CS 1050B: Constructing Proofs

Problem Set 6

Due Wednesday, Nov 3rd, after the class

1. **Rosen 6.1: 36**

   Rolling dice:

   a) What is the total number of possible outcomes of the roll of two dice? and three dice?

   b) How many ways are there to get a total of 8 when two dice are rolled. How many if three dice are rolled?

   c) Which is more likely: rolling a total of 8 when two dice are rolled or rolling a total of 8 when three dice are rolled?

   Answer:

   a) 2 dice: $6^2 = 36$
   3 dice: $6^3 = 216$

   b) 2 dice: $C(5, 1) = 5$. There are only 5 ways since rolling out a one on one of the dice will make it impossible to roll a total of 8.
   3 dice: $C(6, 1) + C(5, 1) + C(4, 1) + C(3, 1) + C(2, 1) + C(1, 1) = 21$. If the first die turn out to be 1, then the rest of the 2 dice have to sum up to be 7 for a total roll of 8. There are $C(6, 1)$ possible ways to roll a total of 7 for two dice. It follows similarly for the cases where the first die is 2, 3, 4, 5, and 6.

   c) 2 dice: $5/36 = 0.139$
   3 dice: $21/216 = 0.097$
   Therefore, rolling a total of 8 is more likely when using two dice than when using three.

2. **Rosen 6.3: 14**
   Suppose that E, F_1, F_2, F_3 are events from a sample space S and that F_1, F_2, and F_3 are mutually disjoint and their union is S. Find $p(F_2|E)$ if $p(E|F_1) = 2/7$, $p(E|F_2) = 3/8$, $p(E|F_3) = 1/2$, $p(F_1) = 1/6$, $p(F_2) = 1/2$, and $p(F_3) = 1/3$.

   Answer By Generalized Bayes' Theorem,

$$p(F_2|E) = \frac{p(E|F_2)p(F_2)}{p(E|F_1)p(F_1) + p(E|F_2)p(F_2) + p(E|F_3)p(F_3)} = \frac{7}{15}$$

3. **Rosen 6.4: 26**
   Provide an example that shows the variance of the sum of two random variables is not necessarily equal to the sum of their variances when the random variables are not independent. (Please show the variance of both random variables, respectively and the variance of the sum.)

   Answer: Let's take X to be the number of heads when a coin is flipped and Y to be be the number of tails. Then $V(X) = V(Y) = 1/4$; but $X+Y = 1$, so $V(X+Y) = 0 \neq V(X)+V(Y)$.

4. **Rosen 6.4: 30**
   Use Chebyshev's Inequality to find an upper bound on the probability that the number of tails that come up when a biased coin (with probability of heads equal to 0.6) is tossed n times deviates from the mean by more than $\sqrt{n}$.

   Answer: $V(X) = np(1-p) = n(0.6)(0.4) = 0.24n$. By Chebyshev's Inequality, we have $p(|X(s) - E(X)| \geq \sqrt{n}) \leq \frac{V(X)}{r^2} = 0.24$.