# [MATH2605] Homework 4

Feb. 2 - Feb. 9

## Problem 1: Matlab Practice #2

We will learn writing matlab scripts and using symbolics. Answer or do questions with $^*$.

1. Try *pwd*. This will show you the current working directory. You can use *cd* command to move to any other directory. You may create a matlab directory in your home directory and move to there.
2. Matlab script is a text file with extension 'm'. Try *edit test1.m*. This will open the matlab editor with a file "test1.m". You can write any matlab command and save the script file. You can execute this script by *test1*. $^*$From internet or matlab help, can you find whether this script is interpreted or compiled?
3. Try *help function*. $^*$Create a script that takes a matrix, and compute and return the largest and smallest eigenvalues. You may submit hand-written (or printed) copy of the script.
4. We now try symbolics. This will allow you to compute partial derivatives quickly. Try *syms x y real;*. This will declare real variables $x$ and $y$.
5. Define a function $f(x,y)$ by *f = (3*(1+x)ˆ2 + x*yˆ3 + yˆ2) / (1+xˆ2+yˆ2)*.
6. Try *help sym/diff* and read help. $^*$What is the command that computes $\frac{\partial f}{\partial x}$ and $\frac{\partial f}{\partial y}$?
7. Try *help pretty* and read help. Try *help simplify* and read help. $^*$Using the simplify and pretty commands, compute and submit the simplified forms of $\frac{\partial f}{\partial x}$ and $\frac{\partial f}{\partial y}$?
8. $^*$What are the commands that compute $\frac{\partial}{\partial y}\frac{\partial f}{\partial x}$ and $\frac{\partial}{\partial x}\frac{\partial f}{\partial y}$?
   $^*$Is $\frac{\partial}{\partial x}\frac{\partial f}{\partial y} = \frac{\partial}{\partial y}\frac{\partial f}{\partial x}$ true?

## Problem 2

Last week, we have computed eigenvalues and eigenvectors of the two matrices $\mathbf{A} = \begin{pmatrix} -1 & 2 & 2 \\ -1 & 2 & 1 \\ -2 & 2 & 3 \end{pmatrix}$, and $\mathbf{B} = \begin{pmatrix} 1 & 4 & 3 \\ -7 & 8 & 6 \\ 2 & 2 & 3 \end{pmatrix}$.
The matrix $\mathbf{A}$ has eigenvalues $\{1,1,2\}$, where 1 is the eigenvalue with multiplicity 2 that has two linearly independent eigenvectors. In contrast, $\mathbf{B}$ has eigenvalues $\{3,3,6\}$, where 3 is the eigenvalue with multiplicity 2, but it has only one linearly independent eigenvector.

Note that $\mathbf{A}$ has three linearly independent eigenvectors: two associated with 1 and another associated with 2. Therefore, we can construct an invertible eigenvector matrix $\mathbf{V}_A$ and the diagonal eigenvalue matrix $\mathbf{D}_A$ so that $\mathbf{AV}_A = \mathbf{V}_A\mathbf{D}_A$. Since $\mathbf{V}_A$ is invertible, we can right multiply $\mathbf{V}_A^{-1}$ to obtain an eigenvalue decomposition $\mathbf{A} = \mathbf{V}_A\mathbf{D}_A\mathbf{V}_A^{-1}$. Since $\mathbf{D}_A$ is diagonal, we say $\mathbf{A}$ is diagonalizable.

However, even though we can construct an eigenvector matrix $\mathbf{V}_B$ and the diagonal eigenvalue matrix $\mathbf{D}_B$ so that $\mathbf{BV}_B = \mathbf{V}_B\mathbf{D}_B$, the eigenvector matrix $\mathbf{V}_B$ is not invertible. There are only two eigenvectors: one associated with 3 and another associated with 6. We cannot construct a full rank, and therefore invertible, matrix $\mathbf{V}_B$. Therefore, we cannot have the eigenvalue decomposition $\mathbf{B} = \mathbf{V}_B\mathbf{D}_B\mathbf{V}_B^{-1}$, with a diagonal matrix $\mathbf{D}_B$. We say that $\mathbf{B}$ is not diagonalizable.

In fact, if allow an off-diagonal term in $\mathbf{D}_B$, we can make $\mathbf{B} = \mathbf{V}_B\mathbf{D}_B\mathbf{V}_B^{-1}$. We can compute this by using the Matlab command *jordan*. Using this command, compute $\mathbf{V}_B$ and $\mathbf{D}_B$ and submit the matrix. Also verify that $\mathbf{B} = \mathbf{V}_B\mathbf{D}_B\mathbf{V}_B^{-1}$. Note that $\mathbf{D}_B$ is not a diagonal matrix anymore. This is called the Jordan decomposition.

## Problem 3

Using Matlab, we can plot graphs of nonlinear surfaces.
Try *help meshgrid*, and read some. Try *[X,Y] = meshgrid(-3:0.2:3, -3:0.2:3);*. This will create an array $X$ that varies along rows and $Y$ that varies through columns.

Now, construct a function $f(x,y) = x^2 + y^2/4 + 3xy + 2$, by *f = X.*X + Y.*Y/4 + 3*X.*Y + 2*.
You can plot the surface by *surf(X,Y,f); xlabel('x'); ylabel('y'); zlabel('f');*. Submit a hand-drawn sketch (or printed copy) of this graph.

The contour can be plotted by *contour(X,Y,f,-40:4:40)*. Submit a hand-drawn sketch (or a printed copy) of this contour graph. What each curves in this contour plot stands for?

The gradient of $f(x,y)$ is $\nabla f = [2x + 3y \quad y/2 + 3x]^T$.
Compute the gradient by *fx = 2*X + 3*Y; fy = Y/2 + 3*X;*.
The gradient vector can be drawn by *quiver(X,Y,fx,fy);*

You can draw the contour and the gradient vectors together by using the 'hold' command.
Try *contour(X,Y,f,-40:4:40); axis equal; xlabel('x'); ylabel('y'); hold on; quiver(X,Y,fx,fy); hold off;*
What is the angle between the gradient vector and the contour line?

Critical point is the point where $\nabla f = 0$. Compute the critical point. Indicate the critical point in your contour graph.