CS 1050A: Constructing Proofs

Problem Set 2 : Induction and Recursion

Due Friday, Sept 22nd, after the class

Problem 1 : Rosen 4.1: 3, 4, 17

1. Let $P(n)$ be the statement that $1^2 + 2^2 + \cdots + n^2 = \frac{n(n+1)(2n+1)}{6}$ for the positive integer n.

   a) What is the statement $P(1)$?
   b) Show that $P(1)$ is true, completing the basis step of the proof.
   c) What is the inductive hypothesis?
   d) What do you need to prove in the inductive step?
   e) Complete the inductive step.

2. Use induction to prove the statement that $1^3 + 2^3 + \cdots + n^3 = (\frac{n(n+1)}{2}^2)$ for every positive integer n.

3. Use induction to prove the statement that $\sum_{j=1}^{n} j^4 = \frac{n(n+1)(2n+1)(3n^2+3n-1)}{30}$ for every positive integer n.

Problem 2 : Rosen 4.2: 4, 9

1. Let $P(n)$ be the statement that a postage of n cents can be formed using just 4-cent stamps and 7-cent stamps. The parts of this exercise outline a strong induction proof that $P(n)$ is true for $n \geq 18$.

   a) Show statement $P(18)$, $P(19)$, $P(20)$, $P(21)$ are true, completing the basis step of the proof.
   b) What is the inductive hypothesis of the proof?
   c) What do you need to prove in the inductive step?
   d) Complete the inductive step for $k \geq 21$.

2. Use strong induction to prove that $\sqrt{2}$ is irrational. [*Hint:* Let $P(n)$ be the statement that $\sqrt{2} \neq n/b$ for any positive integer b.]

Problem 3 : Rosen 4.3: 8, 45

1. Give a recursive definition of the sequence a_n, $n = 1, 2, 3, ...$ if

   a) $a_n = 4n - 2$
   b) $a_n = 1 + (-1)^n$
   c) $a_n = n(n + 1)$
   d) $a_n = n^2$

2. Use generalized induction to show that if $a_{m,n}$ is defined recursively by $a_{0,0} = 0$ and

$$a_{m,n} = \begin{cases} a_{m-1,n} + 1 & \text{if } n = 0 \text{ and } m > 0 \\ a_{m,n-1} + 1 & \text{if } n > 0 \end{cases}$$

   then $a_{m,n} = m + n$ for all $(m, n) \in \mathbf{N} \times \mathbf{N}$

Problem 4: Rosen 4.4: 15, 29, 30

1. Devise a recursive algorithm for computing the greatest common divisor of two nonnegative integers a and b with $a < b$ using the fact that $\gcd(a, b) = \gcd(a, b - a)$.

2. Devise a recursive algorithm to find the nth term of the sequence defined by $a_0 = 1$, $a_1 = 2$, and $a_n = a_{n-1} \cdot a_{n-2}$, for $n = 2, 3, 4, \cdots$.

3. Devise an iterative algorithm to find the nth term of the sequence defined in the above problem.

Puzzle

Puzzle: A hiker started climbing a mountain at 8:00 am and reached the top at 6:00 pm. She spent the night there and started climbing down (on the same trail) the next morning at 8:00 am. She reached the bottom at 6:00 pm. On both days, she hiked at uneven/varying speed and rested several times. Prove that at *some* time on both the days (like 2:13 pm on both days) she was at the same exact spot on the hiking trail. (*Hint: Superimpose the scenarios on the two days).*

Survey

Feedback

1. How did you find this homework? Too easy? Too difficult? Just right?

2. How do you like the teaching so far? Too slow? Too fast? Just right? Any suggestions?