CS 4495/7495 Computer Vision

Assignment 3: Multiview Geometry

23rd September 2006

Introduction

There are three parts in this assignment. The first part gives you the idea of stereo triangulation. The second part gives you hands-on experience with the use of epipolar constraints to recover the fundamental matrix in a 2-view system. The third part is to compute the trifocal tensor for a 3-view system. Each section also has a piece at the end titled 'Deliverables' that contains the thing you should turn in related to that section. The first and second part are due on 10/03 and the third part is due on 10/10.

1 Stereo triangulation

Given a calibtrated stereo rig and two matching image points p and p'(3-vector homogeneous coordinate), we could reconstruct the 3D point P by intersecting the two rays. With an algebraic approach, suppose the 3×4 projection matrices for two cameras are denoted by M and M', we could write the constraints $zp = MP$ and $z'p' = MP$ as:

$$\begin{cases} p \times MP = 0 \\ p' \times M'P = 0 \end{cases} \Leftrightarrow \begin{pmatrix} [p_x]M \\ [p'_x]M' \end{pmatrix} P = 0\,,$$

where $[p_x] = \begin{bmatrix} 0 & -p_z & p_y \\ p_z & 0 & -p_x \\ -p_y & p_x & 0 \end{bmatrix}$. This whole equation could be written as $AP = 0$, which is an overconstrained system of four independent linear equations. To solve P, we could use SVD on A to perform a matrix factorization such that $A = UDV^T$. In which, the diagonal matrix $D = diag\{\delta_1, \delta_2, \delta_3 .. \delta_n\}, \delta_1 \geq \delta_2 \geq \delta_3 \geq .. \geq \delta_n$. Then P will be the last column of V corresponding to the smallest eigenvalue δ_n. Matlab code is provided in `http://swiki.cc.gatech.edu:8080/cs4495-fl04/59`.

1.1 Deliverables

1. Please download the package from the webpage. There are two images, Book1.jpg and Book2.jpg, taken from two calibrated cameras with the projection matrix M_1, M_2 (available in cam_book1.txt and cam_book2.txt). We have manually established a pair of matching feature points in the images. The pixel coordinate of the feature point in Book1.jpg is $(130, 220)$ and the coordinate for corresponding feature in Book2.jpg is $(246, 211)$. Solve the 3D point.

2. Verify it by projecting it to the image Book3.jpg taken from the third camera. Its projection matrix $M_3$is available in cam_book3.txt. Label the point in the image. Is it in the right place?

2 The Eight-point algorithm

2.1 Main Idea

In class we discussed the linear 8-point algorithm to recover the fundamental matrix given corresponding points (p_i, p_i') in two views, where $i \in 1..N$ and $N \geq 8$. In this assignment you are asked to implement and use the 8-point algorithm for manually picked correspondences. The rest of this document describes the process in greater detail.

There are two images, mantle1.jpg and mantle2.jpg, package available from the class website. We have manually established eight features in the images that correspond to each other.

The pixel coordinates of the features in mantle1.jpg are

$$(330, 620), (304, 414), (760, 526), (506, 568), (706, 512), (36, 298), (868, 188), (916, 410)$$

and the corresponding pixel coordinates in mantle2.jpg are

$$(204, 558), (168, 384), (642, 574), (378, 552), (612, 548), (160, 248), (900, 232), (870, 492)$$

The fundamental matrix has rank 2 and 7 degrees of freedom, and hence, can be recovered from only seven correspondences. However, the algorithm is much simpler if we have eight correspondences. If (u, v) and (u', v') are two corresponding points in the two images, and F is the fundamental matrix, the epipolar constraint is

$$(u, v, 1) \begin{pmatrix} F_{11} & F_{12} & F_{13} \\ F_{21} & F_{22} & F_{23} \\ F_{31} & F_{32} & F_{33} \end{pmatrix} \begin{pmatrix} u' \\ v' \\ 1 \end{pmatrix} = 0 \qquad (1)$$

where we have written out the individual elements of the fundamental matrix.

