CS 1050B: Constructing Proofs

Problem Set 7

Due Friday, Nov 20th, after the class

1. **Rosen 3.4: 16**

   Evaluate these quantities.

   a) -17 mod 2
   b) 144 mod 7
   c) -101 mod 13
   d) 199 mod 19

   Answer:

   a) Since $-17 = 2 \cdot (-9) + 1$, the remainder is 1. That is, $-17 \mod 2 = 1$. Note that we do not write $-17 = 2 \cdot (-8) - 1$ since $a \mod b$ is always nonegative, so $-17 \mod 2 \neq 1$.
   b) 4
   c) 3
   d) 9

2. **Rosen 3.4: 22**
   Show that if a, b, c, and m are integers such that $m \geq 2$, $c > 0$, and $a \equiv b \pmod{m}$, where a and b are integers, then $a \equiv b \pmod{m}$.

   Answer: From $a \equiv b \pmod{m}$ we know that $b = a + sm$ for some integer s. Multiplying by c we have $bc = ac + s(mc)$, which means that $ac \equiv bc \pmod{mc}$.

3. **Rosen 3.4: 24**
   Prove that if n is an odd positive integer, then $n^2 \equiv 1 \pmod{8}$.

   Answer: Write $n = 2k + 1$ for some integer k. Then $n^2 = (2k + 1)^2 = 4k^2 + 4k + 1 = 4k(k + 1) + 1$. Since either k or $k + 1$ is even, $4k(k + 1)$ is a m8ultiple of 8. therefore $n^2 - 1$ is a multiple of 8, so $n^2 \equiv 1 \pmod{8}$.

4. **Rosen 3.5: 20**

   What are the greatest common divisors of these pairs of integers

   a) $2^2 \cdot 3^3 \cdot 5^5$, $2^5 \cdot 3^3 \cdot 5^2$

b) $2 \cdot 3 \cdot 5 \cdot 7 \cdot 11 \cdot 13$, $2^{11} \cdot 3^9 \cdot 11 \cdot 17^{14}$

c) 17 , 17^{17}

d) $2^2 \cdot 7$, $5^3 \cdot 13$

e) 0, 5

f) $2 \cdot 3 \cdot 5 \cdot 7$, $2 \cdot 3 \cdot 5 \cdot 7$

Answer:

a) $2^2 \cdot 3^3 \cdot 5^2$

b) $2 \cdot 3 \cdot 11$

c) 17

d) 1

e) 5

f) $2 \cdot 3 \cdot 5 \cdot 7$

5. **Rosen 3.5: 26**

If the product of two integers is $2^7 3^8 5^2 7^{11}$ and their greatest common divisor is $2^3 3^4 5$, what is their least common multiple?

Answer: The product of the greatest common divisor and the least common multiple of two numbers is the product of the two numbers. Therefore the answer is $(2^7 3^8 5^2 7^{11})/(2^3 3^4 5) = 2^4 3^4 5 \cdot 7^{11}$.