[MATH2605] Homework 11

Due: Apr. 13

Problem 1

Consider a matrix

$$\mathbf{A} = \begin{bmatrix} 22 & -4 \\ -13 & 16 \\ 2 & -14 \end{bmatrix}. \tag{1}$$

Compute the SVD by

- Compute $\mathbf{A}^T\mathbf{A}$. Note that $\mathbf{A}^T\mathbf{A} \in \mathbb{R}^{2\times 2}$ is symmetric.
- Compute the eigenvalue matrix $\mathbf{B}$ of $\mathbf{A}^T\mathbf{A}$ and compute the orthogonal eigenvector matrix $\mathbf{U}$ so that $\mathbf{A}^T\mathbf{A} = \mathbf{U}\mathbf{B}\mathbf{U}^T$.
- Since $\mathbf{A}^T\mathbf{A}$ is positive, $\mathbf{B}$ should be a diagonal matrix, whose diagonal entries are positive. Verify that $\mathbf{B}$ has positive diagonals.
- Compute $\mathbf{D} = \sqrt{\mathbf{B}}$.
- Compute $\mathbf{V} = \mathbf{A}\mathbf{U}\mathbf{D}^{-1}$.
- Verify that $\mathbf{V}$ is orthogonal, i.e., check $\mathbf{V}^T\mathbf{V} = \mathbf{I}$.
- Verify that $\mathbf{A} = \mathbf{V}\mathbf{D}\mathbf{U}^T$.

Compute the generalized inverse $\mathbf{A}^\dagger = \mathbf{U}\mathbf{D}^{-1}\mathbf{V}$. Compute the matrix $\left(\mathbf{A}^T\mathbf{A}\right)^{-1}\mathbf{A}^T$ and verify that it is the same as $\mathbf{A}^\dagger$. Obviously, computing the generalized inverse by using $\left(\mathbf{A}^T\mathbf{A}\right)^{-1}\mathbf{A}^T$ is simpler than by using SVD. Unfortunately, $\left(\mathbf{A}^T\mathbf{A}\right)^{-1}\mathbf{A}^T$ cannot be used always. When you cannot use the formula $\left(\mathbf{A}^T\mathbf{A}\right)^{-1}\mathbf{A}^T$?

Let $\mathbf{x} = \begin{bmatrix} x \\ y \end{bmatrix}$. Let $\mathbf{b} = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$. Compute a least square solution to $\mathbf{A}\mathbf{x} = \mathbf{b}$ by using the $\mathbf{A}^\dagger$.