This problem-set on the primal-dual schema for approximation algorithms. Before you start working on this problem-set, make sure you have read and understood the following material:

(a) Vazirani’s book, chapter 15, and in particular paragraph 15.1, which is an overview of the primal-dual schema with relaxed complementary slackness conditions.

(b) Vazirani’s book, chapter 22, which is the Steiner Forest problem, and exercises 22.7, 22.8, 22.9 and 22.12 (for the exercises, you do not have to know how to solve them; you just have to know which problems they refer to, and that these problems can be solved by similar methods as the Steiner Forest problem).

(c) Vazirani’s book, chapter 24, and all the exercises in paragraph 24.5 (again, for the exercises, you do not have to know how to solve them; you just have to know which problems they refer to, and that these problems can be solved by similar methods).

Problem 1 (easy)

(b) Answer exercise 22.4, page 206, in Vazirani’s book.

(c) Look at the definition of a proper function in exercise 22.7, page 207, in Vazirani’s book. Prove that the function $f$ defined by the Steiner Forest Problem 22.1, page 197, is indeed a proper function.

Problem 2 (regular)
Answer exercise 22.10, page 209, in Vazirani’s book. Hint: For all vertices $i$ and $j$, write the requirement $r_{ij}$ between vertices $i$ and $j$ in binary (using at most $\lceil \log_2 k \rceil + 1$ bits), and call Algorithm 22.3 $\lceil \log_2 k \rceil + 1$ times as a subroutine.

Problem 3 (easy)
(a) Consider the general uncapacitated facility location problem in which the connection costs are not required to satisfy triangle inequality. Give a reduction from the set cover problem to show that approximating this problem is as hard as approximating set cover.

(b) Answer exercise 24.6, page 239, in Vazirani’s book.

Problem 4 (regular)