The Design of Teletext Broadcast Cycles

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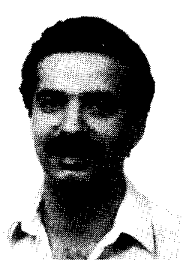
Teletext is a one-way picture information system where pages of information are broadcast to all users in a continuous manner. System response time is an important consideration in designing teletext systems. Factors contributing to system response time include transmission speeds, amount of processing required at user terminals, and efficiency of picture encoding procedures. As important is the design of the teletext broadcast cycle, i.e., the order of pages to be broadcast in a cyclic manner. In this paper, we first derive a formula for the mean response time of a given cycle and a lower bound for the mean response time for any cycle. Next we present a design procedure that yields a cycle with mean response time close to the theoretical lower bound. The use of the results of this paper is demonstrated through a numerical example.

Keywords: Teletext Systems, Broadcast Cycle, Broadcast Delivery, Response Time, Performance Analysis, Information Systems, Computer-Communication Systems.



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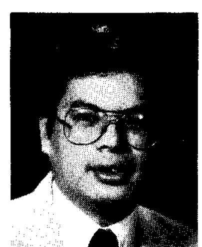
1. Introduction

Teletext is a one-way picture information system where pages of information are broadcast to all users in a continuous manner [1,2,3]. The configuration of a typical teletext system is shown in Fig. 1. In this system, a service computer is connected to the user terminals by a broadcast network. An example of such a network is one-way broadcast using cable television technology [4].

When a page of information is requested by a user, the user terminal examines the incoming stream of data until the desired information page is detected. This page is then captured, stored and displayed. In this system, a request does not propagate beyond the user terminal. Hence, teletext systems are sometimes described as *pseudo-interactive*. Specifics of existing systems may differ, however, they all share the pseudo-interactive and information-broadcasting features.

In a typical teletext system, the service computer maintains a database, and information pages in the database are updated regularly by service providers. These updates are issued locally or remotely by service-provider terminals.

The quality of service provided to a teletext subscriber depends on various aspects of the system. These include: picture quality, information availability, and reliability of transmission. In this paper we focus on yet another aspect of a teletext system that contributes to the perceived quality of service, namely, the system response time.



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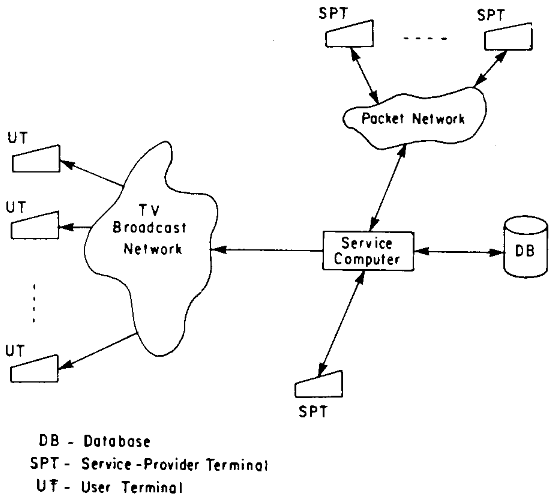

Fig. 1. A typical teletext system.

Several factors of system design can affect the system response time. Among them are: transmission medium speed, amount of processing required at the user terminal, and efficiency of picture encoding procedures. However, these factors are of a "static" nature, since they depend, to a large extent, on decisions made early in the design process.

A more dynamic component of a teletext system which has an impact on response time is the design of the broadcast cycle, i.e., the order of pages to be broadcast in a cyclic manner. Gecsei [4] has investigated the response time of a teletext system where the same probability distribution is used for page selection at each broadcast instance. Conditions under which minimum mean response time is achieved were derived. These results are summarized in Section 2.

It is difficult to talk about a broadcast cycle when a probability distribution is used for page selection at each broadcast instance. One can only achieve the objective that the frequencies of broadcasting the various pages are in agreement with some pre-specified distribution.

