

Homework 1

Lecturer: Sasha Boldyreva

Due: January 22, 2009

Assignment 1.01 Read the syllabus at the course's web page. Do the assigned reading.

Assignment 1.02 Indicate how much time did you spend on this homework.

Problem 1.1, 9 points. Write the converse and contrapositive of each statement:

- a) Alice sings if she feels like it.
- b) If Pete is taller than Mike, then Bob can see Pete behind the gate.
- c) Alex eats pancakes only if Ellen cooks them.

Problem 1.2, 4 points. Prove that the proposition “the theorem has a proof” is equivalent to “if the theorem does not have a proof then it has it” .

Problem 1.3, 12 points. Express each of these system specifications using predicates (that you define), quantifiers and logical connectives.

- a) When there is less than 30 megabytes free on the hard disk, a warning message is sent to all users.
- b) No directories in the file system can be opened and no files can be closed when system errors have been detected.
- c) The file system cannot be backed up if there is a user currently logged in.
- d) Video on demand can be delivered when there are at least 8 megabytes of memory available and the connection speed is at least 56 kilobits per second.

Problem 1.4, 5 points. Determine whether the following argument is valid. Name the rule of inference or explain the fallacy.

Theorem: If n is a real number such that $n > 1$, then $n^2 > 1$. Proof: Suppose that $n^2 > 1$. Then $n > 1$.

Problem 1.5, 4 points. Explain why the negation of “Alice and Bob are good students” is not “Alice and Bob are bad students”.

Problem 1.6, 4 points. Give a proof by cases of the following.
All months have English names consisting of at least 3 letters.

Problem 1.7, 8 points. In the questions below suppose the variables x and y represent real numbers, and $E(x) : x$ is even, $G(x) : x > 0$, $I(x) : x$ is an integer. Write the statement using these predicates and any needed quantifiers.

(a) Some real numbers are not positive.

(b) No even integers are odd.