This is a shortened version of the tutorial given at the ECCV'2002, Copenhagen, and ICPR'2002, Quebec City.

Subspace Methods for Visual Learning and Recognition

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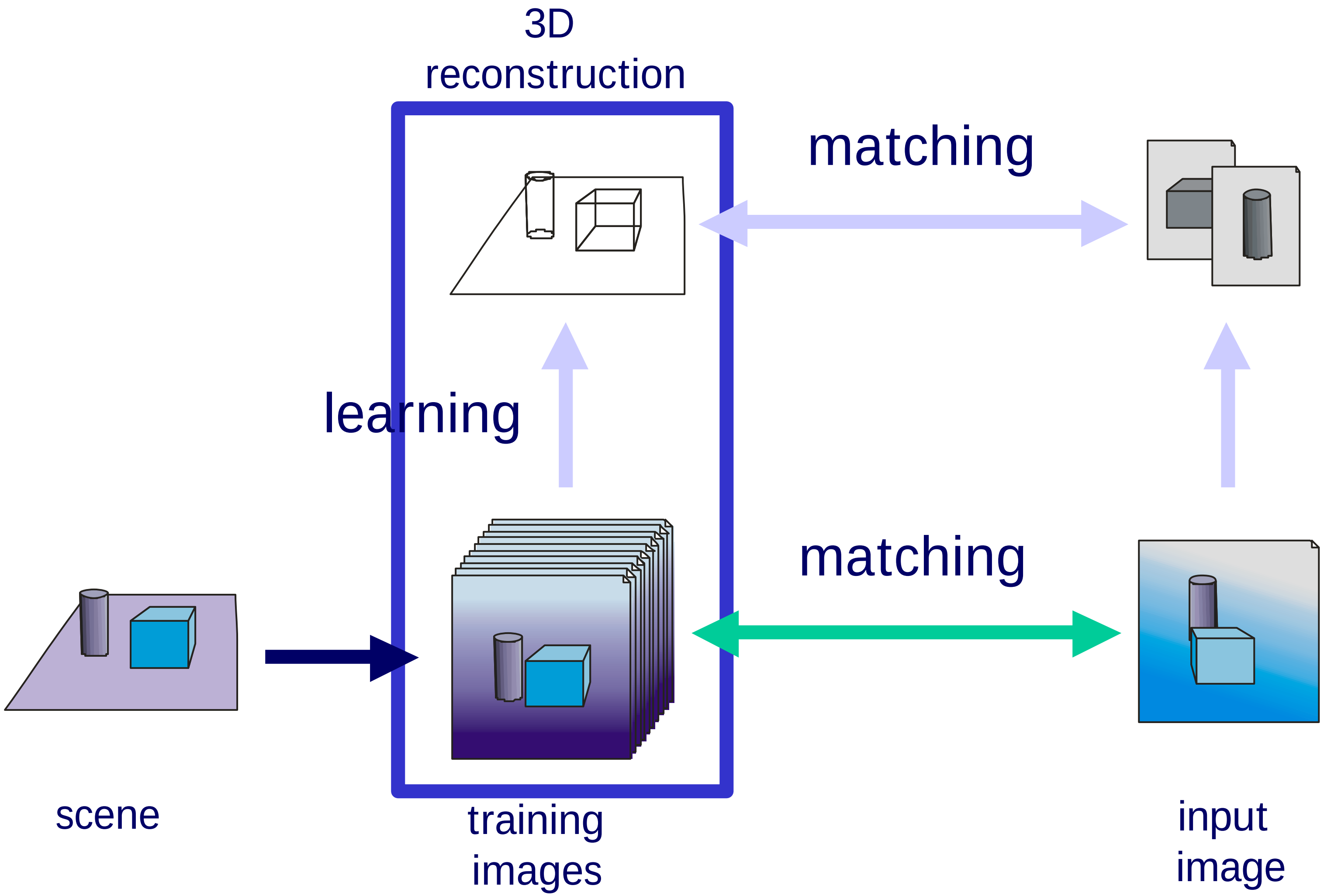
Outline Part 1

- Motivation
- Appearance based learning and recognition
- Subspace methods for visual object recognition
- Principal Components Analysis (PCA)
- Linear Discriminant Analysis (LDA)
- Canonical Correlation Analysis (CCA)
- Independent Component Analysis (ICA)
- Non-negative Matrix Factorization (NMF)
- Kernel methods for non-linear subspaces

Outline Part 2

- Robot localization
- Robust representations and recognition
- Robust PCA recognition
- Scale invariant recognition using PCA
- Illumination insensitive recognition
- Representations for panoramic images
- Incremental building of eigenspaces
- Multiple eigenspaces for efficient representation
- Robust building of eigenspaces
- Research issues

Learning and recognition



Appearance-based approaches

Attention in the appearance-based approaches

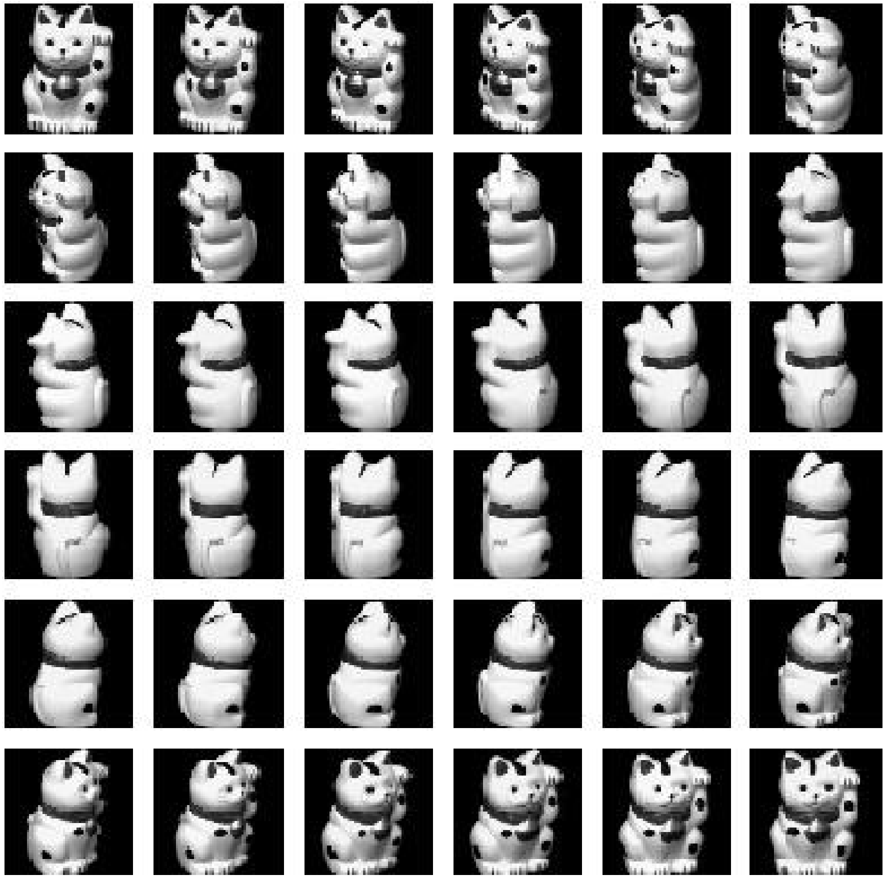
Encompass combined effects of:

- shape,
- reflectance properties,
- pose in the scene,
- illumination conditions.

Acquired through an automatic learning phase.

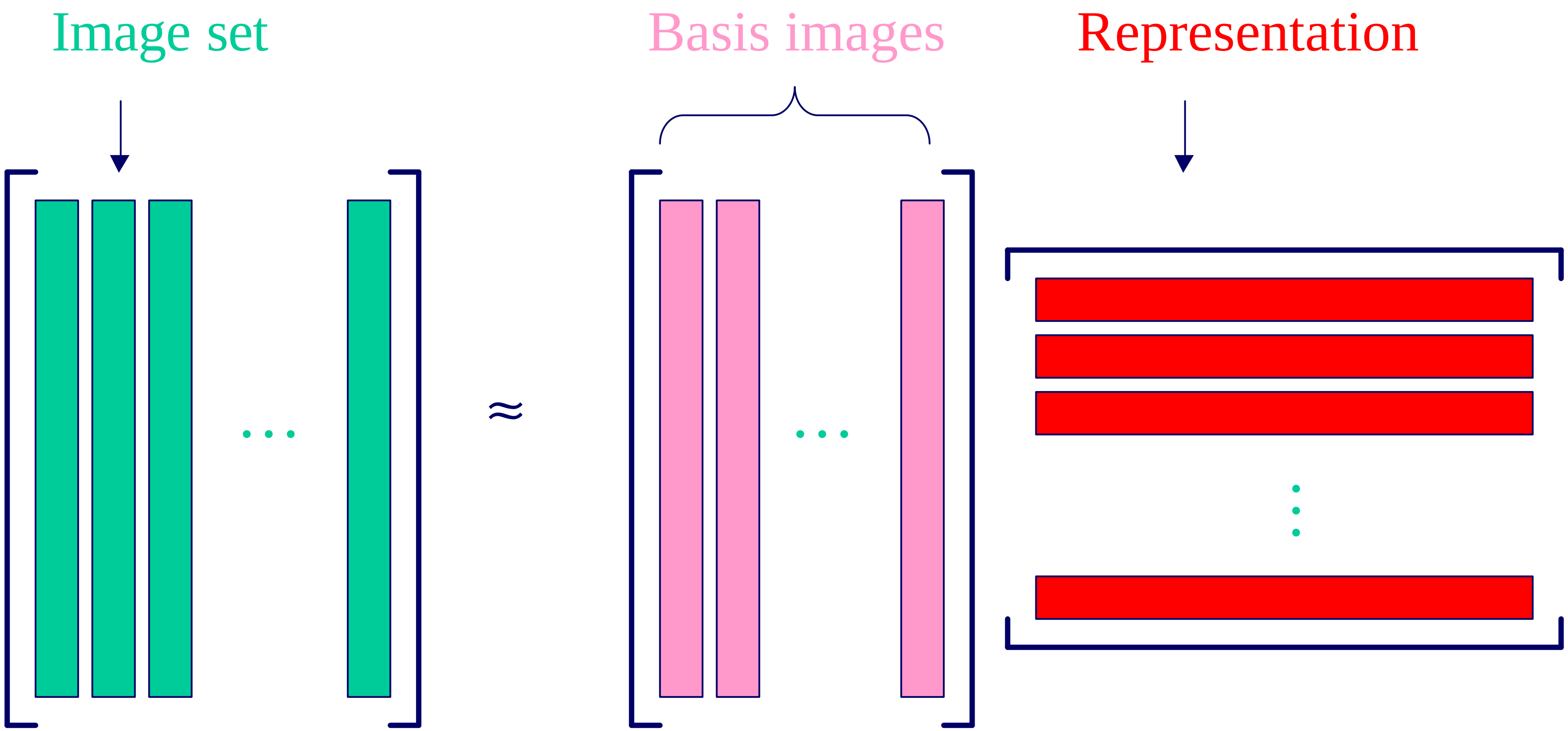
Appearance-based approaches

Objects are represented by a large number of views:



Subspace Methods

- Images are represented as points in the N-dimensional vector space
- Set of images populate only a small fraction of the space
- Characterize subspace spanned by images



Subspace Methods

Properties of the representation:

- Optimal Reconstruction $\Rightarrow$ PCA
- Optimal Separation $\Rightarrow$ LDA
- Optimal Correlation $\Rightarrow$ CCA
- Independent Factors $\Rightarrow$ ICA
- Non-negative Factors $\Rightarrow$ NMF
- Non-linear Extension $\Rightarrow$ Kernel Methods

Eigenspace representation

- **Image set (normalised, zero-mean)**

$$X = [\ \mathbf{x}_0 \quad \mathbf{x}_1 \quad \ldots \quad \mathbf{x}_{n-1}\ ];\ X \in \mathbb{R}^{m\times n}$$

- **We are looking for orthonormal basis functions:**

$$U = [\ \mathbf{u}_0 \quad \mathbf{u}_1 \quad \ldots \quad \mathbf{u}_k\ ];\ k << n$$

- **Individual image is a linear combination of basis functions**

$$\mathbf{x}_i \approx \tilde{\mathbf{x}}_i = \sum_{j=0}^{p} q_j(\mathbf{x}_i)\mathbf{u}_j$$

$$\| \mathbf{x} - \mathbf{y} \|^2 \approx \| \sum_{j=1}^{k} q_j(\mathbf{x})\mathbf{u}_j - \sum_{j=1}^{k} q_j(\mathbf{y})\mathbf{u}_j \|^2 =$$

$$\| \sum_{j=1}^{k} (q_j(\mathbf{x}) - q_j(\mathbf{y}))\mathbf{u}_j \|^2 = \| q_j(\mathbf{x}) - q_j(\mathbf{y}) \|^2$$

Best basis functions n?