In this paper, we consider the case where the order of pages in a broadcast cycle is specified, and the cycle length is finite. We investigate this case by first providing a formula for the mean response time of a given broadcast cycle in Section 3, and then deriving a lower bound for the mean response time in Section 4. These results are then used in Section 5 to develop a heuristic design procedure for determining a cycle with near-optimal mean response time. Finally, some numerical examples are presented in Section 6.

In a related work, Itai and Rosberg [5] have investigated the problem of channel bandwidth allocation to users in a multiple access environment. Time on the channel is slotted, and the design of cycles (i.e., the order in which slots are allocated to the various users) to achieve maximum throughput is considered. In addition, it is shown in [5] that cyclic assignment of slots is optimal from the point of view of maximizing throughput. In [6], a similar result regarding the response time optimality of cyclic transmission in teletext systems is derived.

2. Probabilistic page broadcasting

Let N be the total number of pages, and assume that each user has probability q_i of requesting page i, $i = 1, \ldots, N$. We thus have $\sum_{i=1}^{N} q_i = 1$. We assume that user requests are generated according to a Poisson process, and that all pages require the same transmission time. This latter assumption is true in certain teletext implementations, e.g., Prestel [3]. We will use the page transmission time as our time unit.

At each broadcast instance, let p_i be the probability that page i is selected for transmission to all users. Also let S_i be the mean response time of a request for page i, i.e., the time from when a request for page i is made to when the next broadcast of page i is complete. S_i is given by

$$S_i = \frac{1}{p_i} + \frac{1}{2}. \tag{1}$$

Equation (1) is based on the fact that a random request will on average wait for half the transmission in progress plus the mean number of page transmissions until the end of a page i transmission.

If S is the mean response time over all requests then we have

$$S = \sum_{i=1}^{N} \frac{q_i}{p_i} + \frac{1}{2}. \tag{2}$$

S is minimized when $p_i/p_j = \sqrt{q_i}/\sqrt{q_j}$, for all i, j

[4]. Since $\Sigma_{i=1}^{N} p_i = 1$, we have

$$\min S = \left(\sum_{i=1}^{N} \sqrt{q_i} \right)^2 + \frac{1}{2}. \tag{3}$$

As Gecsei remarked, one drawback to generating a random sequence of pages for broadcast, as described above, is that the response time may become unbounded. This can be avoided by utilizing a fixed broadcast cycle. The balance of this paper is devoted to the issue of designing a broadcast cycle with good response time characteristics.

3. Cyclic page broadcasting

In this section we discuss various aspects of the teletext broadcast cycle. We use the same assumptions as those made in the previous section about user request probabilities and page transmission time.

3.1. The broadcast cycle

As mentioned previously, a broadcast cycle is the order of pages to be broadcast in a cyclic and continuous manner. A simple example of a broadcast cycle is $\{1, 2, \ldots, N\}$. In this example each page appears once and the mean response time is $N/2 + 1$, for any page i, $i = 1, \ldots, N$. In general, the broadcast cycle under consideration has the following two properties:

(i) The cycle length, L, is finite.

(ii) Each page appears at least once in the cycle; this implies that $L \geq N$.

For a given broadcast cycle, the following useful parameters are identified:

(i) Page frequencies, k_i $(i = 1, \ldots, N)$—the number of appearances of page i in the broadcast cycle ($\Sigma_{i=1}^{N} k_i = L$).

(ii) Inter-appearance gaps, T_r^i $(r = 1, \ldots, k_i$, and $i = 1, \ldots, N)$—the number of pages between the beginning of the rth and the $(r+1)$st appearances of page i in the cycle, $r = 1, \ldots, k_i - 1$ (see Fig. 2). $T_{k_i}^i$ is the number of pages between the beginning of the k_ith appearance of page i in the cycle and the beginning of its 1st appearance in the next cycle. Note that $\Sigma_{r=1}^{k_i} T_r^i = L$.

