[MATH2605] Homework 12

Due: Apr. 20

Problem 1

Let $\mathbf{u} \in \mathbb{R}^n$ is a unit vector.

- Consider the Householder reflection matrix $\mathbf{M} = \mathbf{I} - 2\mathbf{u}\mathbf{u}^T$, where $\mathbf{M} \in \mathbb{R}^{n \times n}$.
  Show that $\mathbf{M}^2 = \mathbf{I}$.
  Show that $\mathbf{u}$ is an eigenvector of $\mathbf{M}$. Compute the engenvalue associated with $\mathbf{u}$.
- Consider a projection matrix $\mathbf{P} = \mathbf{I} - \mathbf{u}\mathbf{u}^T$, where $\mathbf{P} \in \mathbb{R}^{n \times n}$.
  Show that $\mathbf{P}^2 = \mathbf{P}$ and therefore $\mathbf{P} = \mathbf{P}^2 = \mathbf{P}^3 = \mathbf{P}^4 = \mathbf{P}^5 = \ldots$
  Show that $\mathbf{u}$ is an eigenvector of $\mathbf{P}$. Compute the engenvalue associated with $\mathbf{u}$.
- Consider a vector $\mathbf{x} \in \mathbb{R}^n$. Geometrically speaking, $\mathbf{M}$ inverts the $\mathbf{u}$ component of $\mathbf{x}$, i.e., $\mathbf{M}\mathbf{x} \cdot \mathbf{u} = -\mathbf{x} \cdot \mathbf{u}$.
  Then, what do you think the projection matrix $\mathbf{P}$ do for the $\mathbf{u}$ component of $\mathbf{x}$?