- **Optimisation problem**

$$\sum_{i=0}^{n-1} ||\mathbf{x}_i - \sum_{j=0}^{k} q_j(\mathbf{x}_i)\mathbf{u}_j||^2 \rightarrow \min$$

- **Taking the *k* eigenvectors with the largest eigenvalues of**

$$C = XX^T = \left[\begin{array}{cccc} \mathbf{x}_0 & \mathbf{x}_1 & \dots & \mathbf{x}_{n-1} \end{array} \right] \left[\begin{array}{c} \mathbf{x}_0^\top \\ \mathbf{x}_1^\top \\ \dots \\ \mathbf{x}_{n-1}^\top \end{array} \right]$$

- **PCA or Karhunen-Loéve Transform (KLT)**

$$C\mathbf{u}_i = \lambda_i \mathbf{u}_i$$

Efficient eigenspace computation

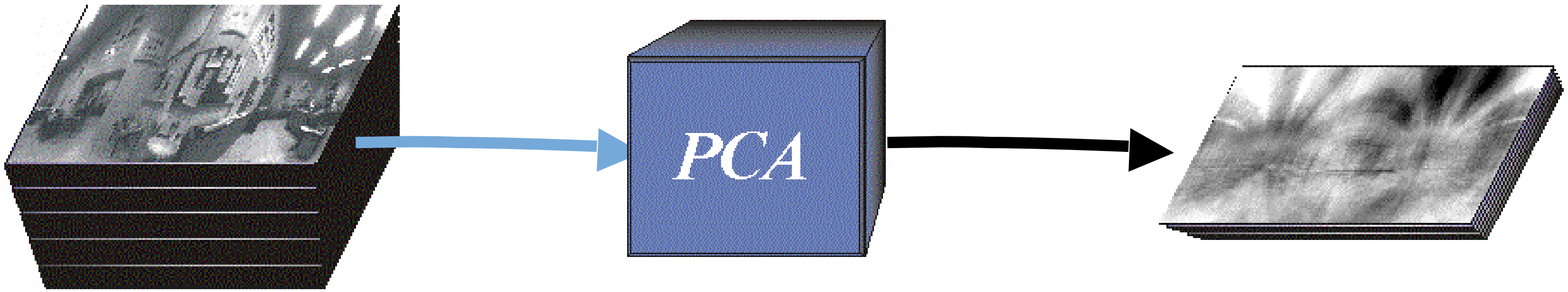
- $n << m$
- Compute the eigenvectors u'_i, $i = 0,\ldots,n-1$, of the inner product matrix

$$Q = X^\top X = \begin{bmatrix} \mathbf{x}_0^\top \\ \mathbf{x}_1^\top \\ \cdots \\ \mathbf{x}_{n-1}^\top \end{bmatrix} \begin{bmatrix} \mathbf{x}_0 & \mathbf{x}_1 & \ldots & \mathbf{x}_{n-1} \end{bmatrix}; \; Q \in \mathbb{R}^{n\times n}$$

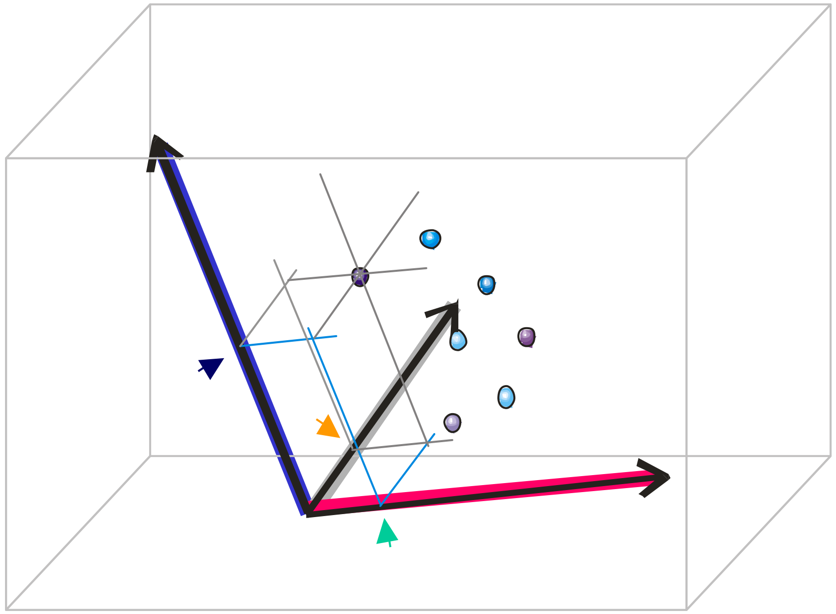
- The eigenvectors of $XX^\top$ can be obtained by using $XX^\top X\nu'_i = \lambda'_i X\nu'_i$:

$$\mathbf{u}_i = \frac{1}{\sqrt{\lambda'_i}} X\mathbf{u}'_i$$

Principal Component Analysis



Principal Component Analysis



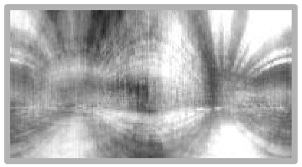
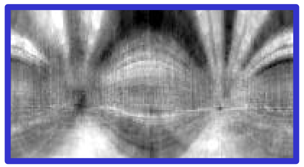
 $= q_1 \cdot$  $+ q_2 \cdot$  $+ q_3 \cdot$  $+ \ldots$

Image representation with PCA

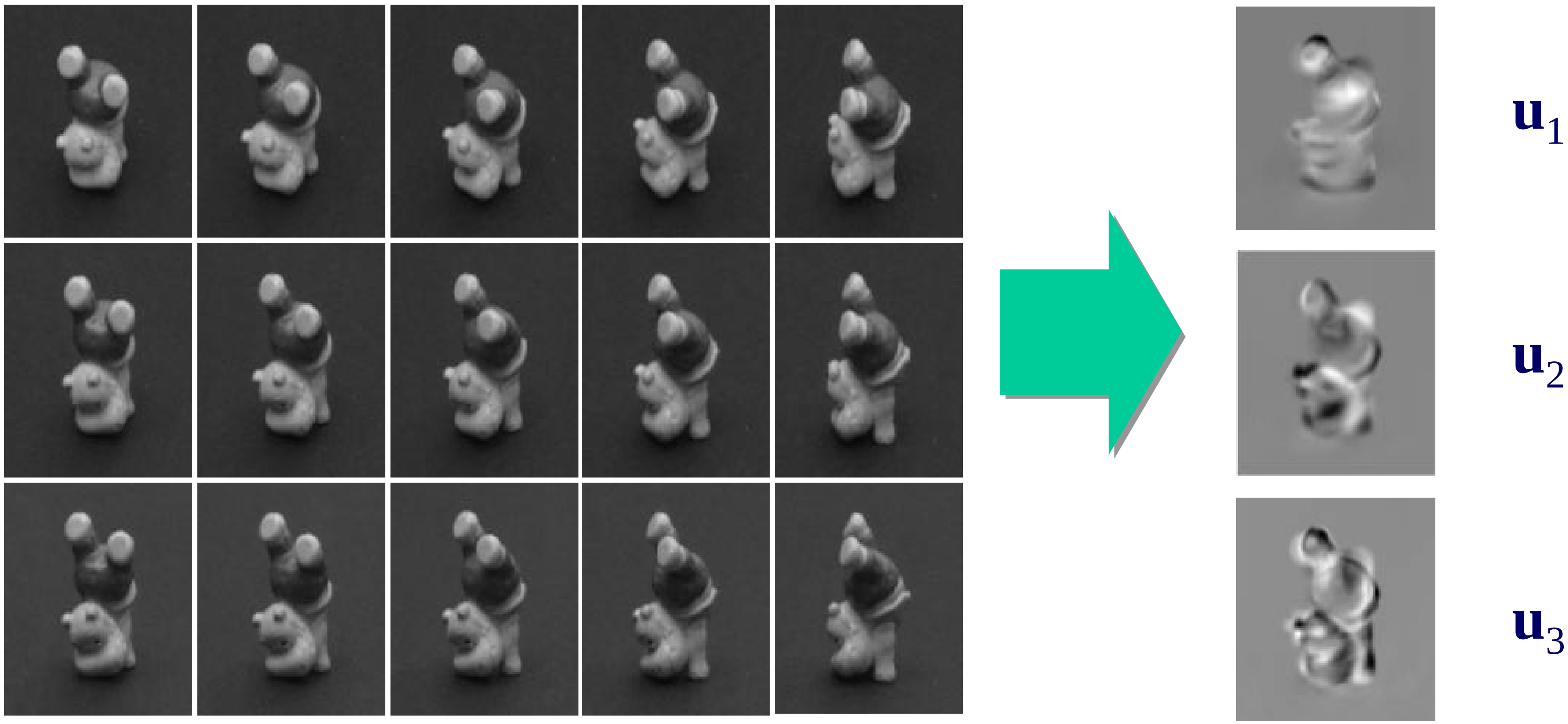
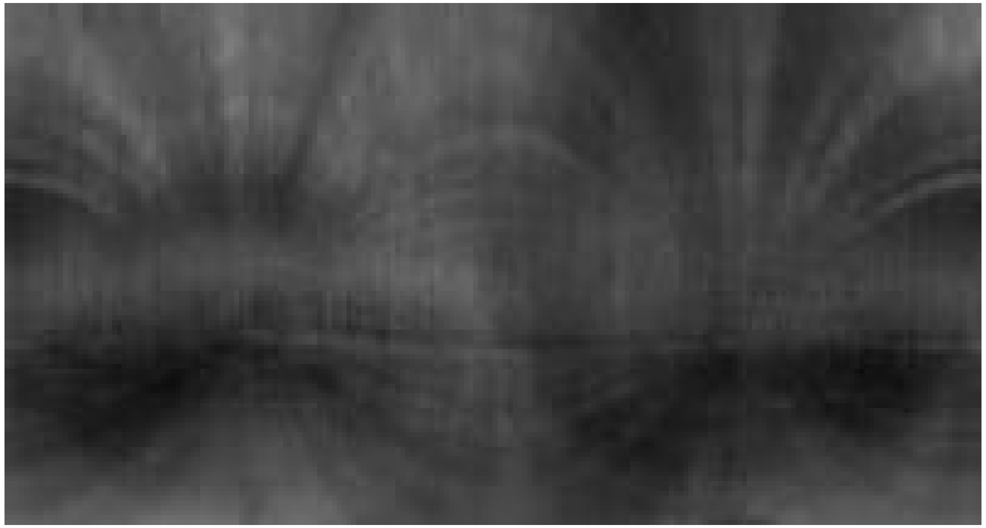


Image presentation with PCA



Properties PCA

- It can be shown that the mean square error between x_i and its reconstruction using only m principle eigenvectors is given by the expression :

$$\boxed{\sum_{j=1}^{N} l_j - \sum_{j=1}^{m} l_j = \sum_{j=m+1}^{N} l_j}$$

- PCA minimizes reconstruction error

- PCA maximizes variance of projection

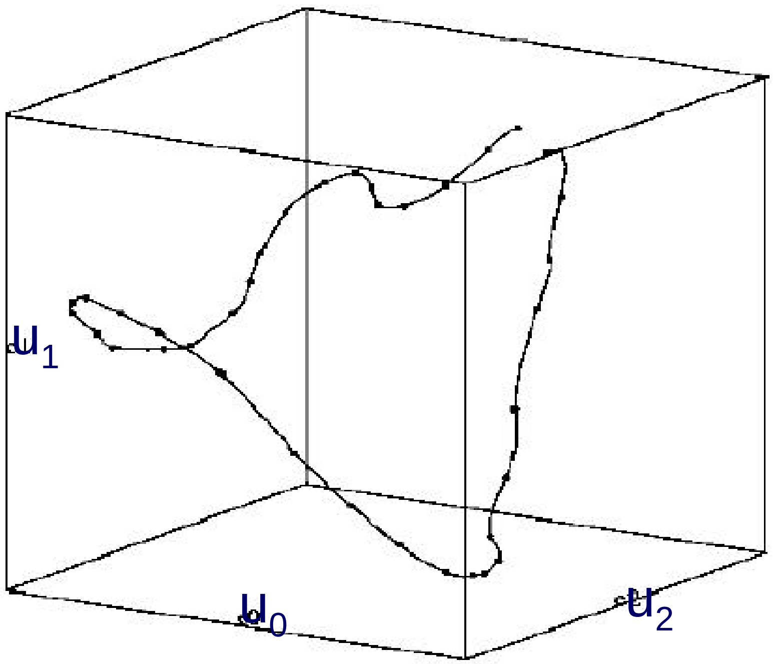
- Finds a more “natural” coordinate system for the sample data.

PCA for visual recognition and pose estimation

Objects are represented as coordinates in an n-dimensional eigenspace.

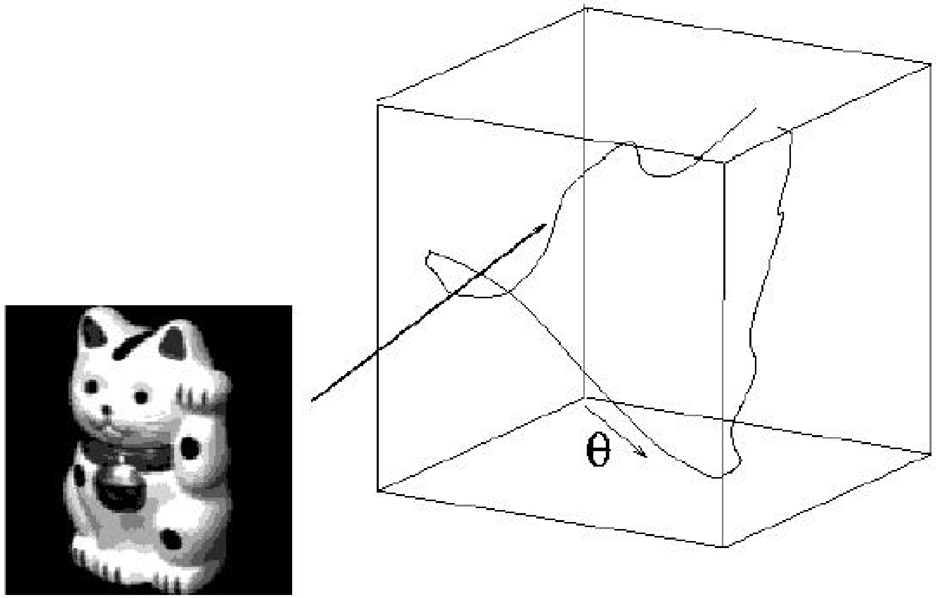
An example:

3-D space with points representing individual objects or a manifold representing **parametric eigenspace** (e.g., orientation, pose, illumination).