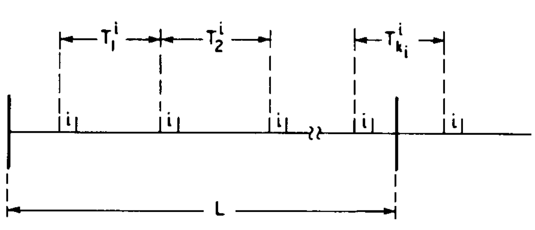

Fig. 2. A teletext broadcast cycle.

3.2. Analysis of mean response time

Here we analyze the performance of a given broadcast cycle. Since the parameters T_r^i are known for $r = 1, \ldots, k_i$, and $i = 1, \ldots, N$, the following observations can be made:

(i) The probability that a request for page i (made at a random point in time) will fall into the gap represented by T_r^i is T_r^i / L (see Fig. 2).

(ii) Given that a request falls into the above mentioned gap, the expected time until the start of the next full page i broadcast is $T_r^i/2$.

The above observations imply that the mean response time for page i is given by:

$$S_i = \sum_{r=1}^{k_i} \frac{T_r^i}{L} \left(\frac{T_r^i}{2} + 1 \right)$$

or

$$S_i = \frac{1}{2L} \sum_{r=1}^{k_i} \left(T_r^i \right)^2 + 1. \tag{4}$$

Note that S_i depends only on the inter-appearance gaps of page i in the cycle. Finally, the mean response time over all requests is given by:

$$S = \sum_{i=1}^{N} q_i S_i = \frac{1}{2L} \sum_{i=1}^{N} q_i \sum_{r=1}^{k_i} \left(T_r^i \right)^2 + 1. \tag{5}$$

4. A lower bound for the mean response time

In this section, we study properties of an ideal cycle and obtain a lower bound for the mean response time. We first consider the case where the cycle length L and the page frequencies k_i, $i = 1, \ldots, N$ are given. The minimization of S_i can be achieved by solving the following problem (see (4)):

$$\min \sum_{r=1}^{k_i} \left(T_r^i \right)^2 \tag{6}$$

subject to

$$\sum_{r=1}^{k_i} T_r^i = L, \quad T_r^i > 0 \text{ and } T_r^i \text{ integer.}$$

To solve the above problem, we first relax the integer restriction. The function in (6) is minimized when all the T_r^i's are equal, i.e.

$$T_r^{i*} = \frac{L}{k_i}, \quad r = 1, \ldots, k_i. \tag{7}$$

Since the function in (6) is convex, re-introducing the integer restriction would result in choosing the nearest integer point to (7) above. The optimal solution then becomes:

$$T_r^{i*} = \begin{cases} \left\lceil \dfrac{L}{k_i} \right\rceil, & r = 1, \ldots, L - k_i \left\lfloor \dfrac{L}{k_i} \right\rfloor, \\ \left\lfloor \dfrac{L}{k_i} \right\rfloor, & r = L - k_i \left\lfloor \dfrac{L}{k_i} \right\rfloor + 1, \ldots, k_i, \end{cases} \tag{8}$$

where $\lceil X \rceil$ represents the smallest integer $\geq X$, and $\lfloor X \rfloor$ denotes the largest integer $\leq X$.

From the above discussion, we can see that for a given L and k_i, the minimum response time for page i is given by

$$\min S_i = \frac{1}{2L} \sum_{r=1}^{k_i} \left(T_r^{i*} \right)^2 + 1. \tag{9}$$

We now derive a lower bound for the mean response time. This bound will be independent of k_i ($i = 1, \ldots, N$) and L. We note from the development leading to (9) that

$$\begin{aligned} S &\geq \frac{1}{2L} \sum_{i=1}^{N} q_i \sum_{r=1}^{k_i} \left(T_r^{i*} \right)^2 + 1 \\ &\geq \frac{1}{2L} \sum_{i=1}^{N} q_i \sum_{r=1}^{k_i} \left(\frac{L}{k_i} \right)^2 + 1 \\ &= \frac{L}{2} \sum_{i=1}^{N} \frac{q_i}{k_i} + 1. \end{aligned} \tag{10}$$

The right-hand side of (10) is smallest when we choose the k_i's such that [4]:

$$\frac{k_i}{k_j} = \frac{\sqrt{q_i}}{\sqrt{q_j}} \tag{11}$$

Substituting (11) into (10) and noting that $\sum_{i=1}^{N} k_i = L$, we get

$$S \geq \frac{1}{2} \left(\sum_{i=1}^{N} \sqrt{q_i} \right)^2 + 1. \tag{12}$$

The lower bound in (12) is useful for evaluating the relative performance of any particular cycle.

