

Homework 1

Lecturer: Sasha Boldyreva

Due: January 24, 2008

Assignment 1.01 Read the syllabus at the course's web page. Do the assigned reading.

Assignment 1.02 Indicate how much time did you spend on this homework.

Problem 1.1, 4 points. (There was a typo here.) Prove that the proposition “if it is not possible to solve the problem, then it is possible to solve it” is equivalent to “it is possible to solve the problem”.

Problem 1.2, 9 points. Write the converse and contrapositive of each statement:

- a) If the hose is 60 ft long, then the hose will reach the tomatoes.
- b) Alice goes for a walk only if Mary goes for a walk.
- c) Bob tells a story if Mike asked for it.

Problem 1.3, 12 points. Express each of these system specifications using predicates (that you define), quantifiers and logical connectives.

- a) When there is less than 30 megabytes free on the hard disk, a warning message is sent to all users.
- b) No directories in the file system can be opened and no files can be closed when system errors have been detected.
- c) The file system cannot be backed up if there is a user currently logged in.
- d) Video on demand can be delivered when there are at least 8 megabytes of memory available and the connection speed is at least 56 kilobits per second.

Problem 1.4, 4 points. Explain why an argument of the following form is not valid:
 $((p \rightarrow q) \wedge \neg p) \rightarrow \neg q$.

Problem 1.5, 5 points. Determine whether the following argument is valid. Name the rule of inference or explain the fallacy.

Theorem: If n is a real number such that $n > 1$, then $n^2 > 1$. Proof: Suppose that $n^2 > 1$. Then $n > 1$.

Problem 1.6, 18 points. Consider the following theorem: If x is an odd integer,

then $x+2$ is odd. Give (a) a direct proof, (b) a proof by contradiction and (c) a proof by contraposition of this theorem.

Problem 1.7, 4 points. Give a proof by cases of the following.
All digits have English names consisting of at most 6 letters.