PCA for visual recognition and pose estimation

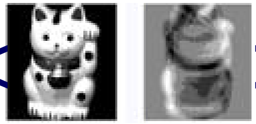
- Calculate coefficients
- Search for the nearest point (individual or on the curve)
- Point determines object and/or pose

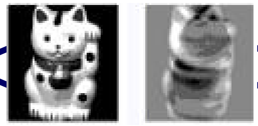
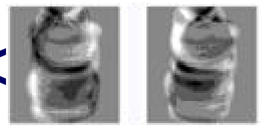
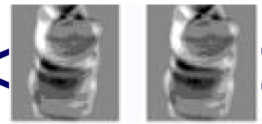


Calculation of coefficients

To recover q_i the image is projected onto the eigenspace

$$q_i(\mathbf{x}) = <\mathbf{x},\mathbf{u_i}> = \sum_{j=1}^{n-1} x_j u_{ij} \qquad 1 \le i \le k$$

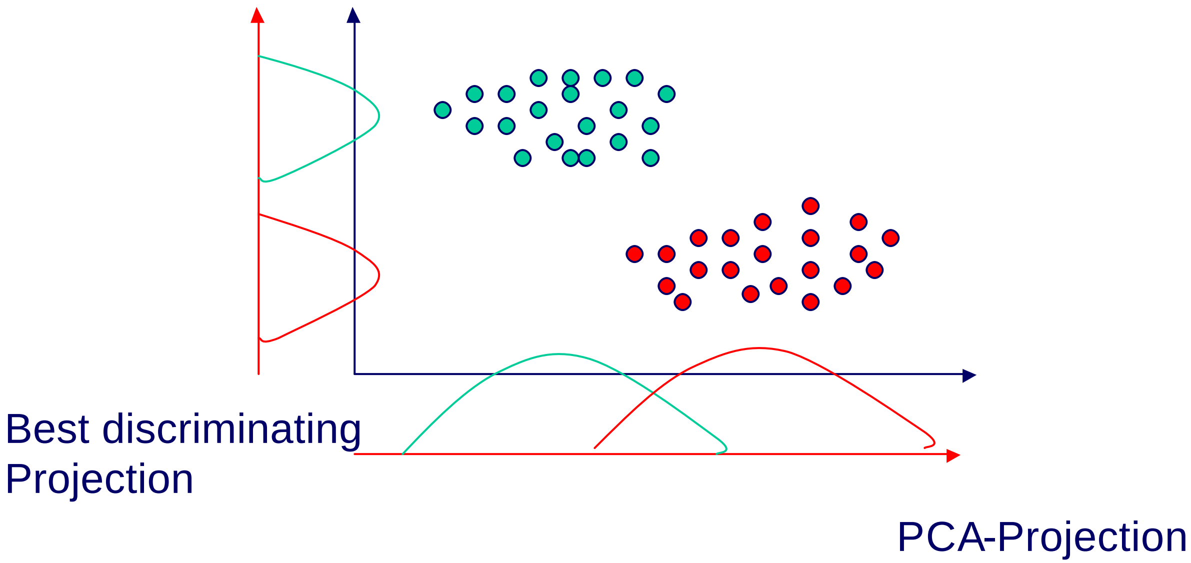
$<$  $> = q_1 <$  $> + q_2 <$  $> + \ldots = q_1$

$<$  $> = q_1 <$  $> + q_2 <$  $> + \ldots = q_2$

- Complete image **x** is required to calculate q_i.
- Corresponds to Least-Squares Solution

Linear Discriminant Analysis (LDA)

- **PCA minimizes projection error**



- **PCA is „unsupervised“ no information on classes is used**
- **Discriminating information might be lost**

LDA

- **Linear Discriminance Analysis (LDA)**
  - Maximize distance between classes
  - Minimize distance within a class

⇨ Fisher Linear Discriminance

$$r(\mathbf{w}) = \frac{\mathbf{w}^T \mathbf{S_B} \mathbf{w}}{\mathbf{w}^T \mathbf{S_W} \mathbf{w}}$$

LDA: Problem formulation

- n Sample images: $\{x_1, \ldots, x_n\}$
- c classes: $\{c_1, \ldots, c_c\}$
- Average of each class: $$m_i = \frac{1}{n_i} \sum_{x_k \in c_i} x_k$$
- Total average: $$m = \frac{1}{n} \sum_{k=1}^{N} x_k$$

LDA: Practice

- **Scatter of class i:**

$$\mathbf{S}_i = \sum_{x_k \in c_i} (\mathbf{x}_k - m_i)(\mathbf{x}_k - m_i)^T$$

- **Within class scatter:**

$$\mathbf{S}_W = \sum_{i=1}^{c} \mathbf{S}_i$$

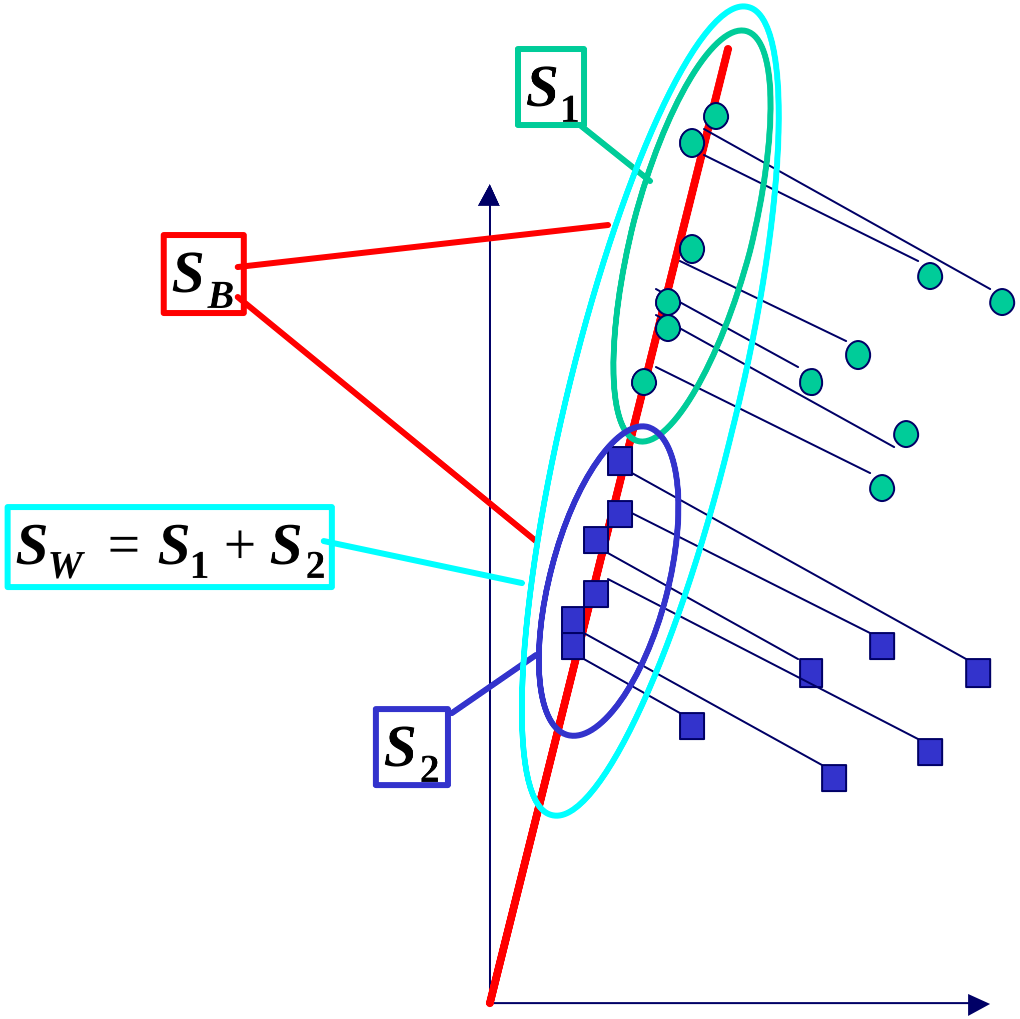
- **Between class scatter:**

$$\mathbf{S}_B = \sum_{i=1}^{c} |c_i| (m_i - m)(m_i - m)^T$$

- **Total scatter:**

$$\mathbf{S}_T = \mathbf{S}_W + \mathbf{S}_B$$

LDA



Good separation

LDA

- **Maximization of**

$$r(\mathbf{w}) = \frac{\mathbf{w}^T \mathbf{S_B} \mathbf{w}}{\mathbf{w}^T \mathbf{S_W} \mathbf{w}}$$

- **is given by solution of generalized eigenvalue problem**

$$\mathbf{S_B}\mathbf{w} = l\,\mathbf{S_w}\mathbf{w}$$

- **For the c-class case we obtain (at most) c-1 projections as the largest eigenvalues of**

$$\mathbf{S_B}\mathbf{w}_i = l\,\mathbf{S_w}\mathbf{w}_i$$

LDA

- **Example Fisherface of recognition Glasses/NoGlasses (Belhumeur et.al. 1997)**

Canonical Correlation Analysis (CCA)

- **Also „supervised" method but motivated by regression tasks, e.g. pose estimation.**

- **Canonical Correlation Analysis relates two sets of observations by determining pairs of directions that yield maximum correlation between these sets.**

- **Find a pair of directions (canonical factors)** $w_x \in \Re^p$, $w_y \in \Re^q$, **so that the correlation of the projections c =** $w_x^T x$ **and d =** $w_y^T y$ **becomes maximal.**

What is CCA?

Canonical Correlation $0 \leq r \leq 1$

$$r = \frac{E[cd]}{\sqrt{E[c^2]E[d^2]}} =$$

$$\frac{E[\mathbf{w}_x^T \mathbf{x}\mathbf{y}^T \mathbf{w}_y]}{\sqrt{E[\mathbf{w}_x^T \mathbf{x}\mathbf{x}^T \mathbf{w}_x]E[\mathbf{w}_y^T \mathbf{y}\mathbf{y}^T \mathbf{w}_y]}} =$$

Between Set Covariance Matrix

$$\frac{\mathbf{w}_x^T \mathbf{C}_{xy} \mathbf{w}_y}{\sqrt{\mathbf{w}_x^T \mathbf{C}_{xx} \mathbf{w}_x \mathbf{w}_y^T \mathbf{C}_{yy} \mathbf{w}_y}}$$

What is CCA?

- Finding solutions

$$\mathbf{w} = \begin{pmatrix} \mathbf{w}_x \\ \mathbf{w}_y \end{pmatrix} \qquad \mathbf{A} = \begin{pmatrix} 0 & \mathbf{C}_{xy} \\ \mathbf{C}_{yx} & 0 \end{pmatrix}, \mathbf{B} = \begin{pmatrix} \mathbf{C}_{xx} & 0 \\ 0 & \mathbf{C}_{yy} \end{pmatrix}$$

Rayleigh Quotient

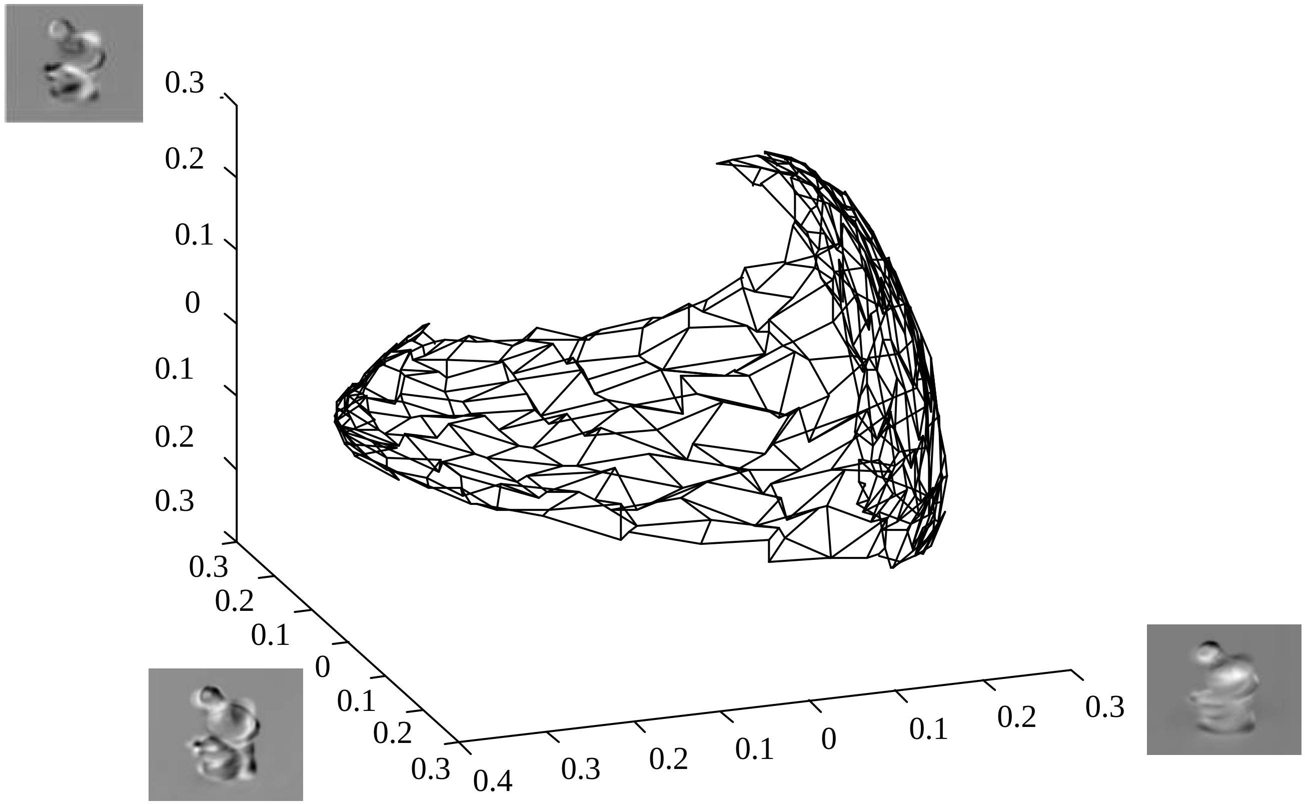
$$r = \frac{\mathbf{w}^T \mathbf{A} \mathbf{w}}{\mathbf{w}^T \mathbf{B} \mathbf{w}}$$

