Fast Mean Shift with Accurate and Stable Convergence

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Mean Shift

Input: X_Q, X_R, ε

Output: the converged X_Q

while $\max(dist) \geq \varepsilon$ do

for $x_q \in X_Q$ do — Dual-tree approximation

$$m(x_q) = \frac{h(x_q)}{f(x_q)} = \frac{\sum_{x_r \in X_R} K_h(x_r - x_q) w(x_r) x_r}{\sum_{x_r \in X_R} K_h(x_r - x_q) w(x_r)}$$

$$dist = \left\| m(X_Q) - X_Q \right\|_2$$

$$X_Q \leftarrow m(X_Q)$$

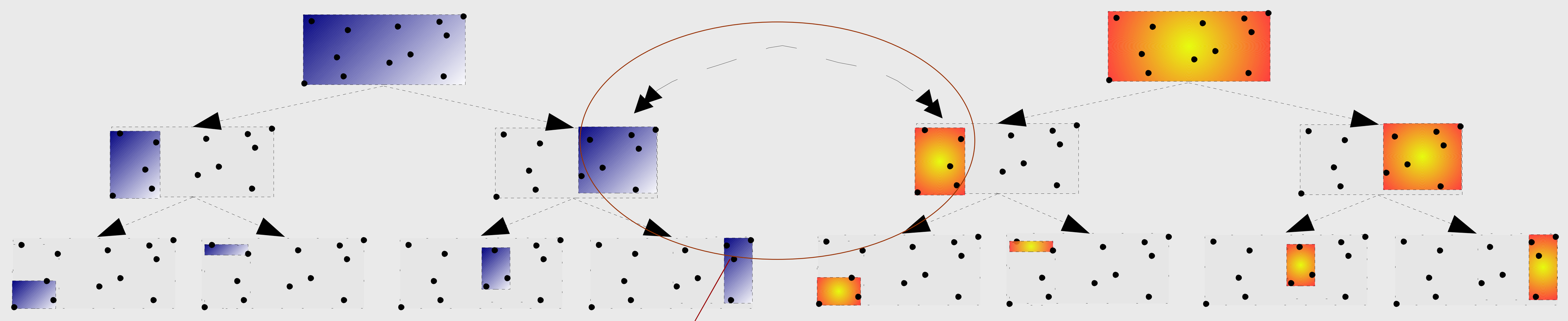
Return X_Q

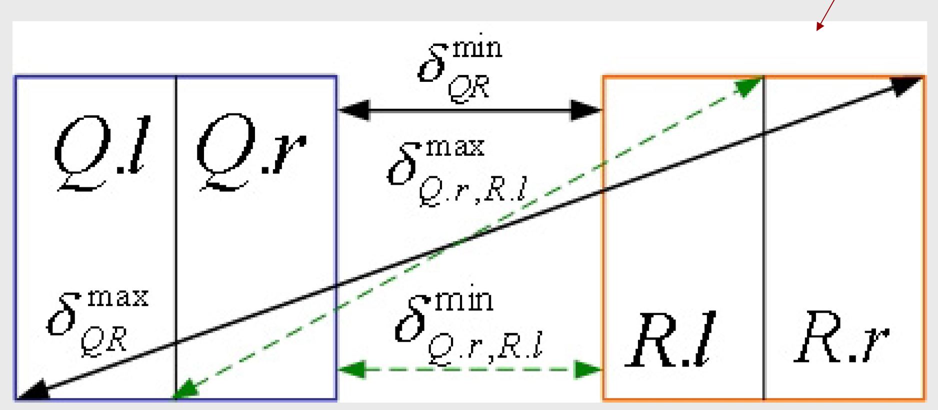
Each iteration requires O(NM) operations!

Approach: Approximate MS

- Accuracy: an explicit error bound on the approximation error of the mean vector
- Stability
- Parameters: as few as possible

Dual-tree Methodology





$$\delta_{QR}^{min} \leq \delta_{Q.l/r,R.l/r}^{min} \leq \delta_{Q.l/r,R.l/r}^{max} \leq \delta_{QR}^{max}$$

Dualtree(Q,R)

if **Can-approximate**(Q,R,τ), **Approximate**(Q,R), return;

if leaf(Q) and leaf(R), **DualtreeBase**(Q,R)

else **Dualtree**(Q.l, R.l), **Dualtree**(Q.l, R.r),

Dualtree(Q.r, R.l), **Dualtree**(Q.r, R.r)

How DT ensures approximation error

- Distribute global error bound into node-node pruning criterion
- Finite difference approximation with updated bounds

