

[MATH2605] Homework 1

Jan. 12 - Jan. 19

Problem 1

Let $\mathbf{a} = \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}$.

- 1.1 Compute a vector \mathbf{v}_1 that is perpendicular to \mathbf{a} .
- 1.2 Compute another vector \mathbf{v}_2 that is perpendicular to \mathbf{a} and \mathbf{v}_1 .
- 1.3 Compute the following:

$$\begin{aligned}\mathbf{a} \cdot \mathbf{v}_1 &= \\ \mathbf{a} \cdot \mathbf{v}_2 &= \\ \mathbf{v}_1 \cdot \mathbf{v}_2 &= \end{aligned}$$

Problem 2

Consider a plane given as $x + 2y - 3z = 2$.

Compute the minimum distance from $\mathbf{x}_0 = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$.

Problem 3

Let $\mathbf{x} = \begin{pmatrix} 1 \\ -1 \\ 2 \\ -3 \\ -5 \end{pmatrix}$ and $\mathbf{y} = \begin{pmatrix} 1 \\ -2 \\ 4 \\ -6 \\ -5 \end{pmatrix}$. Remember that $\|\mathbf{x}\|_p = \left(\sum_{i=1}^5 |x_i|^p \right)^{\frac{1}{p}}$.

- 3.1 Compute $\|\mathbf{x}\|_1$, $\|\mathbf{x}\|_2$, $\|\mathbf{x}\|_3$, $\|\mathbf{x}\|_4$, $\|\mathbf{x}\|_5$, $\|\mathbf{x}\|_6$, $\|\mathbf{x}\|_7$, $\|\mathbf{x}\|_8$, $\|\mathbf{x}\|_9$, $\|\mathbf{x}\|_{10}$, and $\|\mathbf{x}\|_\infty$.
- 3.2 Compute $\|\mathbf{x}\|_1$, $\|\mathbf{y}\|_1$, and $\|\mathbf{x} + \mathbf{y}\|_1$. Does the triangle inequality ($\|\mathbf{x} + \mathbf{y}\|_1 \leq \|\mathbf{x}\|_1 + \|\mathbf{y}\|_1$) hold?
- 3.3 Compute $\|\mathbf{x}\|_{0.5}$, $\|\mathbf{y}\|_{0.5}$, and $\|\mathbf{x} + \mathbf{y}\|_{0.5}$. Does the triangle inequality ($\|\mathbf{x} + \mathbf{y}\|_{0.5} \leq \|\mathbf{x}\|_{0.5} + \|\mathbf{y}\|_{0.5}$) hold? If not, can you say why?

Problem 4

Let $\mathbf{x} \in \mathbb{R}^n$ and $\mathbf{y} \in \mathbb{R}^n$. Show that $\|\mathbf{x} - \mathbf{y}\| \geq | \|\mathbf{x}\| - \|\mathbf{y}\| |$ by using the triangle inequality $\|\mathbf{x} + \mathbf{y}\| \leq \|\mathbf{x}\| + \|\mathbf{y}\|$.
Hint: Use $\mathbf{x} = (\mathbf{x} - \mathbf{y}) + \mathbf{y}$.