5. Cycle design procedure

Our discussion of cycle design is based on the following considerations:

(i) The cycle length L must not exceed a prespecified value L^*. The maximum cycle length is determined by the amount of memory used to store the broadcast cycle.

(ii) The response time of any given request must not exceed $T^* + 1$. This implies that the inter-appearance gaps T_r^i must not exceed T^* for all r and i.

(iii) The mean response time over all requests is te be minimized.

Note that L^* and T^* must not be smaller than N, the total number of pages.

5.1. Optimization problem

For a given frequency vector $\boldsymbol{k} = (k_1, \ldots, k_N)$ where $\sum_{i=1}^{N} k_i = L$, the number of possible broadcast cycles is

$$J(\boldsymbol{k}) = \frac{L!}{k_1! \cdots k_N!}.$$

Let $S(\boldsymbol{k}, j)$ and $T_r^i(\boldsymbol{k}, j)$ ($r = 1, \ldots, k_i$; $i = 1, \ldots, N$) be the mean response time and the inter-appearance gaps respectively, when the jth possible cycle is used, $j = 1, \ldots, J(\boldsymbol{k})$. The optimal broadcast cycle is the solution to the following problem:

$$\min_{\boldsymbol{k}, j} S(\boldsymbol{k}, j)$$

subject to

$$k_i \geq 1, \quad i = 1, \ldots, N,$$

$$\sum_{i=1}^{N} k_i = L \leq L^*,$$

$$\max_{r,i} T_r^i(\boldsymbol{k}, j) \leq T^*.$$

The above problem is extremely complex and an exact solution is not possible in general. In what follows we present a heuristic technique that can be used to obtain a near-optimal solution.

5.2. Heuristic solution procedure

We note from the development leading to the lower bound in (12) that the mean response time is minimized if (11) and (7) are satisfied. For our heuristic, we first develop a method to obtain the page frequencies, k_i, $i = 1, \ldots, N$, such that the ratio k_i/k_j is as close to $\sqrt{q_i}/\sqrt{q_j}$ as possible and $\sum_{i=1}^{N} k_i = L$ for any given L.

Without loss of generality, we assume that the page indices are such that $q_1 \geq q_2 \geq \cdots \geq q_N$. Since $\sum_{i=1}^{N} k_i = L$, a good initial guess for k_1 is

$$k_i = \left\lfloor L \frac{\sqrt{q_1}}{\sum_{i=1}^{N} \sqrt{q_i}} \right\rfloor. \tag{13}$$

We then choose

$$k_i = \max\left(1, \left\lfloor k_1 \frac{\sqrt{q_i}}{\sqrt{q_1}} + 0.5 \right\rfloor\right), \quad i = 2, \ldots, N. \tag{14}$$

The maximum function is used to insure that $k_i \geq 1$ for all i.

The above steps may lead to $\sum_{i=1}^{N} k_i \neq L$. To obtain a cycle of length L, the following method is used:

(i) If $\sum_{i=1}^{N} k_i < L$, increase the values of k_i, $i = 1, \ldots, L - \sum_{j=1}^{N} k_j$ by 1, so that the cycle length is equal to L.

(ii) If $\sum_{i=1}^{N} k_i > L$, repeatedly decrement k_1 by 1 and re-evaluate k_i $(i = 2, \ldots, N)$ using (14) until $\sum_{i=1}^{N} k_i \leq L$; apply (i) above if the resulting k_i's are such that $\sum_{i=1}^{N} k_i < L$.

