CS 6260
Applied Cryptography

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Introduction, perfect (Shannon) secrecy.

- All the information is at www.cc.gatech.edu/classes/AY2005/cs6260_fall/
- Office hours: CoC 254, M 2-3 pm, Th 2-3 pm
- Please answer the following questions:
  - Your name
  - Your department/major, Ph.D., M.S. or B.S., year in the program
  - Why you are taking this class (required, curious, etc.)
  - Your plans after the graduation (industry, research, etc.)

Cryptography is very old and very new

- Crypto is an ancient discipline
  - Recall Julius Caesar, Enigma,...
- Crypto as a science (modern cryptography) has short but exciting history
  - Most of it happened in the last 30 years!
- This course will be an introduction to modern cryptography

Main goals of cryptography are

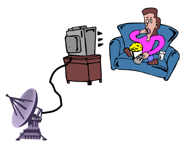
- data privacy
- data authenticity (it came from where it claims)
- data integrity (it has not been modified on the way)

in the digital world

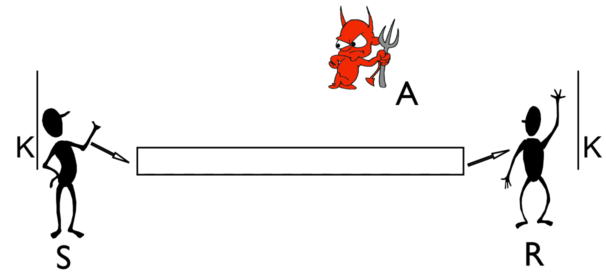
Who used some cryptography recently?

Crypto is used by most people when

- Doing on-line shopping and banking 
- Talking on a cell phone 
- Watching satellite TV and pay-per-view movies

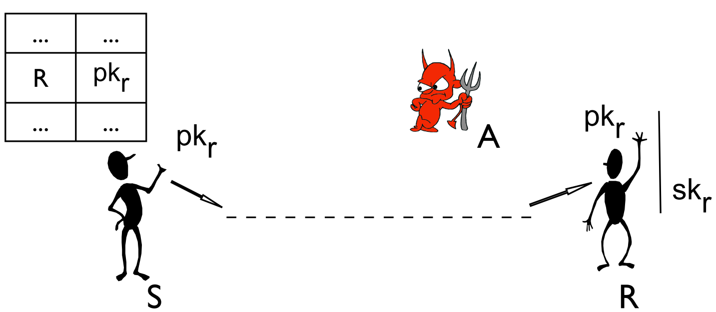


Players and settings



1. Symmetric-key setting

Players and settings



2. Asymmetric (public)-key setting

Goals and primitives

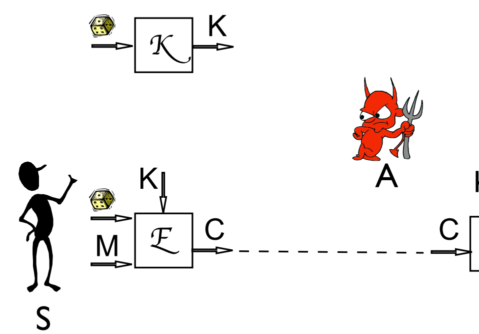
goal \ setting	symmetric-key	asymmetric-key
data privacy	symmetric (secret-key) encryption	asymmetric (public-key) encryption
data authenticity/ integrity	message authentication code (MAC)	digital signature scheme

How good is a scheme?

- "Trial-and-error" approach:
  1. Try to find an attack
  2. If an attack found then the scheme is insecure, fix the scheme, repeat step 1.
  3. If no attack found then ….?
- "Provable security" approach:
  - show that if an attack found (a scheme is insecure), then one can break some trusted assumption (e.g. factoring)
  - requires a definition of what "secure" means

Symmetric encryption schemes

- A scheme SE is specified by 3 algorithms $\mathcal{K}$,$\mathcal{E}$,$\mathcal{D}$.



MsgSp-message space
KeySp-key space
SE=($\mathcal{K}$,$\mathcal{E}$,$\mathcal{D}$) or
SE=(KeySp,$\mathcal{E}$,$\mathcal{D}$)

It is required that for every M∈MsgSp and every K∈KeySp,

$\mathcal{D}$(K, $\mathcal{E}$(K, M))=M

One Time Pad

- OneTimePad=($\mathcal{K}$,$\mathcal{E}$,$\mathcal{D}$), MsgSp=$\{0,1\}^n$:
  - $\mathcal{K}$: return a random n-bit string K (KeySp=$\{0,1\}^n$)
  - $\mathcal{E}$(K,M): C←M⊕K, return C
  - $\mathcal{D}$(K,C): M←C⊕K, return M
- Example: M=011111111011101
  K=110010011010100
  C=101101100001001
- A new key must be used to encrypt a new message

Perfect (Shannon) security

- Def 1. An encryption scheme SE=(K,E,D) is perfectly secure if for every probability distribution PD $\{0,1\}^n$→]0,1] on a MsgSp=$\{0,1\}^n$, for every ciphertext C and message M
  Pr[message is M | ciphertext is C] = PD(M)
  over the choices of K
- Def 2. An encryption scheme SE=(K,E,D) is Shannon-secure if for every ciphertext C and messages M1,M2
- Pr[E(K1,M1)=C] = Pr[E(K2,M2)=C]
  over the choices of K1,K2
- Claim. Def 1 and Def 2 are equivalent, i.e. a scheme is perfectly secure iff it is Shannon-secure.

- Th.1 OneTimePad is a Shannon-secure encryption scheme.
  - Proof. Fix any ciphertext $C \in \{0,1\}^n$. For every M $Pr[E(K,M)=C] = Pr[K=M \oplus C] = 2^{-n}$

- Th.2 [Shannon's theorem, optimality of OneTimePad] If a scheme is Shannon-secure, then the key space cannot be smaller than the message space (if $KeySp=\{0,1\}^k$ and $MsgSp=\{0,1\}^m$, then $k \geq m$ and a key must be as long as the message we want to encrypt).
  - Proof. We prove that |KeySp| cannot be smaller that |MsgSp|. Fix a ciphertext C (by picking M1,K1 and setting C=E(K1,M1)). Thus Pr[E(K,M1)=C]>0. Assume there exists M2 such that Pr[D(K,C)=M2]=0. By the correctness requirement Pr[E(K,M2)=C]=0. Therefore Pr[E(K,M1)=C]≠ Pr[E(K,M2)=C] that violates Shannon secrecy. Thus for every M2∈MsgSp there exists K∈KeySp s.t. D(K,C)=M2, and thus |KeySp|>= |MsgSp|.

- Th.3 If a scheme is perfectly secure, then the key space cannot be smaller than the message space.
  - Proof. We prove that |KeySp| cannot be smaller that |MsgSp|. Assume |KeySp|<|MsgSp|. Fix C. Let's count messages to which C can decrypt to under various keys: $S=\{M_1,..M_{|KeySp|}\}$. |S|<|MsgSp|, thus there exists M_i st Pr[message is M_i|ciphertext is C]=0 while $PD(M_i)>0$. A contradiction. Thus |KeySp|≥|MsgSp|.

- So we cannot do better than OneTimePad. But it is impractical (very fast, but we need a very long key). Is it the end? Yes, of the information-theoretic crypto. No, if we relax the security requirement and assume that adversaries are computationally bounded. We will also assume that
  - There are some "hard" problems
  - Secret keys are secret
  - All algorithms are public (Kerckhoff's principle)
- We move to the area of computational-complexity crypto, that opens a lot of possibilities.