Given 8 such correspondences, we can recover the matrix F from the homogeneous system

$$\begin{pmatrix} u_1u'_1 & u_1v'_1 & u_1 & v_1u'_1 & v_1v'_1 & v_1 & u'_1 & v'_1 & 1 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ u_8u'_8 & u_8v'_8 & u_8 & v_8u'_8 & v_8v'_8 & v_8 & u'_8 & v'_8 & 1 \end{pmatrix} \begin{pmatrix} F_{11} \\ F_{12} \\ F_{13} \\ F_{21} \\ F_{22} \\ F_{23} \\ F_{31} \\ F_{32} \\ F_{33} \end{pmatrix} = AF = \quad 0$$

The least-squares solution for F is the eigenvector corresponding to the smallest eigenvalue of A, that is, the last column of V in the SVD, $A = UDV^T$. This is the uncorrected fundamental matrix $\tilde{F}$, as the rank 2 constraint is not enforced using this procedure.

2.1.1 Deliverables

You should implement and run the eight-point algorithm using the provided test images to obtain the fundamental matrix $\tilde{F}$. Plot 15-20 epipolar lines in mantle2.jpg, obtained by varying the pixel location x in the equation $l = \tilde{F}^T x$, in mantle1.jpg. Your deliverables are the code for the eight-point algorithm, the uncorrected fundamental matrix $\tilde{F}$, and the images respectively with your chosen features and the epipolar lines drawn on it.

2.2 Rank Correction

To obtain the correct rank 2 fundamental matrix, we again use SVD on $\tilde{F}$ to perform a matrix factorization (as in assignment 1). Let the diagonal matrix obtained from the SVD be written as $D = \mathrm{diag}(r,s,t)$. Then the correct rank 2 fundamental matrix $\hat{F}$ is given by $\hat{F} = U \times \mathrm{diag}(r,s,0) \times V^T$.

2.2.1 Deliverables

The deliverables are exactly the same, but now computing (showing) the rank-corrected fundamental matrix $\hat{F}$.

2.3 Normalization of Measurements

The eight-point algorithm is sensitive to scaling and translation in the measurements. Hence, normalization of the input measurements is required. The normalization is performed to center the measurements on the origin and make the mean distance of the measurements from the origin to be $\sqrt{2}$. *Note that the normalization is performed separately for measurements from each image as follows*

- Center the measurements at origin by pre-multiplying each measurement in homogeneous coordinates by $T = \begin{pmatrix} 1 & 0 & -m_x \\ 0 & 1 & -m_y \\ 0 & 0 & 1 \end{pmatrix}$ where (m_x, m_y) is the mean of the measurements.
- Scale the modified measurements to have mean length $\sqrt{2}$ by pre-multiplying with the matrix $S = \begin{pmatrix} \frac{\sqrt{2}}{d} & 0 & 0 \\ 0 & \frac{\sqrt{2}}{d} & 0 \\ 0 & 0 & 1 \end{pmatrix}$, where d is the average length of the measurements after the above step.

After the normalization, the eight-point algorithm can be run on the measurements. After obtaining the fundamental matrix, we have to de-normalize it. This can be done by the step $F'' = T_1^T S_1 F' S_2 T_2$, where T_1 and S_1 are the normalization matrices for the features from the first image, T_2 and S_2 are the corresponding matrices for the second image, and F'' is the true fundamental matrix obtained after the de-normalization.

2.3.1 Deliverables

Obtain the fundamental matrix using the provided test images, but this time with the normalization implemented. You should plot the epipolar lines in mantle2.jpg as before.

3 Triple Delight !

Obtain (individually or with another student) 3 test images that form a 3-view system, and compute the trifocal tensor for this system.

3.1 Deliverables

Turn in the test images (with measurement points marked) and the trifocal tensor. That will get you about half the extra credit. To get the full extra credit, show one point-line-point correspondence obtained by choosing one 3D line in image A, then picking one point on it in image B, and showing its actual and computed position in image C.

4 How to turn in

Put all your code, images, and computed matrices in a directory named <name-emailPrefix>, tar it or zip it, and email it to the TA (cynthia@cc.gatech.edu). You should also hand in during class a hard copy of your test images and all the images with epipolar lines drawn.