After determining the frequency vector $\boldsymbol{k}$, we next consider the problem of selecting a cycle from among the $J(\boldsymbol{k})$ possibilities, such that the mean response time is minimized. Our approach is to construct one such cycle by assigning pages to cycle positions such that the inter-appearance gaps are as close as possible to those characterized by (7). It would be ideal if a cycle can be constructed such that the integer solution for the inter-appearance gaps in (8) is satisfied for all i. However, this is not always possible, because the same position in the cycle may be needed by more than one page. The task of constructing a good cycle for given k_i's can be accomplished by using the following procedure, where the cycle positions are numbered starting from zero.

Step 1. Use (8) to obtain T_r^{i*} for $r = 1, \ldots, k_i$, and $i = 1, \ldots, N$.

Step 2. Assign page 1 to positions: 0, T_1^{i*}, $T_1^{1*} + T_2^{1*}, \ldots, \sum_{r=1}^{k_1-1} T_r^{1*}$.

Step 3. For pages $i = 2$ to $N - 1$

Step 3a. For $j = 0$ to $L - 1$
Obtain d_j, the number of already assigned positions from the list:

$$\left\{(j) \bmod L, \left(j + T_1^{i*}\right) \bmod L, \ldots, \left(j + \sum_{r=1}^{k_i-1} T_r^{i*}\right) \bmod L\right\}$$

Step 3b. Choose j^* such that $d_{j^*} = \min_j \{d_j\}$.

Step 3c. For each of the positions:

$$(j^*) \bmod L, \left(j^* + T_1^{i*}\right) \bmod L, \ldots, \left(j^* + \sum_{r=1}^{k_i-1} T_r^{i*}\right) \bmod L,$$

if the position is free then assign page i to it, else assign page i to the *closest* free position.

Step 4. Assign page N to the remaining free positions in the cycle.

By assigning the popular pages first, the above cycle construction procedure attempts to reduce the major contributions to the overall mean response time. The less popular pages receive less favourable treatment, and ultimately the least popular page gets assigned to the remaining slots, which may not be a good assignment for that page.

Since the cycle length L can range from N to L^*, a complete cycle design process consists of the application of the above procedure, i.e., determination of the k_i's and cycle construction, for each L in that range. For each cycle constructed, the mean response time is evaluated. Note that Step 3c above may lead to a cycle with $T_r^i > T^*$ for some i and r. We must, therefore, select the cycle where the mean response time is lowest and $\max_{r,i}\{T_r^i\} \leq T^*$. Observe that we are guaranteed to find at least one cycle satisfying this condition (e.g., when $L = T^*$).

5.3. Computational requirement

To analyze the complexity of the cycle design procedure outlined in Section 5.2, we first observe that in constructing a near-optimal cycle for a given cycle length L, most of the time is spent in the execution of Step 3. Furthermore, we note that, for a given L, cycle construction is the most time consuming phase of the design procedure.

For a given i, Step 3a requires $k_i L$ operations. Since j^* can be determined while the d_j's are obtained in Step 3a, we do not consider the computational requirements for Step 3b. The operation count for Step 3c is not known in general. However, each placement of page i requires at most L operations. Hence, for a total of k_i placements, at most $k_i L$ operations are needed.

Let $C(L)$ be the operation count for Step 3 when the cycle length is L; then we have

$$C(L) \leq \sum_{i=2}^{N-1} 2k_i L < 2L^2.$$

For a complete cycle design L will range from N to L^*. Thus, a measure of the computational requirement for the cycle design heuristic will be given by:

$$C = \sum_{L=N}^{L^*} C(L) < \sum_{L=N}^{L^*} 2L^2.$$

If $M = L^* - N$, we have

$$C = \begin{cases} O(M^3) & \text{if } M \geq N \text{ (or } L^* \geq 2N) \\ O(N^2 M) & \text{if } M \leq N \text{ (or } L^* \leq 2N). \end{cases}$$

6. Numerical examples and discussion

Consider a teletext system with a 100 pages. The broadcast cycle length is restricted to be not more than 1000, and the maximum acceptable response time is 301. The request probabilities for the pages are assumed to follow Zipf's law [7]. That is, if the pages are ordered such that $q_1 \geq q_2 \geq \cdots \geq q_N$ then $q_i = c/i$ where c is a normalizing constant. This particular distribution has been shown to closely approximate real user behaviour in teletext systems [4].

