) between the predicted values and the actual measurements.
- Results:** 2D CT-HMM outperforms LR (t-test,  $p < 0.001$ )

	MAE	2D CT-HMM	Linear Regression	t-test
VFI	4.88 +- 8.44	5.95 +- 9.79		$p < 0.001$
RNFL	8.25 +- 7.89	16.34 +- 19.65		$p < 0.001$

### The trend of learned state transition intensity



## Conclusion and Future Work

- Conclusion:** the proposed state-based model resulted in more accurate estimates of future observations (VFI and RNFL thickness) compared to linear regression method.
- Future work:** incorporate covariates (age, treatment, etc.) for individual-level prediction.

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