

CS 1050B: Constructing Proofs

Problem Set 5

Due Wednesday, Oct 25th, after the class

1. Rosen 5.3: 30

Seven women and nine men are on the faculty in the mathematics department at a school

- How many ways are there to select a committee of five members of the department if at least one woman must be on the committee?
- How many ways are there to select a committee of five members of the department if at least one woman and at least one man must be on the committee?

Answer:

- There are $C(16, 5)$ ways to select a committee if there are no restriction. There are $C(9, 5)$ ways to select a committee from just the 9 men. Therefore, there are $C(16, 5) - C(9, 5) = 4242$ committees with at least one woman.
- There are $C(9, 5)$ ways to select a committee from just the 9 men and there are $C(7, 5)$ ways to select a committee from just the 7 women. These two possibilities do not overlap, since there are no ways to select a committee containing neither men or women. Therefore, there are $C(16, 5) - C(9, 5) - C(7, 5) = 4221$ committees with at least one woman and at least one man.

2. Rosen 6.1: 10

What is the probability that a five-card poker hand contains the two of diamonds and the three of spades?

Answer There is no choice about choosing the two of diamonds and the three of spades. To form the rest of the hand, we need to choose 3 cards from the 50 remaining cards.

$$\frac{C(50, 3)}{C(52, 5)} = \frac{5}{663}$$

3. Rosen 6.1: 20

What is the probability that a five-card poker hand contains a royal flush, that is, the 10, jack, queen, king, and ace of one suit?

Answer: There are 4 royal flushes, one in each suit. There fore the answer is $4/C(52, 5) = 1/649740$.

4. **Rosen 6.2: 20**

Find the smallest number of people you need to choose at random so that the probability that at least one of them has a birthday today exceeds $1/2$ (We assume 366 equally likely and independent birthdays).

Answer: If n people are chosen at random, then the probability that none of them has a birthday today is $(365/366)^n$. The question asks for the smallest n such that this quantity is less than $1/2$. We can solve the equation $(365/366)^n = 1/2$ using logarithms. We find that when $n \leq 253$, $(365/366)^n > 1/2$, but $(365/366)^{254} \approx .4991$. Therefore, the answer is 254.