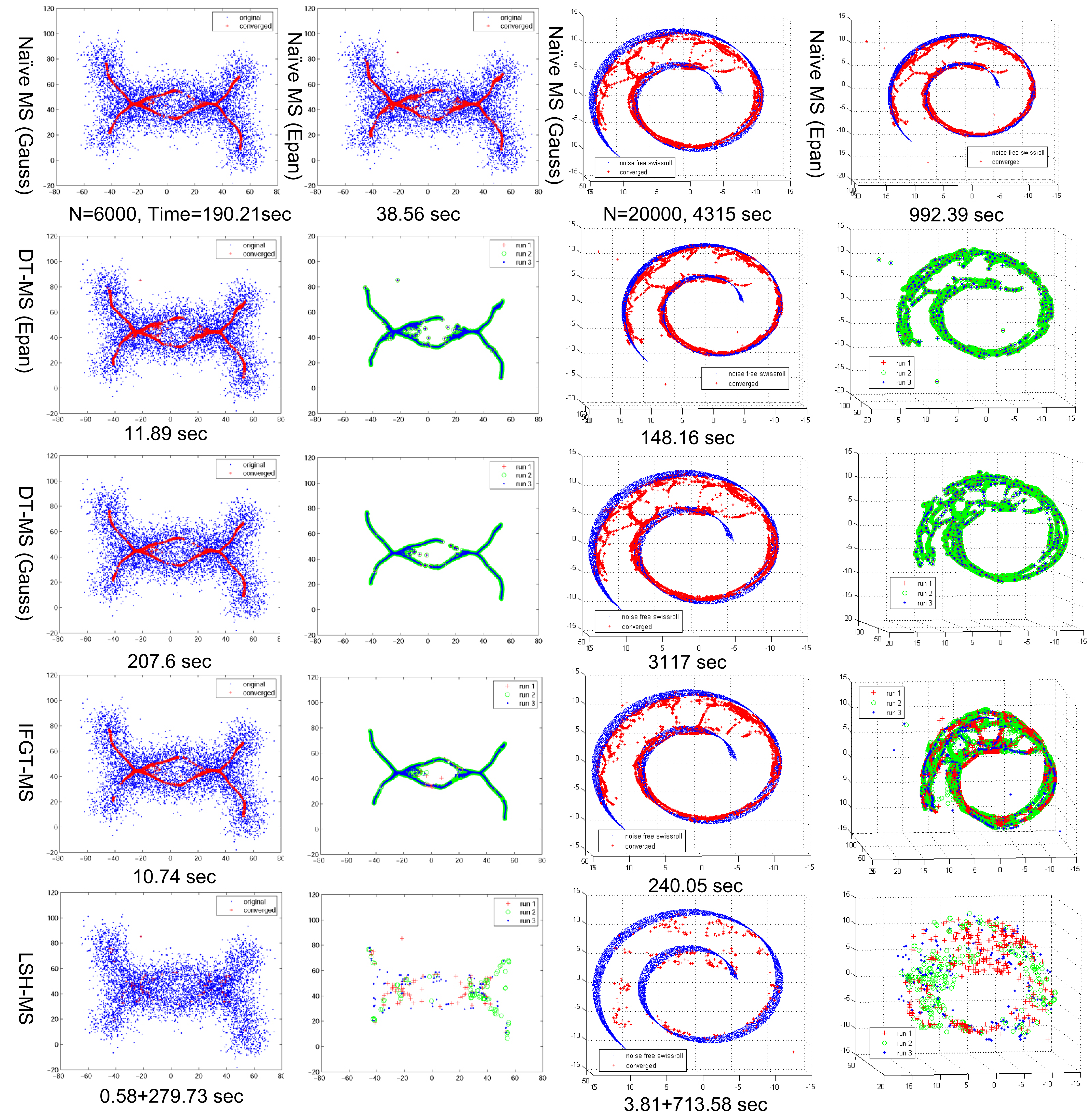
Examples	DT-KDE $f(x_q)$	DT-MS $m(x_q) = h(x_q)/f(x_q)$
τ	$\left\| f(x_q) - \hat{f}(x_q) \right\| / \left\| f(x_q) \right\| \leq \tau$	$\left\| h(x_q)/f(x_q) - \hat{h}(x_q)/\hat{f}(x_q) \right\|_1 / \left\| h(x_q)/f(x_q) \right\|_1 \leq \tau$
Bounds	$f_q^{min}, f_q^{max}, f_Q^{min}, f_Q^{max}$	$f_q^{min}, f_q^{max}, f_Q^{min}, f_Q^{max}, h_{q,d}^{min}, h_{q,d}^{max}, h_{Q,d}^{min}, h_{Q,d}^{max}$
Approx.	$N_R \overline{K}_h, \overline{K}_h = (K_h(\delta_{QR}^{min}) + K_h(\delta_{QR}^{max}))/2$	$\hat{h}_{R,d}(x_q) = (h_{R,d}^{min} + h_{R,d}^{max})/2 = S_{R,d} \overline{K}_h$
Can-approx.	$K_h(\delta_{QR}^{min}) - K_h(\delta_{QR}^{max}) \leq 2\tau f_Q^{min} / N, N = \left\| X_R \right\|$	$K_h(\delta_{QR}^{min}) - K_h(\delta_{QR}^{max}) \leq \min\{\tau f_Q^{min} L_Q / N U_Q, \tau L_Q / \sum_d S_d^A\}$

Speedup: DT-MS over naïve MS

Images	Speedup	Time (DT/Naive)	Nit	hg
Fox	44.74	155.22/6944.54	1/1	0.0166
Snake	136.51	39.71/5420.36	1/1	0.0065
Cowboys	1.75	3059.38/5352.24	2/1	0.0172
Vase	19.06	300.66/5729.44	1/1	0.0163
Plane	32.86	187.54/6162.65	1/1	0.0102
Hawk	48.88	127.35/6224.48	1/1	0.0136

N=481*321, D=3

Comparison among DT-MS, IFGT-MS and LSH-MS



Contributions

- DT-MS, a novel MS approximation based on the dual-tree (DT)
- Extend DT method to signed vector computation
- Compare 3 fast MS algorithms on a standardized dataset and Highlight for the first time the issue of **stability** in MS approximation

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