[MATH2605] Exam 1

Feb. 12. 2007

Problem 1

Consider a vector $\mathbf{x} = \begin{pmatrix} x \\ y \\ z \end{pmatrix}$. Provide an algorithm to compute a vector orthogonal to $\mathbf{x}$. The subroutine should work for all the following vectors.

$$\mathbf{x} = \begin{pmatrix} 0 \\ 1 \\ 2 \end{pmatrix} \quad , \quad \mathbf{x} = \begin{pmatrix} 2 \\ 0 \\ 1 \end{pmatrix} \quad , \quad \mathbf{x} = \begin{pmatrix} 1 \\ 2 \\ 0 \end{pmatrix} \quad , \quad \mathbf{x} = \begin{pmatrix} 2 \\ 0 \\ 0 \end{pmatrix} \quad , \quad \mathbf{x} = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} \quad , \quad \mathbf{x} = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} \qquad (1)$$

Problem 2

Let $\mathbf{x}_0 = \begin{pmatrix} 1 \\ -1 \\ 2 \end{pmatrix}$, $\mathbf{x}_1 = \begin{pmatrix} 3 \\ 1 \\ 3 \end{pmatrix}$, $\mathbf{p} = \begin{pmatrix} 2 \\ 3 \\ 1 \end{pmatrix}$.

Compute the minimum distance between $\mathbf{p}$ and the line passing $\mathbf{x}_0$ and $\mathbf{x}_1$.

Compute the minimum distance point $\mathbf{q}$, i.e., compute $\mathbf{q}$ such that $\mathbf{q}$ is on the line and $||\mathbf{p}-\mathbf{q}||$ is minimum.

Problem 3

Compute $\begin{pmatrix} 3 & -2 \\ 1 & 0 \end{pmatrix}^{100}$.

Problem 4

Let $\mathbf{A} = \mathbf{V}\mathbf{D}\mathbf{V}^{-1}$, where $\mathbf{V} = \begin{pmatrix} 1 & 2 & -1 & 2 \\ 2 & 1 & 2 & 3 \\ 1 & -2 & -1 & 1 \\ 2 & 1 & 1 & 3 \end{pmatrix}$, and let $\mathbf{D} = \begin{pmatrix} 1 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 2 \end{pmatrix}$.

Compute the trace and determinant of $\mathbf{A}$.

Problem 5

Let $f(x,y) = x^4 + y^3 + 2xy^2$.

Compute ∇f and H_f.

Compute the quadratic approximation to $f(x,y)$ at $(x,y) = (1,1)$.

Compute the maximum and minimum curvature at $(x,y) = (1,1)$.

Problem 6

Let $\mathbf{F}(x,y) = \begin{pmatrix} 3x^3 \sin(y^2) \\ \cos(xy) \end{pmatrix}$. Compute the Jacobian $J_{\mathbf{F}}$.

Next exam will be on Mar. 7.