1 GENERAL

1.1 Shadow
The scene comprises two vertical poles on a horizontal floor. Let $T_i$ and $B_i$ be the projections on the screen of the top and bottom of pole $i$. There is a point light source $L$ somewhere. You see on the screen the shadow case by each pole $i$. It stretches from $B_i$ to $S_i$. You are given the 2D locations of points $T_1$, $B_1$, $S_1$, $T_2$, $B_2$, and $S_2$ on the screen, but no information about $L$. You draw a third vertical pole $T_3$, $B_3$ on the screen. Explain how to compute the point $S_3$ on the screen where its shadow ends. You may use a drawing to explain your construction. Would your solution still be valid if the poles were not vertical?

1.2 Reflection
Let $L$ be a point light source and $V$ be the viewpoint. There are only two triangles $(A,B,C)$ and $(D,E,F)$ in the scene. They are highly specular. Explain in details how you would compute the precise location $H$ where an observer at $V$ would see a highlight on $(D,E,F)$ where light from $L$ would reach $H$ after a first reflection on $(A,B,C)$. Explain also how you would test whether such a highlight will indeed appear on $(D,E,F)$. Then, comment on how such a solution could be used to accelerate approximate photorealistic rendering.

1.3 Silhouettes
You are given a set of triangles that form a connected water-tight manifold surface. They are represented by a table $G[]$ of vertices (each defined by its 3 coordinates) and by a table $V[]$ of triangle/vertex indices arranged in triplets (3 entries per triangle). There is no consistency for ordering the vertex indices in a triplet (i.e., no consistent triangle orientation). Provide (1) a mathematically precise definition of a consistent orientation for the triangles, (2) a description of how it is represented, and (3) an efficient algorithm for computing it. Then, provide (4) a mathematically precise definition of which edges are silhouettes from a given view-point $P$ and an (5) argument that these edges form closed loops. Finally, propose (6) a high-level algorithm explaining how to detect and trace these loops.

1.4 Stereo
A point $X$ in 3D is visible from two views. In view one, it is seen from viewpoint $V_1$ through pixel $P_1$. In view two, it is seen from viewpoint $V_2$ through pixel $P_2$. Points $P_1$, $P_2$, $V_1$ and $V_2$ are all known in the global coordinate system. Unfortunately, the lines $\text{Line}(P_1,V_1)$ and $\text{Line}(P_2,V_2)$, each defined by two interpolated points, do not quite intersect. Nevertheless, (1) provide the detailed geometric construction of the best estimation for $X$. Now, to put a bound on the uncertainty, we assume that $P_2$ and $P_1$ are each confined to lie in a small ball of radius $r$. Explicitly, we are given points $C_1$ and $C_2$ and are told that $|P_1C_1|<r$ and that $|P_2C_2|<r$. Explain (2) how you would represent and compute the precise 3D region $R$ where $X$ is guaranteed to lie. Explain the nature of the surfaces that bound $R$ and say how you would compute them and how you would compute the edges and vertices of $R$. In particular, explain (3) how you would test whether $R$ is empty.

1.5 Ray Acceleration
The naïve method of implementing a ray tracer intersects each ray against each object in the scene. Give three acceleration techniques that eliminate some of the ray/object intersection tests during rendering. Analyze the performance of each of these methods for a scene consisting of a digitized statue of one million triangles, where all of the triangles are roughly the same size. Perform a similar analysis for rendering a building that has one million polygons of widely varying sizes (e.g. from post-it-note sized to wall-sized polygons). Try to make reasonable assumptions about the scene for analysis purposes, and state these assumptions.

1.6 Bezier Curves
The family of Bezier curves begins with line segments (polynomials of degree one) and includes quadratic, cubic and higher degree curves. This question centers on the cubic Bezier curves.

a) What properties do cubic Bezier curves have that makes them particularly attractive for many applications?

b) Give pseudo-code for two distinct algorithms for drawing 2D cubic Bezier curves in the plane.

c) For each of the two algorithms in (b), give the number of arithmetic operations per segment and/or point that is drawn.
2 MODELING

2.1 Path
Consider a ball of fixed radius r in 3D. Its center is initially at point P and you wish to move it to point Q. The motion must avoid a set of zero-dimensional point obstacles V_i. Explain how you would test whether a valid path exists (we do not need to know the path, just to know whether it exists). Outline a geometrically precise algorithm first and then explain whether approximating the boundary of the ball with a triangle mesh would simplify or accelerate the test.

2.2 Fill
You are given a simply-connected polygonal curve P in 3D. It is represented by an ordered cycle of v vertices. You wish to compute a triangle mesh M that has P as its border and that has no interior vertices (i.e., its vertices are the v vertices of P). We assume that M is a manifold with boundary and that M is its only boundary. What constraints can you impose on the number t of triangles in M? Is t fixed for a given v? If so, prove it. If not, show two examples with same v and different t. We wish to compute the mesh M (as specified above) that has minimum area. Provide a naïve, but simple algorithm or doing so and explain its complexity. Then, discuss strategies/heuristics for accelerating it.

2.3 Sweep
This is a 2D problem. A planar region R is bounded by circular arcs, although it may have sharp corners. It is rotated in the plane by an angle t around a fixed point C. Explain how you would compute the precise representation of the boundary of the region S swept by R during that rotation.

2.4 Compression
Consider a connected manifold triangle mesh M represented in a Corner Table and having about 100,000 triangles. It represents the shape of a part P in a large mechanical assembly. Design an approach that would compute a compressed representation of an approximation of P that would take less than 1000 bits of storage (when compressed) and would vaguely resemble the original. Mention other approaches in lossy compression, simplification, representation conversion and comment on how effective they would be on this challenge.

3 RENDERING

3.1 Diffuse Interreflection
Your company is designing a new entrance and lobby for your building, and you have been given the task of evaluating what kind of global illumination software should be used to create images from a model. All of the surfaces in the model are diffuse. Indirect illumination (diffuse interreflections) will be important to the look of the rendered images.

a) Describe three different image synthesis algorithms that could be used to provide such images that incorporate global illumination. Do not just give the names of the approaches – give an outline of the important parts of each method.

b) Characterize the different methods in terms of their computational costs and the possible visual artifacts of each approach.

c) Give an indication of what would be necessary to incorporate specular effects for each of the three methods.

3.2 Anti-Aliasing Procedural Textures
Perlin-style procedural texture synthesis creates color variations across a surface using function composition. Like all textures, procedural textures are prone to aliasing. Say that you are writing a renderer and you wish to anti-alias the procedural textures that are described in your shading language. Describe how you would render each of the following textures so that they are anti-aliased:

a) A sum of sines and cosines that are functions of the texture position.

b) Sums of Perlin-style band-limited noise of different frequencies and amplitudes.

c) Parallel black-and-white stripes.

d) White noise.
3.3 REYES Architecture
The REYES rendering architecture is a production-quality approach to rendering scenes for motion pictures. Several choices were made in the original micropolygon based system. For each of the following decisions, discuss why they were made and whether those assumptions are still valid today:

a) All geometry is broken down into micropolygons before being rasterized.
b) Shading is performed before visibility calculations.
c) No ray tracing is used.

3.4 Distributed Ray Tracing
The Distributed Ray Tracing approach is to take a given rendering effect that requires evaluating an integral and casting many rays to evaluate such integrals. For each of the effects listed below, state what integral is being evaluated, how the rays should be distributed and how the ray values should be combined to achieve the proper effect. Pick a manner of ray distribution that will give high quality results, and explain what considerations guided your choices.

a) Anti-aliasing.
b) Glossy reflection.
c) Motion blur.
d) Depth-of-field.