Generalized Eigenproblem

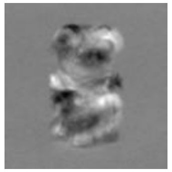
$$\mathbf{A}\mathbf{w} = m\mathbf{B}\mathbf{w}$$

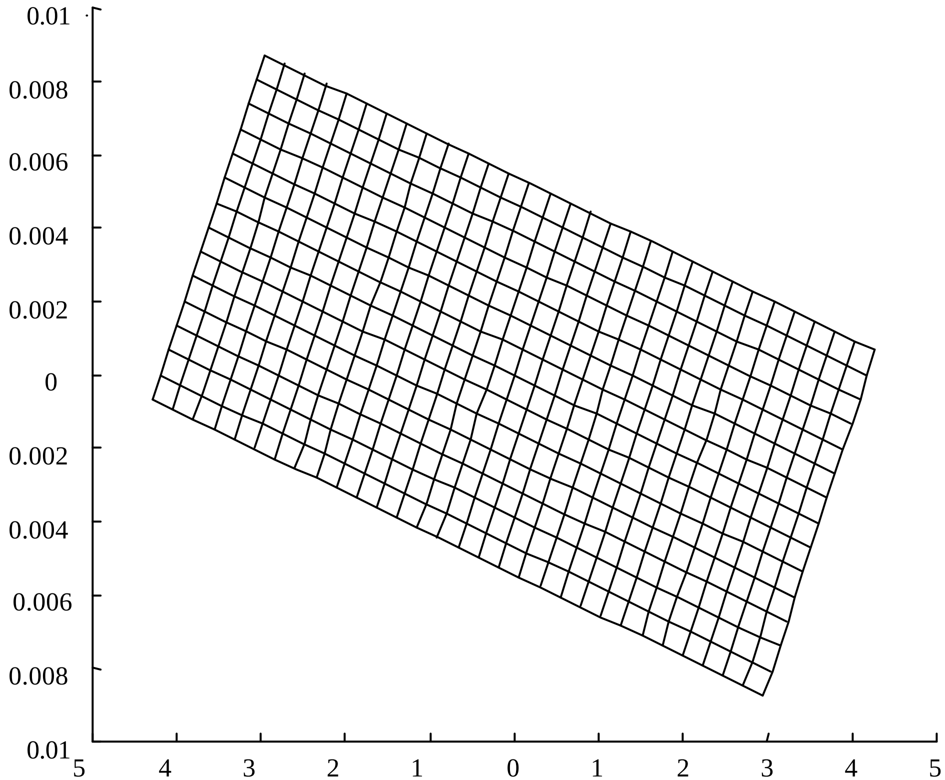
CCA Example

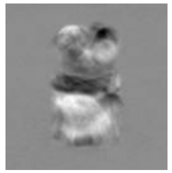
Parametric eigenspace obtained by PCA for 2DoF in pose



CCA representation
(projections of training images onto $\mathbf{w}_{x1}$, $\mathbf{w}_{x2}$)







Independent Component Analysis (ICA)

- ICA is a powerful technique from signal processing (Blind Source Separation)
- Can be seen as an extension of PCA
- PCA takes into account only statistics up to 2nd order
- ICA finds components that are statistically independent (or as independent as possible)

Independent Component Analysis (ICA)

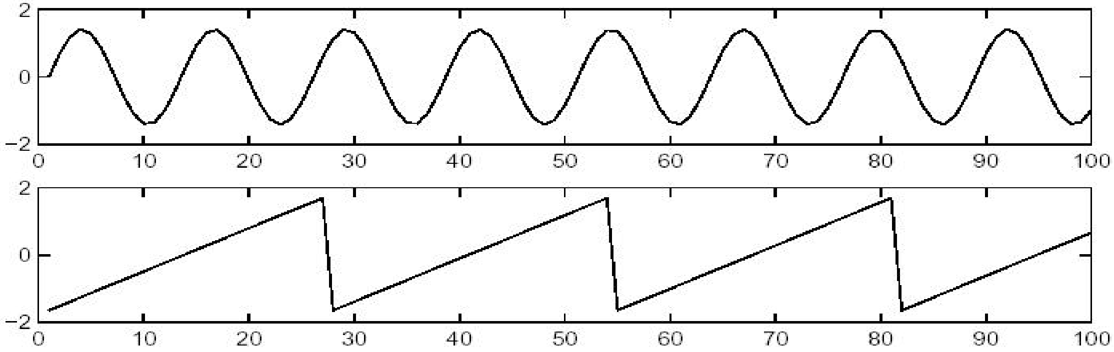
- m scalar variables $X=(x_1 \ldots x_m)^T$
- They are assumed to be obtained as linear mixtures of n sources $S=(s_1 \ldots s_n)^T$

$$\mathbf{X} = \mathbf{A}\mathbf{S}$$

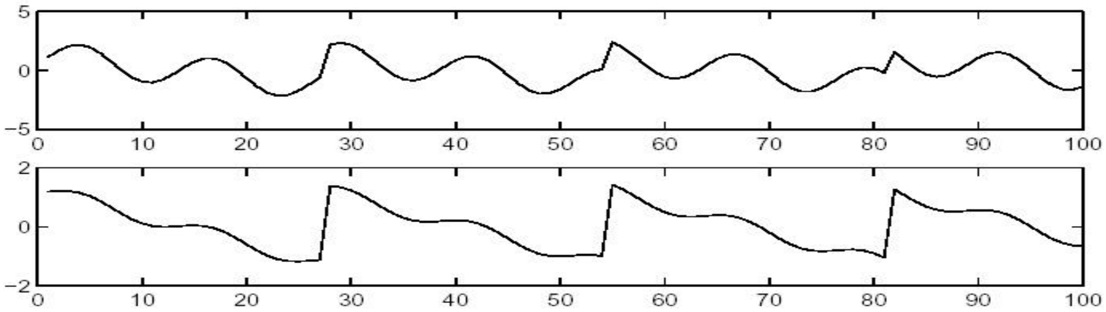
- Task: Given X find A, S (under the assumption that S are independent)

ICA Example

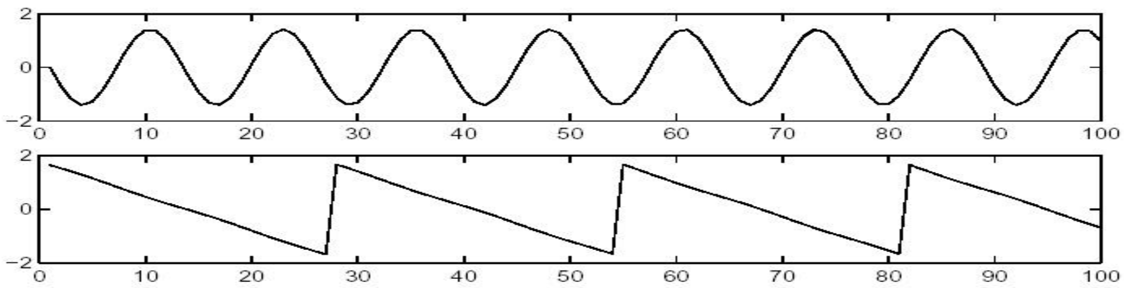
Original Sources



Mixtures

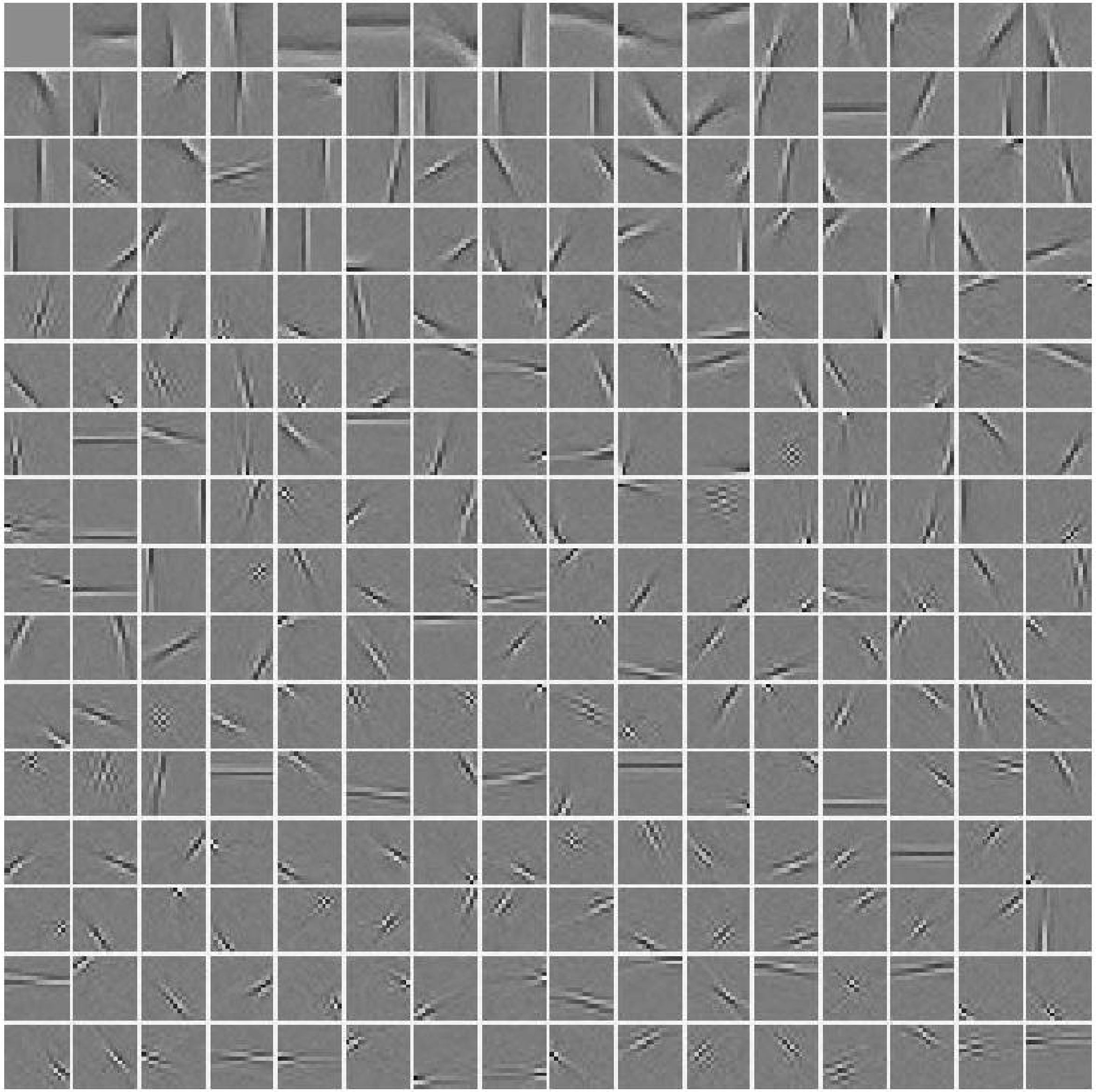


Recovered Sources



ICA Example

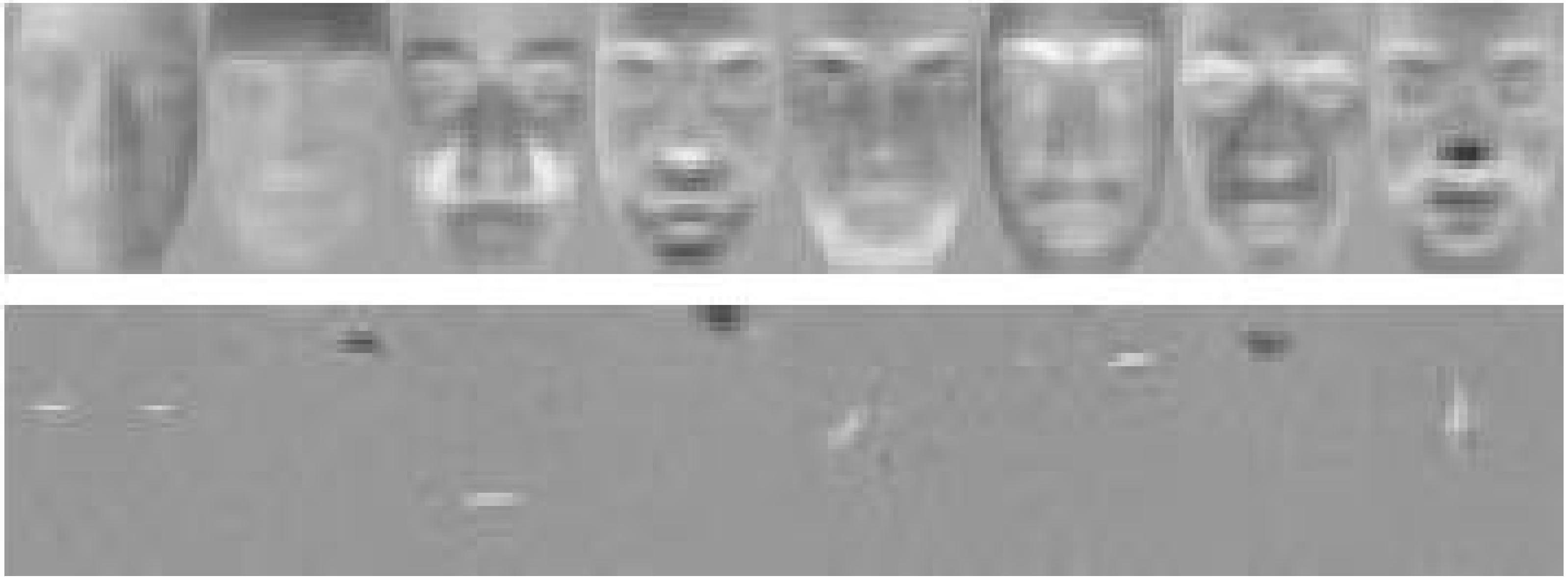
ICA basis obtained from 16x16 patches of natural images (Bell&Sejnowski 96)



