[MATH2605] Exam 3a

Name:_____
Please print your full name.

Apr. 30. 2007

Problem 1 (20pt)

Let $\mathbf{F}(x,y) = \begin{bmatrix} x^3+3xy-5 \\ x^3y-1 \end{bmatrix}$. Compute $\mathbf{J_F}(x,y)$. Let $(x_0,y_0) = (1,1)$. Compute $\mathbf{J_F}(x_0,y_0)$. Using (x_0,y_0) as the initial guess, perform one Newton iteration to compute (x_1,y_1).

Problem 2 (20pt)

Let $\mathbf{A} = \begin{bmatrix} 2 & 2 & 3 \\ 2 & 1 & 1 \\ 3 & 1 & 2 \end{bmatrix}$. Compute one iteration of the Jacobi algorithm to zero the largest off-diagonal element. Let the resulting matrix be $\mathbf{A}_1$. Compute the three Gershgorin disks of $\mathbf{A}_1$, and discuss locations of eigenvalues of $\mathbf{A}_1$.

Problem 3 (20pt)

Let $\mathbf{A} = \begin{bmatrix} 6 & -4 & 14 \end{bmatrix}$. Compute the singular decomposition of $\mathbf{A}$ and compute the pseudo inverse $\mathbf{A}^{\dagger}$.

Let $\mathbf{B} = \begin{bmatrix} 2 \\ 5 \\ -4 \end{bmatrix}$. Compute the singular decompositions of $\mathbf{B}$ and compute the pseudo inverse $\mathbf{B}^{\dagger}$.

Problem 4 (20pt)

Let $\mathbf{A} = \begin{bmatrix} -2 & 3 \\ -7 & 9 \end{bmatrix}$. Compute the Schur factorization of $\mathbf{A}$.

Problem 5 (20pt)

Let $\mathbf{A} = \begin{bmatrix} 6 & 6 & 15 \\ 6 & 15 & 6 \\ 16 & 6 & 6 \end{bmatrix}$. An eigenvalue of $\mathbf{A}$ is $\lambda = -9$ with eigenvector $\mathbf{v} = \begin{bmatrix} -\frac{1}{\sqrt{2}} & 0 & \frac{1}{\sqrt{2}} \end{bmatrix}^T$. Compute an orthogonal matrix whose first column is $\mathbf{v}$. Compute the first iteration in the Schur factorization of $\mathbf{A}$, i.e., compute $\mathbf{B}$ so that the first column is parallel to $\mathbf{e}_1$. (This is the first step to make $\mathbf{A}$ a triangular matrix.)