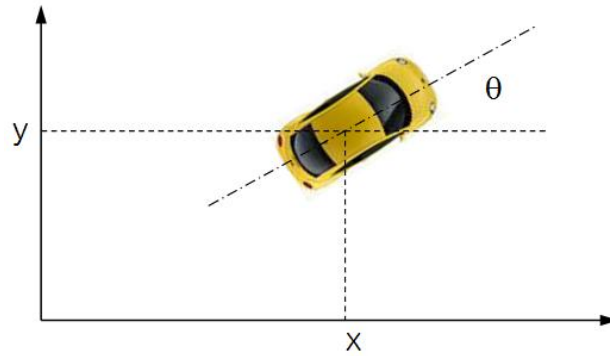


[MATH2605] Project 1

A nonlinear ODE used to move a robot car.

A background

Consider a car. Let its position be (x, y) , and let its orientation angle be θ .



We define the state variables x_1, x_2 , and x_3 as

$$\begin{aligned} x_1 &= x \cos(\theta) + y \sin(\theta) \\ x_2 &= r \\ x_3 &= x \sin(\theta) - y \cos(\theta) \end{aligned} \tag{1}$$

Since the car can move forward only, the constraint is $\dot{y}/\dot{x} = \tan(\theta)$. The derivatives of (1) can be written as

$$\begin{aligned} \dot{x}_1 &= v - x_3 \omega \\ \dot{x}_2 &= \omega \\ \dot{x}_3 &= x_1 \omega \end{aligned} \tag{2}$$

where v is the forward/backward velocity (gas pedal), and ω is the angular velocity (steering wheel). We can now choose v and ω to move the car. The derivation of (2) is not important for this project.

ODE#1

Let $s = x_3 - x_1 x_2 / 2$ and $r = \sqrt{x_1^2 + x_2^2}$. We first choose v and ω as

$$\begin{aligned} v &= -k_1 x_1 + \mu_1 \frac{s}{r^2} x_2 \\ \omega &= -k_1 x_2 - \mu_1 \frac{s}{r^2} x_1 \end{aligned} \tag{3}$$

where k_1 and μ_1 are some fixed constants. I chose k_1 and μ_1 by trial and error, and provided them in the source code. By plugging this into (2), we obtain an ODE

$$\begin{aligned} \dot{x}_1 &= -k_1 x_1 + \mu_1 \frac{s}{r^2} x_2 - x_3 \left(-k_1 x_2 - \mu_1 \frac{s}{r^2} x_1 \right) \\ \dot{x}_2 &= -k_1 x_2 - \mu_1 \frac{s}{r^2} x_1 \\ \dot{x}_3 &= x_1 \left(-k_1 x_2 - \mu_1 \frac{s}{r^2} x_1 \right) \end{aligned} \quad (4)$$

This ODE can be used to move the car. Implement a forward-Euler solver by completing the missing five lines in the provided processing file “robot1.pde”. Once done, you will see a car showing a parking maneuver. Post the applet in a web page and email me the link to it.

ODE#2

Let $s = x_3 - x_1 x_2 / 2$ and $r = \sqrt{x_1^2 + x_2^2}$. We now choose velocities in a different way.

$$\begin{aligned} v &= -k_2 \frac{x_1}{\sqrt{r^2 + 1}} + \mu_2 \text{sat}_2(s, r) \\ \omega &= -k_2 \frac{x_2}{\sqrt{r^2 + 1}} - \mu_2 \text{sat}_1(s, r) \end{aligned} \quad (5)$$

where the $\text{sat}_{1,2}$ are defined by

$$\begin{aligned} \text{sat}_i(s, r) &= \begin{cases} \text{sat}\left(\frac{s}{r}\right) \frac{x_i}{r} & , \text{ if } r \geq 10^{-10} \\ \text{sgn}(s) & , \text{ if } r < 10^{-10} \end{cases} \\ \text{sat}(x) &= \begin{cases} x & , \text{ if } |x| \leq 1 \\ 1 & , \text{ if } x > 1 \\ -1 & , \text{ if } x < -1 \end{cases} \\ \text{sgn}(x) &= \begin{cases} 0 & , \text{ if } x = 0 \\ 1 & , \text{ if } x > 0 \\ -1 & , \text{ if } x < 0 \end{cases} \end{aligned} \quad (6)$$

Copy the provided “Robot1.pde” file as “Robot2.pde” (of course in a folder named “Robot2”). Implement the forward Euler solver of (5) on this copied file. You may see a car motion with less oscillation.

Submission

Create an web page that contains “Robot1.pde” and “Robot2.pde” as applets, and then email the link to the web page. If you cannot make a web page, please email these files directly to me, but the grading may be delayed. The due date is Apr. 2nd, Monday.

Extra credit

You can work on anything you want. The due date for the extra credit part is Apr. 30th. Possible extra credit works are

- Make robot move to anywhere on the plane in any orientation. Some user interface will be a plus: You may click and draw a direction to specify the destination and orientation.
- Compare the two ODE by showing two cars.
- Visualize path.
- Any user interface.