Face Recognition using ICA

- **PCA vs. ICA on Ferret DB (Baek et.al. 02)**

PCA



ICA

Non-Negative Matrix Factorization (NMF)

- How can we obtain part-based representation?
- Local representation where parts are added
- E.g. learn from a set of faces the parts a face consists of, i.e. eyes, nose, mouth, etc.
- Non-Negative Matrix Factorization (Lee & Seung 1999) lead to part based representation

Matrix Factorization - Constraints

V* » *WH

- **PCA: *W* are orthonormal basis vectors**

$$W = [\vec{w}_1, \vec{w}_2, \Lambda\ , \vec{w}_n], \qquad \vec{w}_i \cdot \vec{w}_j = d_{ij}$$

- **VQ : *H* are unity vectors**

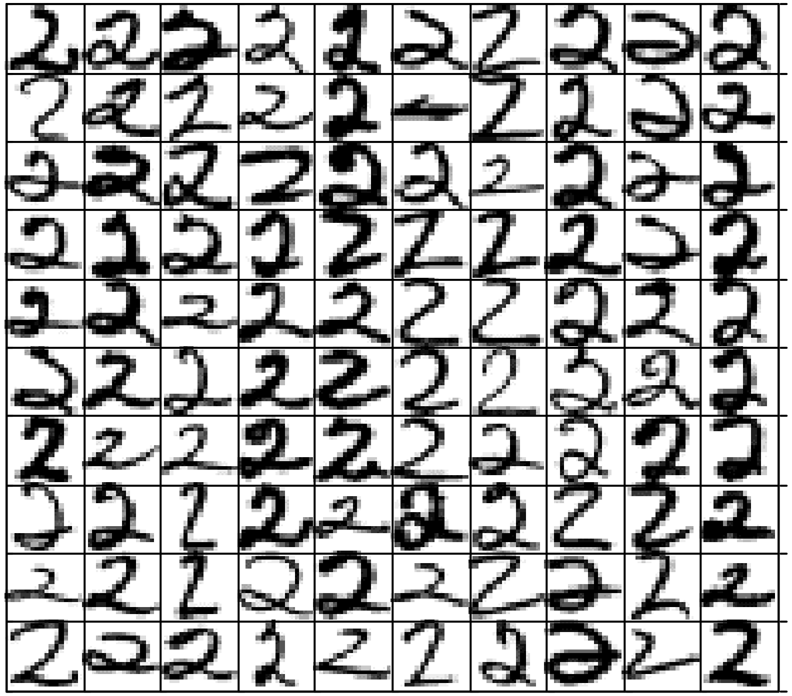
$$H = [\vec{h}_1, \vec{h}_2, \Lambda\ , \vec{h}_n], \qquad \vec{h}_j^{\,T} = [0,0,1,0,\Lambda\ ,0]$$

- **NMF: *V,W,H* are non-negative**

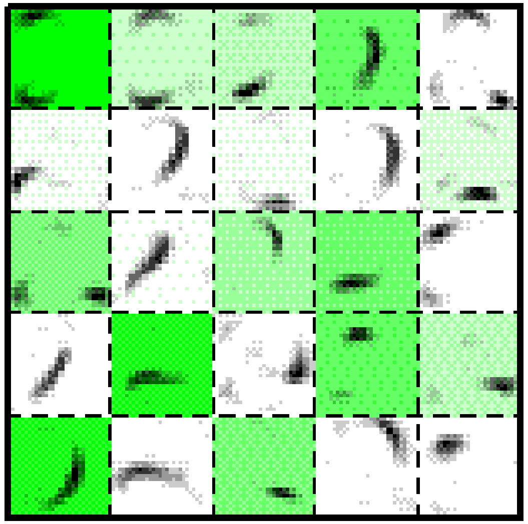
$$V_{ij}, W_{ij}, H_{ij} \geq 0 \qquad \forall i, j$$

Learning

Find basis images from the training set

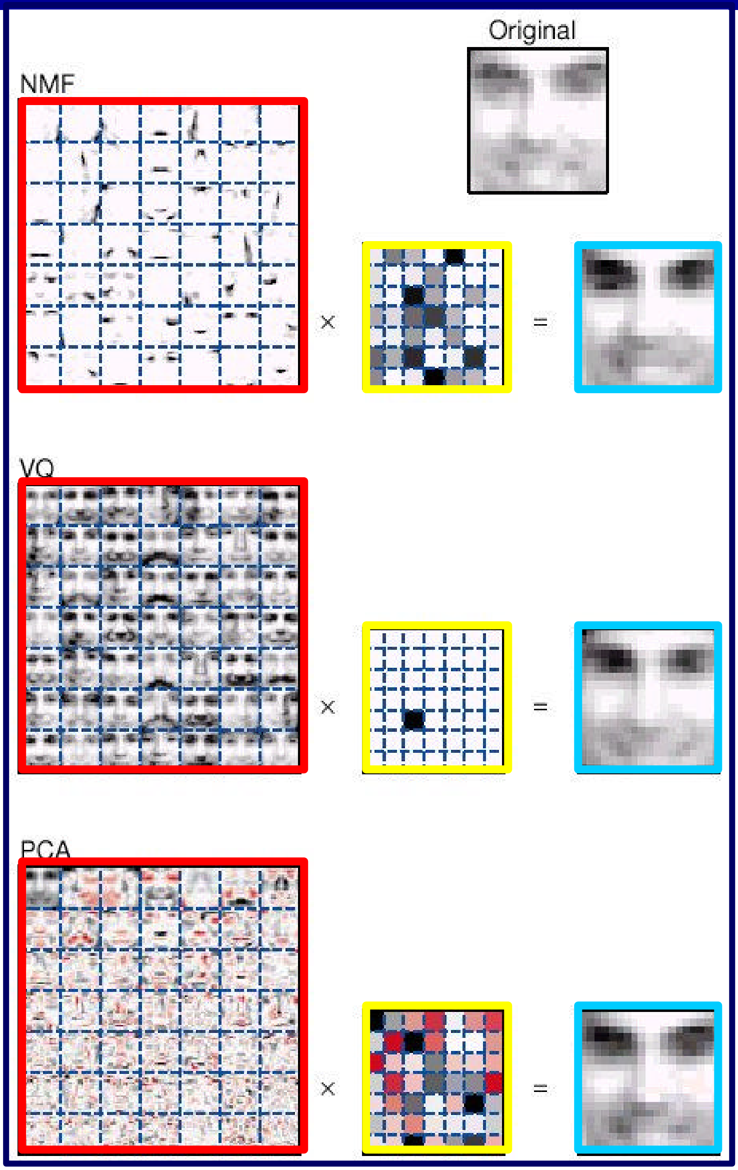


Training images



Basis images

Face features



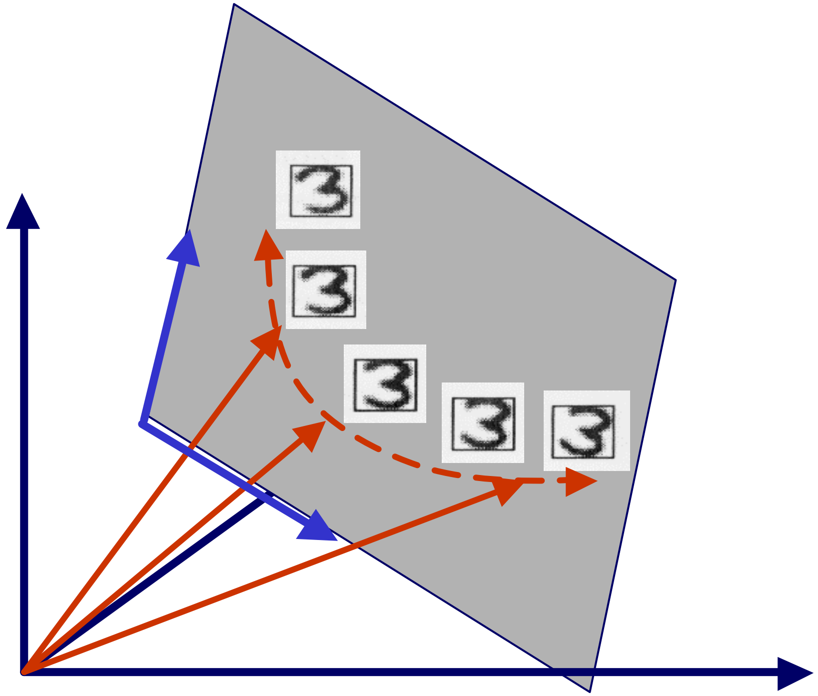
Basis images

Encoding (Coefficients)

Reconstructed image

Kernel Methods

- All presented methods are linear



- Can we generalize to non-linear methods in a computational efficient manner?

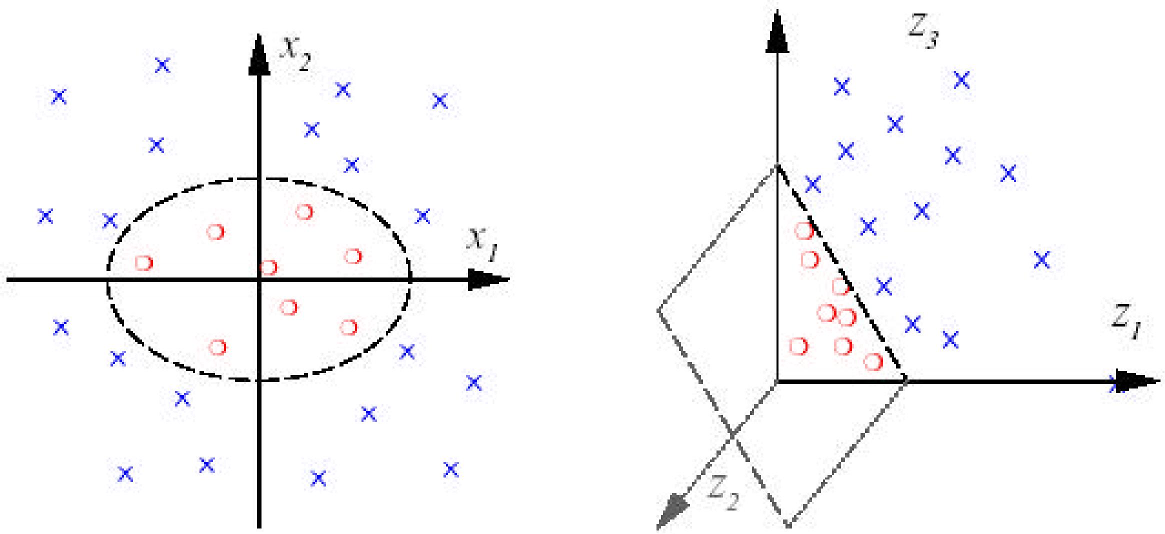
Kernel Methods

- **Kernel Methods are powerful methods (introduced with Support Vector Machines) to generalize linear methods**

BASIC IDEA:

1. **Non-linear mapping of data in high dimensional space**
2. **Perform linear method in high-dimensional space**

   ➡ **Non-linear method in original space**



Outline Part 2

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- Robust representations and recognition
- Robust recognition using PCA
- Scale invariant recognition using PCA
- Illumination insensitive recognition
- Representations for panoramic images
- Incremental building of eigenspaces
- Multiple eigenspaces for efficient representation
- Robust building of eigenspaces
- Research issues

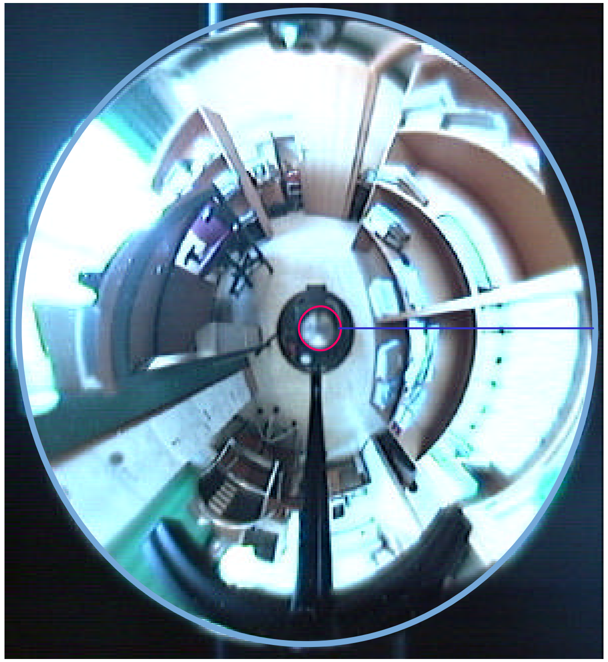
Appearance-based approaches

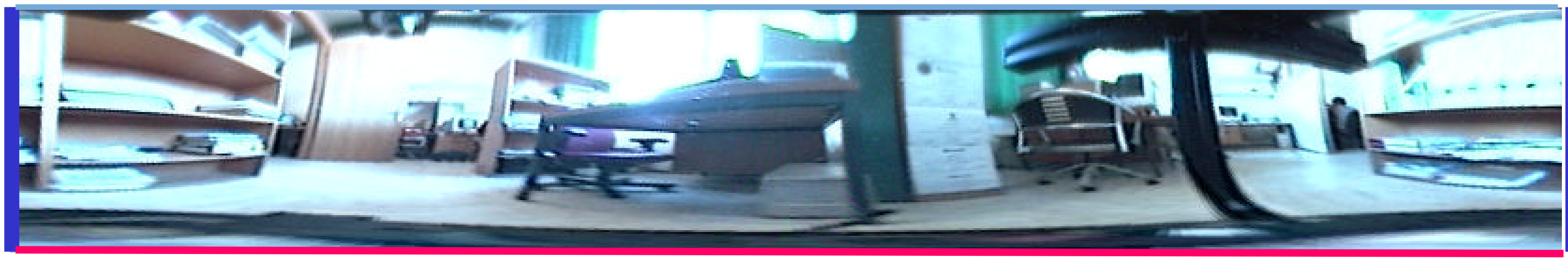
A variety of successful applications:

- Human face recognition e.g. [Turk & Pentland]
- Visual inspection e.g. [Yoshimura & Kanade]
- Visual positioning and tracking of robot manipulators, e.g. [Nayar & Murase]
- Tracking e.g., [Black & Jepson]
- Illumination planning e.g., [Murase & Nayar]
- Image spotting e.g., [Murase & Nayar]
- Mobile robot localization e.g., [Jogan & Leonardis]
- Background modeling e.g., [Oliver, Rosario & Pentland]

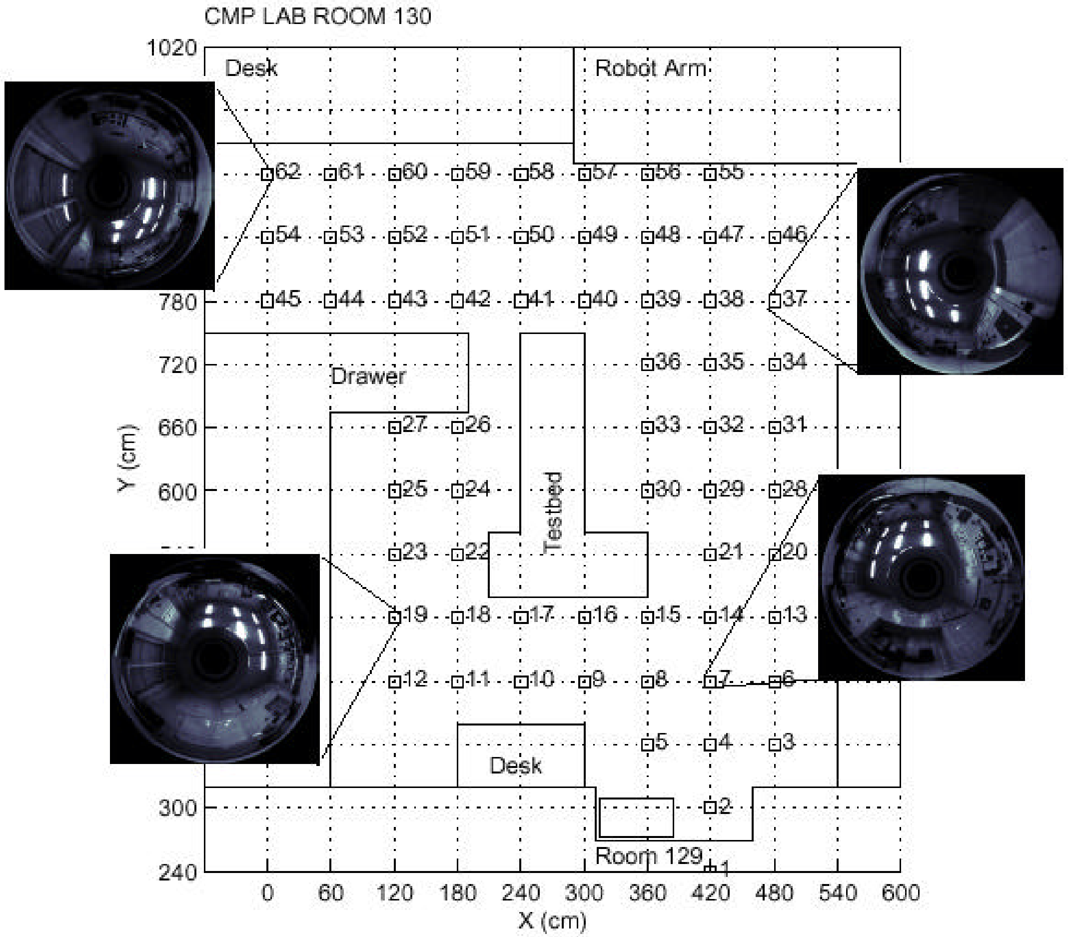
Mobile Robot

Panoramic image





Environment map



- ♦ **environments are represented by a large number of views**
- ♦ **localisation = recognition**

Compression with PCA

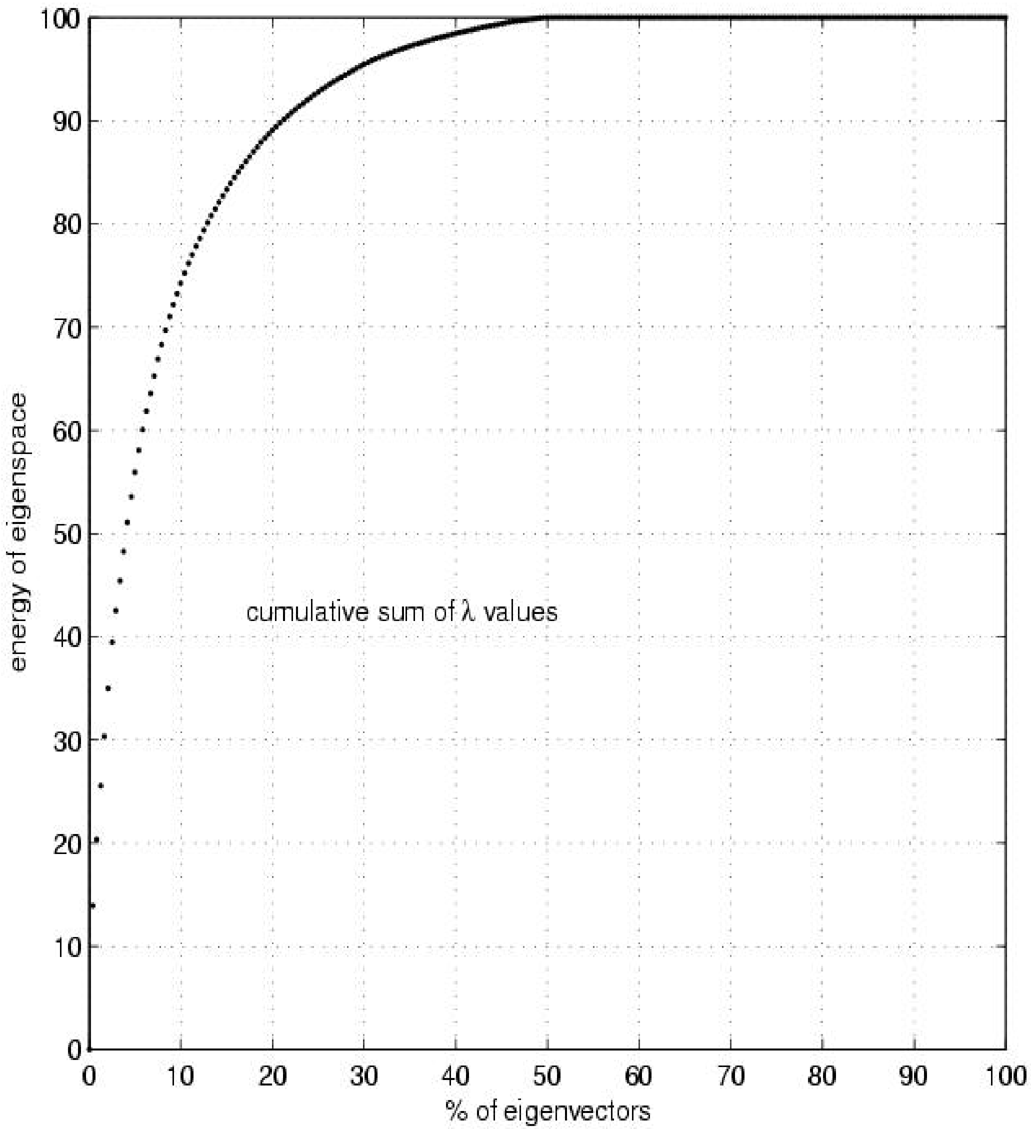
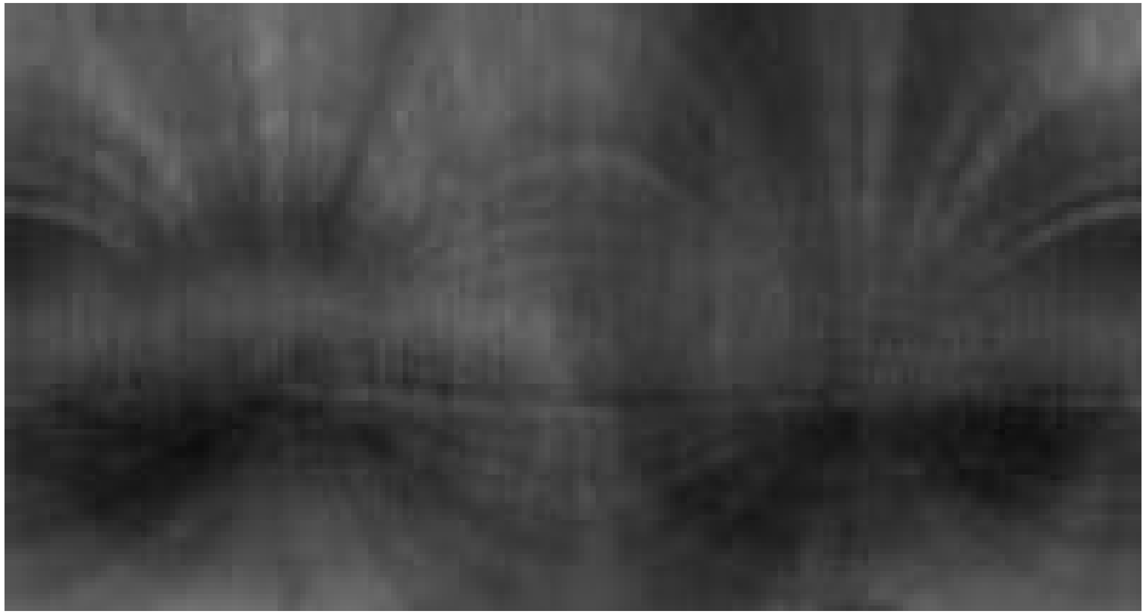
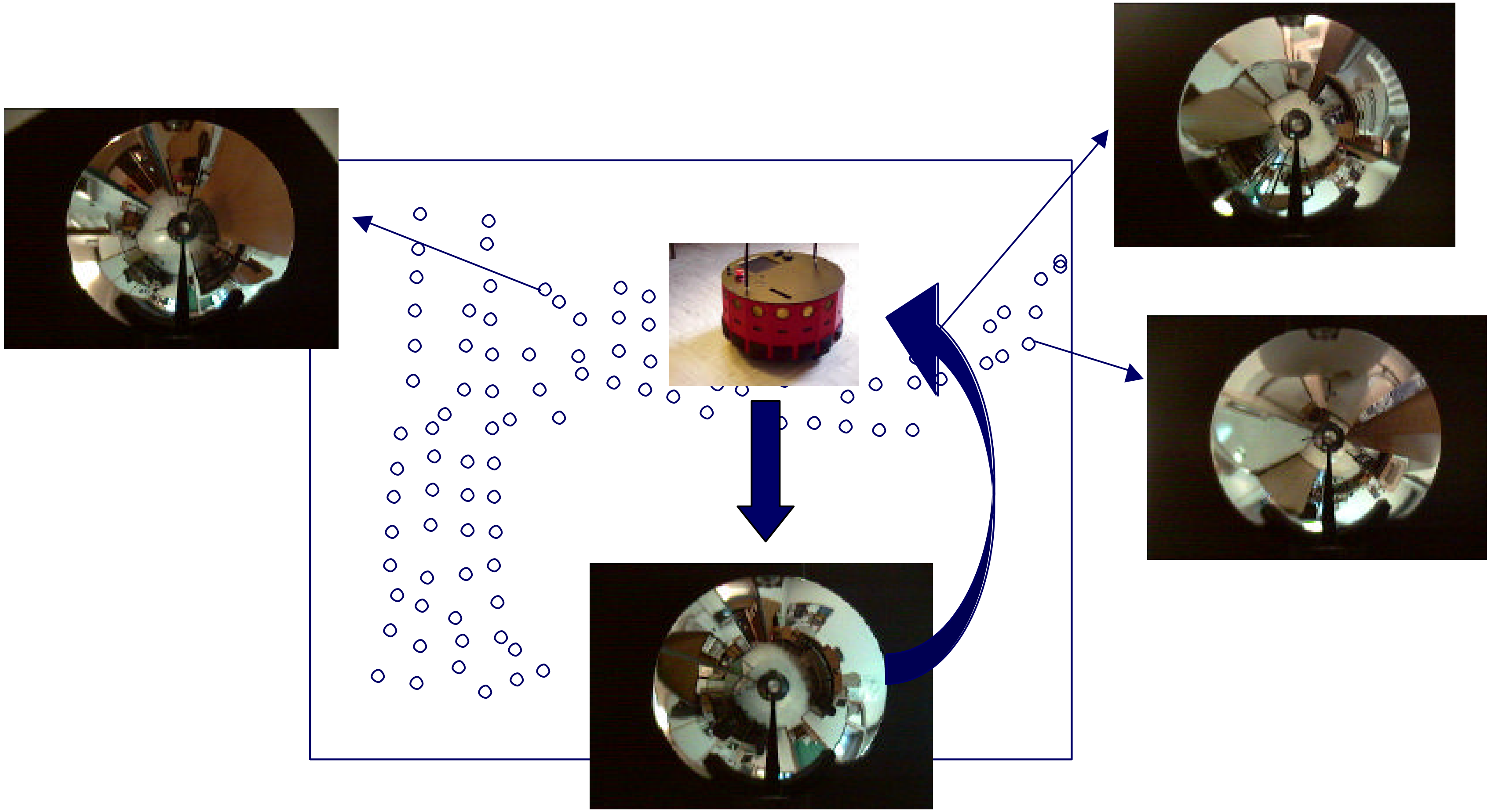


