[MATH2605] Homework 3

Jan. 27 - Feb. 2

Problem 1

Let $\mathbf{A} = \begin{pmatrix} 1 & 2 & 3 \\ 5 & 3 & 1 \\ 3 & 1 & -1 \end{pmatrix}$.

1. Show that columns of **A** are linearly dependent, and show that rank(A)=2.
2. Since rank(A)=2, the range of **A** will be a plane. Compute equation of that plane.
   The null space will be a line. Compute the equation of that line.
3. Do you think that for some matrix $\mathbf{B} \in \mathbb{R}^{3\times3}$, rank($\mathbf{AB}$) $>$ rank($\mathbf{A}$)?
   Try with some matrix **B** and answer. You don't have to prove it.

Problem 2

Let $\mathbf{A} = \begin{pmatrix} 1 & -2 & 2 \\ -2 & 4 & -4 \\ 1 & -2 & 2 \end{pmatrix}$.

1. Show that columns of **A** are linearly dependent, and show that rank(A)=1.
2. Since rank(A)=1, the range of **A** will be a line. Compute the equation of that line.
   The null space will be a two dimensional plane. Compute the equation of that plane.
3. Do you think that for some matrix $\mathbf{B} \in \mathbb{R}^{3\times3}$, rank($\mathbf{AB}$) $>$ rank($\mathbf{A}$)?
   Try with some matrix **B** and answer. You don't have to prove it.

Problem 3

This problem is to show that, sometimes, an eigenvalue can have a multiplicity greater than one, and to show that the number of linearly independent eigenvectors is not always the same as the multiplicity.

Let $\mathbf{A} = \begin{pmatrix} -1 & 2 & 2 \\ -1 & 2 & 1 \\ -2 & 2 & 3 \end{pmatrix}$, and $\mathbf{B} = \begin{pmatrix} 1 & 4 & 3 \\ -7 & 8 & 6 \\ 2 & 2 & 3 \end{pmatrix}$.
The characteristic equation $\det(\mathbf{A} - \lambda\mathbf{I}) = 0$ has the solution 1 with the multiplicity 2, and $\det(\mathbf{B} - \lambda\mathbf{I}) = 0$ has the solution 3 with the multiplicity 2. Check if this is true. Compute other engenvalues of **A** and **B**.

The rank of $\mathbf{A} - \mathbf{I}$ is 1. Verify this. Eigenvectors of **A**, associated with the eigenvalue 1, are the basis of the null space of $\mathbf{A} - \mathbf{I}$. Compute the two basis vectors of $\mathbf{A} - \mathbf{I}$. Verify that they are indeed the engenvectors of **A** associated with the eigenvalue '1'.

The rank of $\mathbf{B} - 3\mathbf{I}$ is 2. Verify this. The eigenvector of **B**, associated with the eigenvalue 3, is the basis of the null space of $\mathbf{B} - 3\mathbf{I}$. Compute the basis vector of $\mathbf{B} - 3\mathbf{I}$. Verify that it is indeed the engenvector of **B** associated with the eigenvalue '3'.

Problem 4: Matlab Practice #1

These are made of trivial steps. Answer or do questions with *.

1. Run Matlab. You should see a matlab command window. Start using matlab as a calculator. Try "1+2", "sin(pi/4)", etc.

2. Now, use a variable. Try "a=2+3". This will make a variable a. Try "who", which should display the variable a. Try "a", which should display the value of a. Of course, a should be 5.

3. Now, define a matrix by "A = [1, 2; 3, 4]". Compute the eigenvalues by "eig(A)".
   For the eigen decomposition, try "[V,D] = eig(A)". Check if you indeed have $\mathbf{A} = \mathbf{VDV}^{-1}$, by "A - V*D*inv(V)".
   * Did you get exactly zero? * If not, what do you think is the reason?

4. A transpose of **A** is computed as A'. Try "AT = A' ". Now, you have a new vatiable AT that is the transpose of **A**. Compute eigenvalues of AT. Are they the same as A?

5. Try "help elmat". You will see a list of functions about elementaty matrix operations.
   Find a function that constructs an identity matrix. *What is the name of this function? By using that function, construct an identity matrix of dimension 10×10. *What are the eigenvalues of it? *What do you think is(are) the multiplicit(y,ies) of this(these) eigenvalue(s)?
   Find a function that constructs a random matrix. *What is the name of this function? By using that function, build a random matrix of dimension 10×20

6. Try "help ops". You will see a list of operators such as +, -, etc. Among them, the colon operator ':' is somewhat important. Type "help colon" and read the help.
   *Given a matrix $A \in \mathbb{R}^{3\times 3}$, what is the command that extracts the first column from A?
   *Given a matrix $A \in \mathbb{R}^{3\times 3}$, what is the command that extracts the second row from A?
   *Given a matrix $A \in \mathbb{R}^{3\times 3}$, what is the command that extracts first and second columns from A?
   *Given a matrix $A \in \mathbb{R}^{3\times 3}$, what is the command that extracts first and third columns from A?

7. Among the operators, learn the '.' operator. Try 'help punct' and read the 'Field Access' part.
   Let A,B be matrices. *Tell the difference between the two commands "A*B" and "A.*B".
   Let A be a matrix. *Tell the difference between the two commands "A^2" and "A.^2".

8. Try "help matfun". You will see a list of functions about matrix operations.
   Find a function that computes the singular value decomposition. *Construct random 3×3, 2×3, and 3×2 matrices and submit your matrices. *Compute their singular value decompositions, and submit singular values and vectors.

9. Let's draw a graph. First, try "x = 0:0.01:3". Check what numbers are generated.
   Try "plot(x, x.^2, 'r')".
   Try "help plot". *What is the command that plots the same graph in green color?

10. Finally, try "help rank" and "help null". You may verify some of your answers for Problems 1, 2, and 3.