Quiz 2, Mon 11-14-11
CS 2050, Intro Discrete Math for Computer Science

This quiz has 9 pages (including this cover page) and 4 Problems (2 pages for each problem). You have 50 minutes to work on the 4 problems.

Fill out your information on this cover page FIRST.

Last Name ........................................................................

First Name ........................................................................

email .............................................................................
Problem 1: 25 Points
Prove that, for every positive integer $n$, $5n^2 + 5n$ is divisible by 10.
State clearly which proof method you are using.
Problem 2: 25 Points
Parts (a) and (b) on this page. Part (c) on the next page.

Where \( n \) is a positive integer, define

\[
S_n = \sum_{i=1}^{n} i2^{i-1}.
\]

(a) True or False? (No explanation needed, simply circle true or false.)

\[
S_n = 1 \cdot 2^0 + 2 \cdot 2^1 + 3 \cdot 2^2 + \ldots + n2^{n-1}.
\]

(b1) \( S_1 = ? \) (No explanation needed, you may simply answer with a number.)

(b2) \( S_2 = ? \) (No explanation needed, you may simply answer with a number.)

(b3) What is the last term of \( S_n \)?
(No explanation needed, simply express the last term of \( S_n \) in terms of \( n \).)
(c) Prove that, for all positive integers $n$,

$$\sum_{i=1}^{n} i2^{i-1} = (n - 1)2^n + 1.$$
Problem 3: 25 Points

Define the function $f$ recursively over the non-negative integers as follows:

\[ f(n) = f(n-1) + 1 \quad , \quad \forall n \geq 1 \]

\[ f(0) = 15 \quad . \]

Prove that

\[ f(n) = n + 15 \quad , \quad \forall n \geq 0 \quad . \]
Problem 4: 25 Points
Part (a) on this page. Part (b) on the next page.

(a) Identify integers $p \geq 0$ and $q \geq 0$ such that:

(a1) $20 = 2p + 15q$

(a2) $21 = 2p + 15q$

(a3) $22 = 2p + 15q$

(a4) $23 = 2p + 15q$

(a5) $24 = 2p + 15q$

(a6) $25 = 2p + 15q$

(a7) $26 = 2p + 15q$

(a8) $90 = 2p + 15q$

(a9) $100 = 2p + 15q$

(a10) $101 = 2p + 15q$
(b) Prove that, for every integer \( n \geq 20 \), there exist integers \( p \geq 0 \) and \( q \geq 0 \), such that \( n = 2p + 15q \).