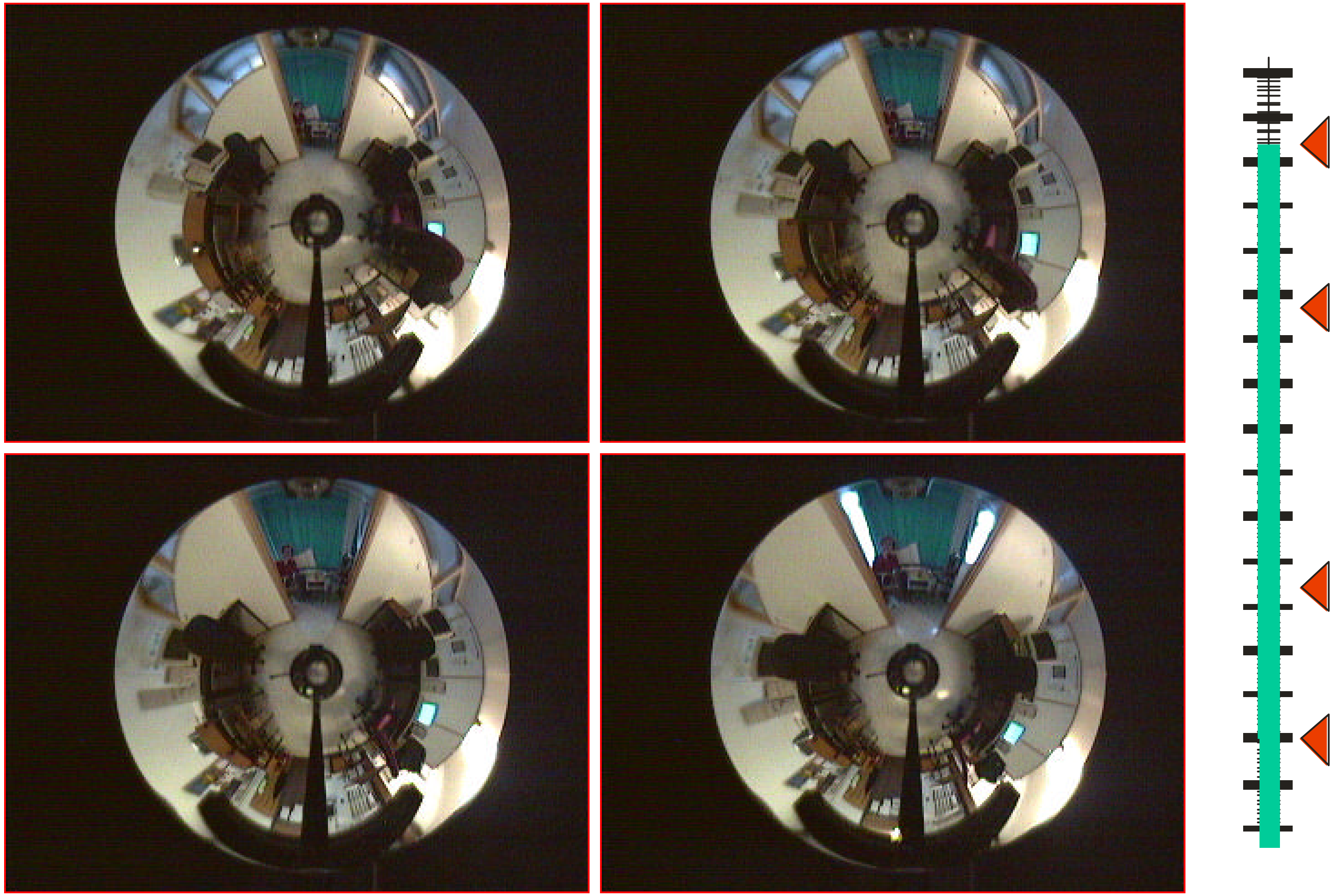
Image representation with PCA



Localisation



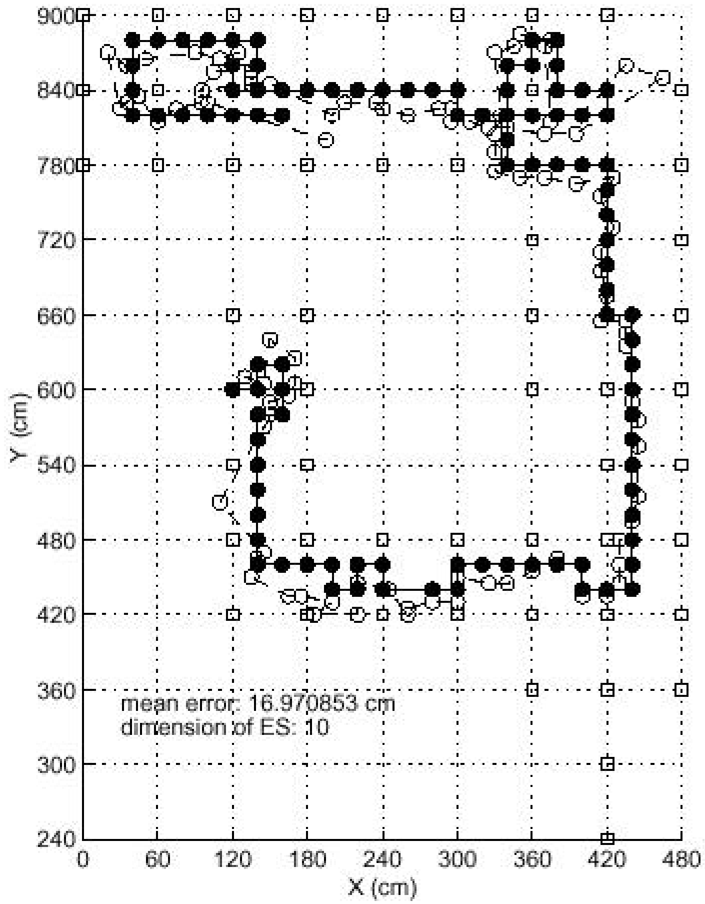
Distance vs. similarity



Robot localisation

- Interpolated hyper-surface represents the memorized environment.
- The parameters to be retrieved are related to position and orientation.
- Parameters of an input image are obtained by scalar product.

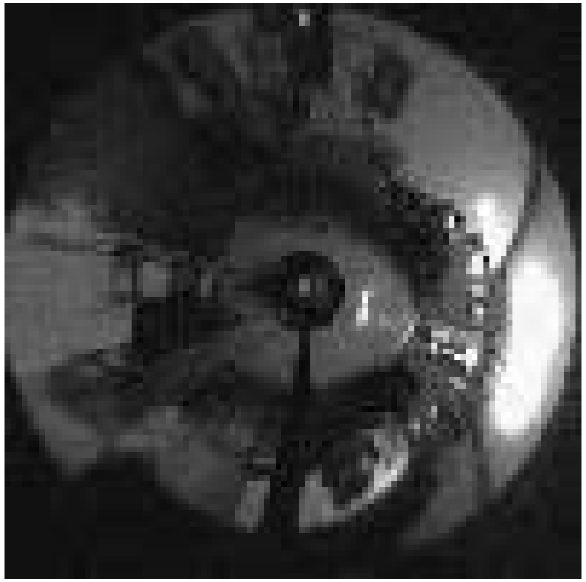
Localisation



Enhancing recognition and representations

- **Occlusions, varying background, outliers**
  - Robust recognition using PCA
- **Scale variance**
  - Multiresolution coefficient estimation
  - Scale invariant recognition using PCA
- **Illumination variations**
  - Illumination insensitive recognition
- **Rotated panoramic images**
  - Spinning eigenimages
- **Incremental building of eigenspaces**
- **Multiple eigenspaces for efficient representations**
- **Robust building of eigenspaces**

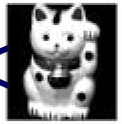
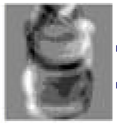
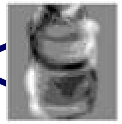
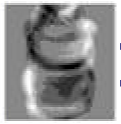
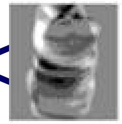
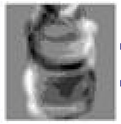
Occlusions

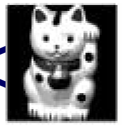
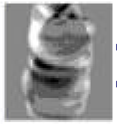
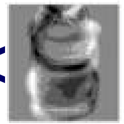
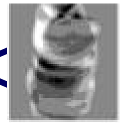
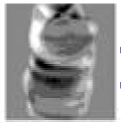


Calculation of coefficients

To recover q_i the image is projected onto the eigenspace

$$q_i(\mathbf{x}) = <\mathbf{x},\mathbf{u_i}> = \sum_{j=1}^{n-1} x_j u_{ij} \qquad 1 \le i \le k$$

$<$   $> = q_1 <$   $> + q_2 <$   $> + \ldots = q_1$

$<$   $> = q_1 <$   $> + q_2 <$   $> + \ldots = q_2$

- Complete image **x** is required to calculate q_i.
- Corresponds to Least-Squares Solution

Non-robustness

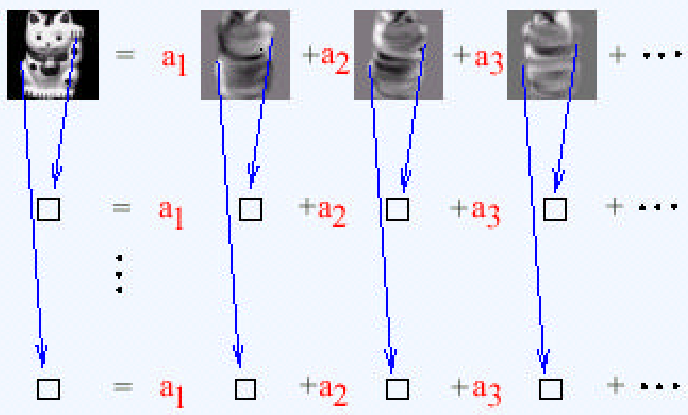
Drawbacks: Prone to errors caused by

- occlusions (outliers)
- cluttered background

Original | Occluded | Reconstruction

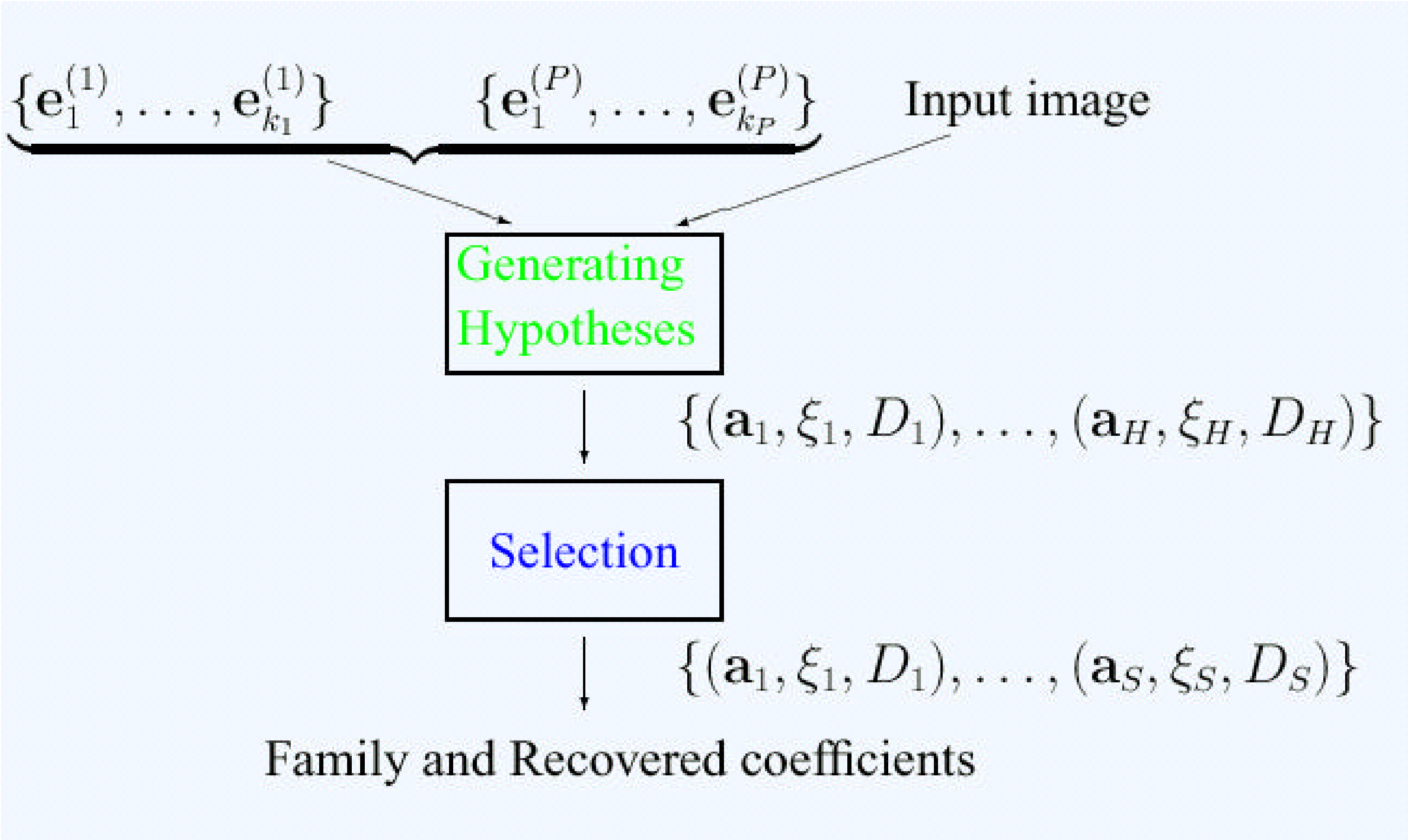
Robust method

- **Major idea:** Instead of using the standard approach we:
  - **subset of data points** $\longrightarrow$ linear system of equations
  - **Robust** solution of this system of equations
  - Perform **multiple hypotheses**



- Hypothesize-and-test paradigm
- Competing hypotheses are subject to a **selection** procedure based on the MDL principle.