We first note that if the probabilistic scheme of Section 2 were used, the optimal mean response time in (3) would be $S = 67.1$. Using a cycle of the form $\{1, 2, \ldots, 100\}$ the overall mean response time would be $S = 51.0$. On the other hand, the lower bound of the overall mean response time in (12) is $S \geq 34.3$. Note that this lower bound is approximately half the optimal mean response time for the probabilistic scheme. One can achieve significant improvement over the probabilistic scheme if a cycle with mean response time close to the lower bound can be designed.

In Fig. 3, we plot the overall mean response time of the cycles constructed for $100 \leq L \leq 1000$. The fact that the mean response time is non-monotonic in L can be explained by considering a simpler case where $q_i = 1/N$ for all i. Clearly, the cycle $\{1, 2, \ldots, N\}$ is optimal in this instance with a mean response time equal to the lower bound in (12). This lower bound is also reached for $L = jN$, $j = 1, 2, \ldots$. Other values of L will yield sub-optimal cycles. The mean response time will therefore be a non-monotonic function of L.

In Fig. 4, we show how the maximum inter-appearance gap varies as a function of cycle length. Note that only a fraction of cycles satisfy the condition that the maximum inter-appearance gap is not longer that 300. It is from this set of cycles that we make our choice.

In the range $100 \leq L \leq 1000$ the best cycle according to our criteria is the one with cycle length $L = 743$. The overall mean response time for that cycle is $S = 34.4$, which is only 0.29% higher than the lower bound. The maximum inter-appearance gap is 223.

From the view point of computational complexity, it is desirable to consider as small a range for L as possible. The results in Fig. 3 indicate that a cycle length L' can be selected such that $L' < L^*$, and the mean response time of the best cycle in the range $N \leq L \leq L'$ is close to the lower bound. This leads us to consider the following stopping rule:

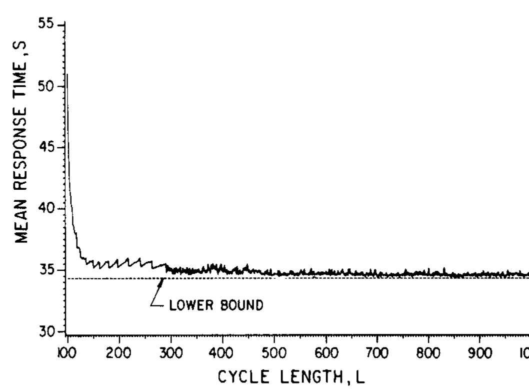

Fig. 3. Overall mean response time VS cycle length.

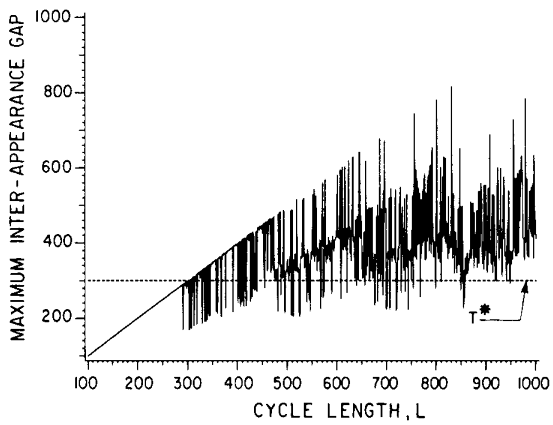

Fig. 4. Maximum inter-appearance gap VS cycle length.

