CS 1050 B: Constructing Proofs	January 22, 2009

Homework 2

Lecturer: Sasha Boldyreva — Due: January 29, 2009

Assignment 2.01 Do the assigned reading.

Assignment 2.02 Indicate how much time did you spend on this homework.

Problem 2.1, 5 points. Use induction (the Principle of Mathematical Induction) to prove that $1 + 2^n \leq 3^n$ for all $n \geq 1$.

Problem 2.2, 5 points. Use the Principle of Mathematical Induction to prove that $1 + 4 + 7 + 10 + \ldots + (3n - 2) = \frac{n(3n-1)}{2}$ for all $n \geq 1$.

Problem 2.3, 5 points. Give a proof by contradiction of the following: "If n is an odd integer, then n^2 is odd".

Problem 2.4, 5 points. Find the error in the following proof of this "theorem":
Theorem: Every positive integer equals the next largest positive integer.
Proof: Let $P(n)$ be the predicate "n = n + 1". To show that $P(k) \rightarrow P(k+1)$, assume that $P(k)$ is true for some k, so that $k = k + 1$. Add 1 to both sides of this equation to obtain $k + 1 = k + 2$, which is $P(k+1)$. Therefore $P(k) \rightarrow P(k+1)$ is true. Hence $P(n)$ is true for all positive integers n.

Problem 2.5, 5 points. Sharing a chocolate bar. Problem 10 from Section 4.2 of Rosen's textbook.

Problem 2.6, 5 points. Prove by contraposition that for all non-negative integers x, y, if $\sqrt{xy} \neq (x + y)/2$ then $x \neq y$.

Problem 2.7, 5 points. Prove by contradiction that if an integer n^2 is a multiple of 3 (divisible by 3), then n is a multiple of 3.