

# [MATH2605] Homework 2

Jan. 19 - Jan. 26

## Problem 1

$$\text{Let } \mathbf{A} = \begin{pmatrix} 1 & 2 & 3 \\ 5 & 3 & 2 \\ 3 & 1 & 4 \end{pmatrix}.$$

1. Check if columns of  $\mathbf{A}$  are linearly independent or not.
2. What is the rank of  $\mathbf{A}$ ? Is  $\mathbf{A}$  invertible?
3. In case  $\text{rank}(\mathbf{A}) = 1$ , the range of  $\mathbf{A}$  will be a line. Compute the equation of that line. The null space will be a two dimensional plane. Compute the equation of that plane.  
In case  $\text{rank}(\mathbf{A}) = 2$ , the range of  $\mathbf{A}$  will be a plane. Compute equation of that plane. The null space will be a line. Compute the equation of that line.  
In case  $\text{rank}(\mathbf{A}) = 3$ , what is the range of  $\mathbf{A}$ ?
4. Do you think that for some matrix  $\mathbf{B} \in \mathbb{R}^{3 \times 3}$ ,  $\text{rank}(\mathbf{AB}) > \text{rank}(\mathbf{A})$ ?  
Try with some matrix  $\mathbf{B}$  and answer. You don't have to prove it.

## Problem 2

Let  $\mathbf{A}, \mathbf{B} \in \mathbb{R}^{n \times n}$ . Show that  $\text{trace}(\mathbf{AB}) = \text{trace}(\mathbf{BA})$ .

Hint: Let  $A_{i,j}$ ,  $B_{i,j}$ , and  $(\mathbf{AB})_{i,j}$  be the elements at  $i^{\text{th}}$  row and  $j^{\text{th}}$  column of  $\mathbf{A}, \mathbf{B}$  and  $\mathbf{AB}$ , respectively. Then

$$(\mathbf{AB})_{i,j} = \sum_{k=1}^n A_{i,k} B_{k,j} \tag{1}$$

## Problem 3

$$\text{Let } \mathbf{A} = \begin{pmatrix} 4 & 3 & -2 \\ 1 & 0 & 1 \\ -4 & 6 & 2 \end{pmatrix}.$$

Compute the eigenvalues by solving the cubic equation  $\det(\mathbf{A} - \lambda \mathbf{I}) = 0$ . Compute the eigenvectors.

Using the eigenvalues and eigenvectors that you computed, write an eigen decomposition, and verify that the eigen decomposition is indeed true. (Use calculator or matlab, etc to invert the eigenvector matrix).

Notice that since you can place eigenvalues in different order, you can make more than one eigen decomposition. If you know how to use matlab, check what decomposition matlab produces.