"For some pre-specified $\alpha < 1$ (e.g., $\alpha = 0.98$) apply the procedure in Section 5.2 starting with $L = N$ until a cycle is obtained where the ratio of the lower bound in (12) to the mean response time of that cycle is greater than or equal to α."

If this stopping rule were used in the above example with $\alpha = 0.98$, we would obtain a cycle with length $L = 291$, overall mean response time $S = 34.8$, and maximum inter-appearance gap of 168. Such a cycle is perfectly acceptable since its mean response time is only 1.16% higher than that for $L = 743$.

Finally, we consider the cycle obtained for $L = 743$. In Table 1 we show, for selected pages, the

Table 1
Page frequencies and inter-appearance gaps of selected cycle

i	k_i	T_r^i, $r = 1, \ldots, k_i$
1	41	19 19 19 19 19 18 18 18 18 18 18 18 18 18 18 18 18 18 18 18 18 18 18 18 18 18 18 18 18 18 18 18 18 18 18 18 18 18 18 18 18
10	13	57 57 57 57 57 57 58 58 57 57 57 57 57
20	9	82 82 82 82 83 83 83 83 83
30	7	107 106 107 105 106 106 106
40	6	123 125 124 124 124 123
50	6	122 124 124 123 124 126
60	5	149 148 146 151 149
70	5	149 140 161 145 148
80	4	185 186 190 182
90	4	169 203 178 193
100	4	168 213 195 167

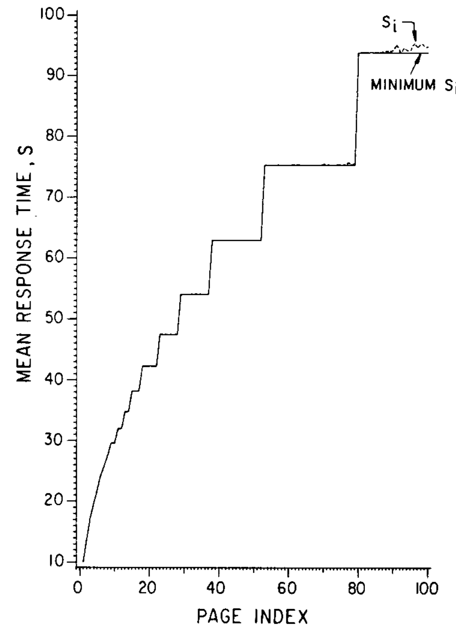

Fig. 5. Mean response time VS page index for selected cycle.

values obtained for k_i and for T_r^i. Note that the deviation of the inter-appearance gaps from the optimal values in (8) increases as the page index increases.

Figure 5 contains a graphical representation of the mean response times S_i, as well as the minimum S_i for a given k_i (see (9)), for $L = 743$. Notice that the cycle construction procedure has the effect of reducing the response time for the most popular pages at the expense of the response time for the less popular ones, thus reducing the overall mean response time. The "staircase" pattern of this graph is due to the fact that the procedure tends to group pages with close request probabilities.

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References

[1] J.R. Storey, A. Vincent, R. Fitzgerald, A description of the broadcast telidon system, IEEE Trans. on Cons. Electronics 26 (3) (1980).

[2] E. Chang, Sending pages, in: D. Godfrey, E. Chang (eds), The Telidon Book (Press Porcepic Ltd., Toronto, 1981).

[3] P. Mothersole, Teletext and viewdata: New information systems using the domestic television receiver, Proc. IEE 126 (12) (1979).

[4] J. Gecsei, The Architecture of Videotex Systems (Prentice-Hall, Englewood Cliffs, NJ, 1983).

[5] A. Itai and Z. Rosberg, A golden ratio control policy for a multiple-access channel, IEEE Trans. on Automatic Control AC-29 (8) (1984).

[6] M.H. Ammar, Performance evaluation of information systems using broadcast delivery, Ph.D. Thesis, Department of Electrical Engineering, University of Waterloo, Waterloo, Ontario, Canada, 1985.

[7] G.K. Zipf, Human Behaviour and the Principle of Least Effort (Addison-Wesley, Reading, MA, 1949).