Robust algorithm



Selection

Three cases:

1. **One object**: Select best match (c_{ii})
2. Multiple **non-overlapping** objects: Select local maximum (c_{ii})
3. Multiple **overlapping** objects: MDL-criterion:

The objective function:

$$F(\mathbf{h}) = \mathbf{h}^T \mathbf{C} \mathbf{h}$$

$\mathbf{h}^T = [h_1, h_2, \ldots, h_R]$ — set of hypotheses

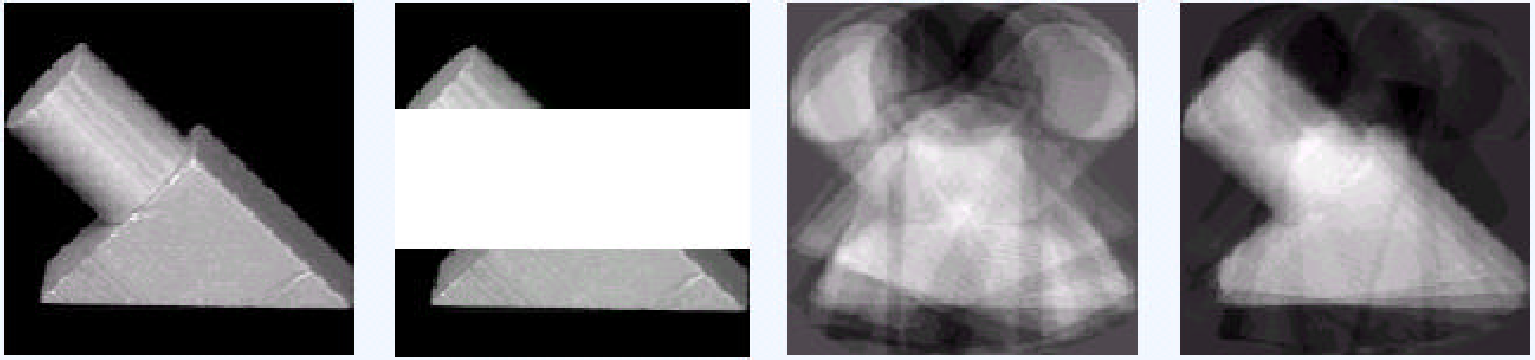
Diagonal terms of $\mathbf{C}$ express the cost-benefit value for hypothesis i

$$c_{ii} = \mathrm{K}_1 |D_i| - \mathrm{K}_2 ||\vec{\xi}_i||_{D_i} - \mathrm{K}_3 N_i$$

Off-diagonal terms handle overlapping hypotheses

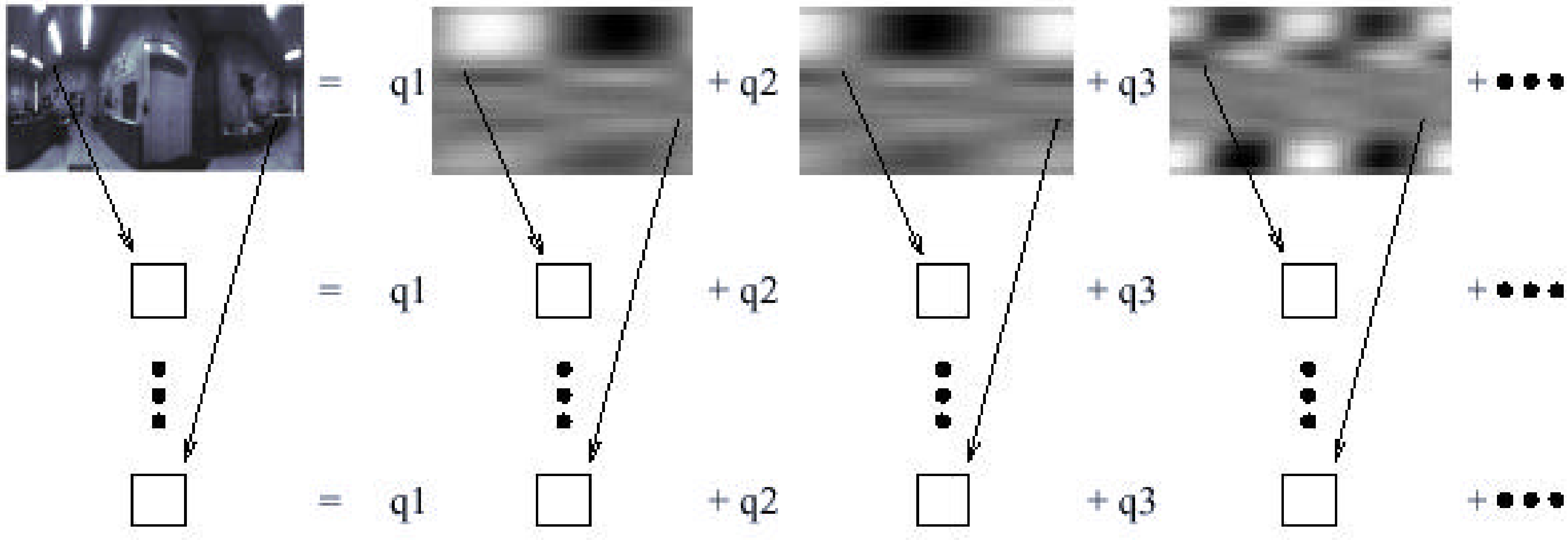
$$c_{ij} = \frac{-\mathrm{K}_1 |D_i \cap D_j| + \mathrm{K}_2 \xi_{ij}}{2}$$

Robust recovery of coefficients

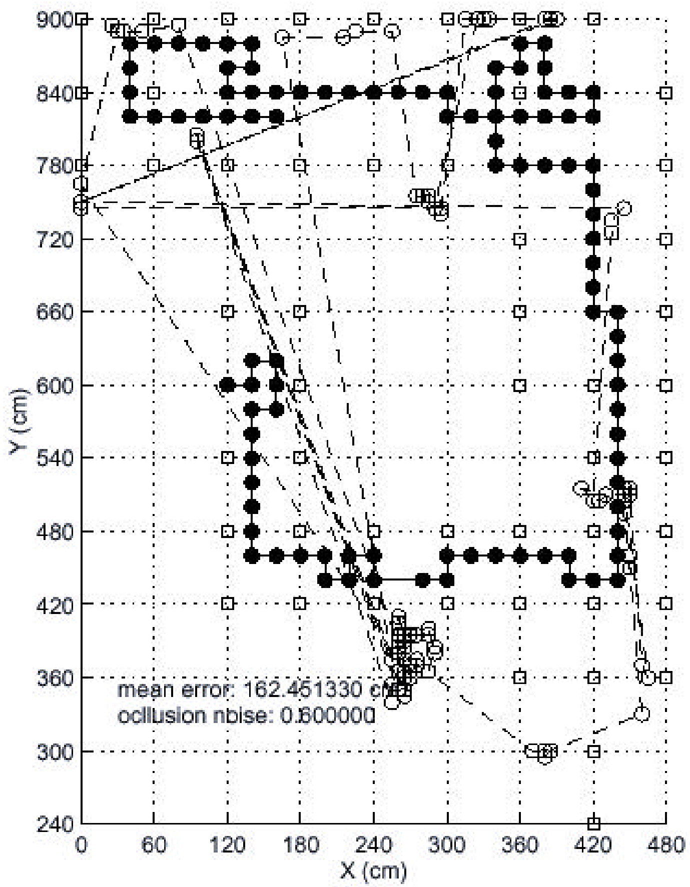


Original	Occluded	Standard	Robust

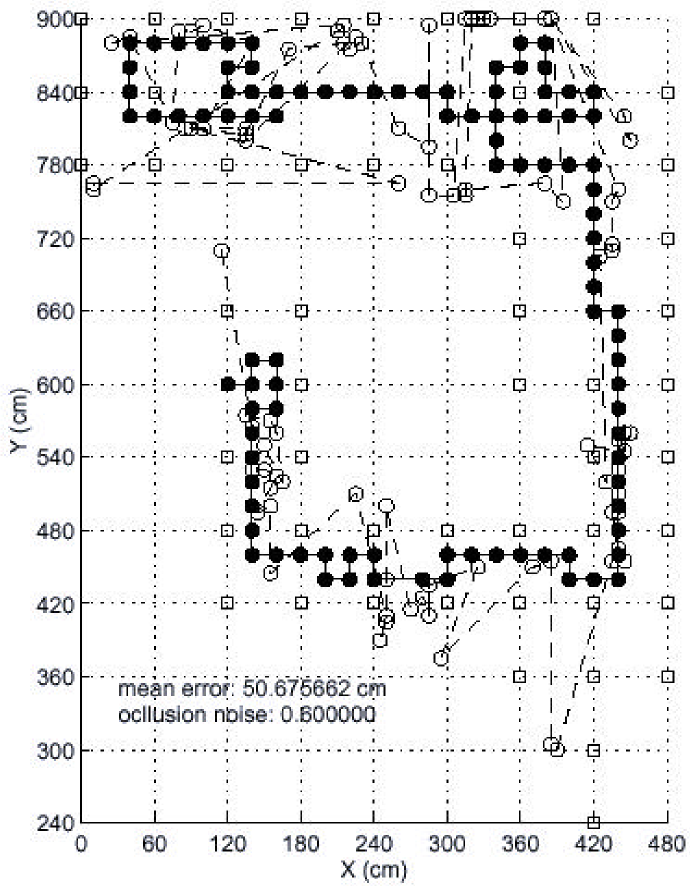
Robust localisation under occlusions



Robust localisation at 60% occlusion



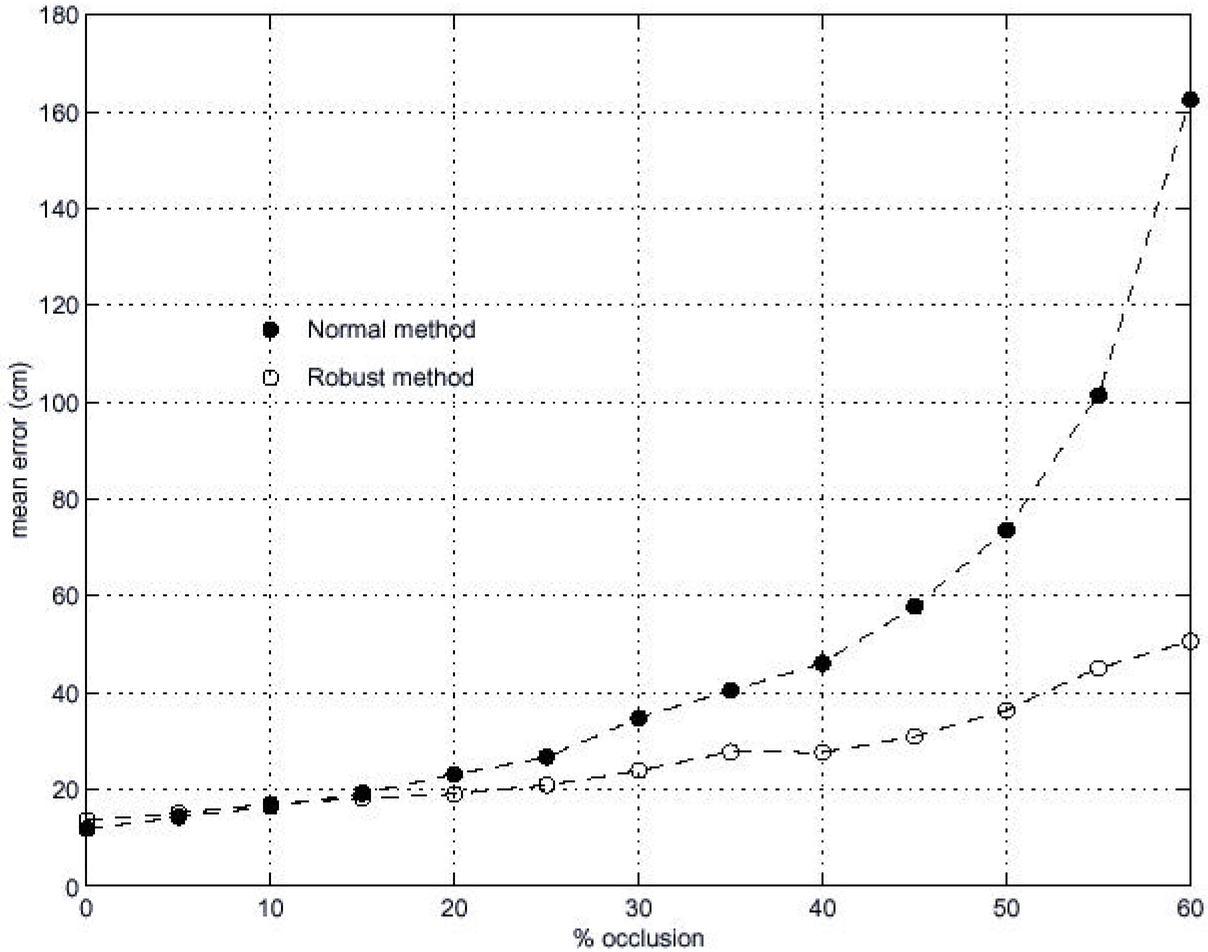
Standard approach



Robust approach

Mean error of localisation

- **Mean error of localisation with respect to % of occlusion**



Illumination insensitive recognition

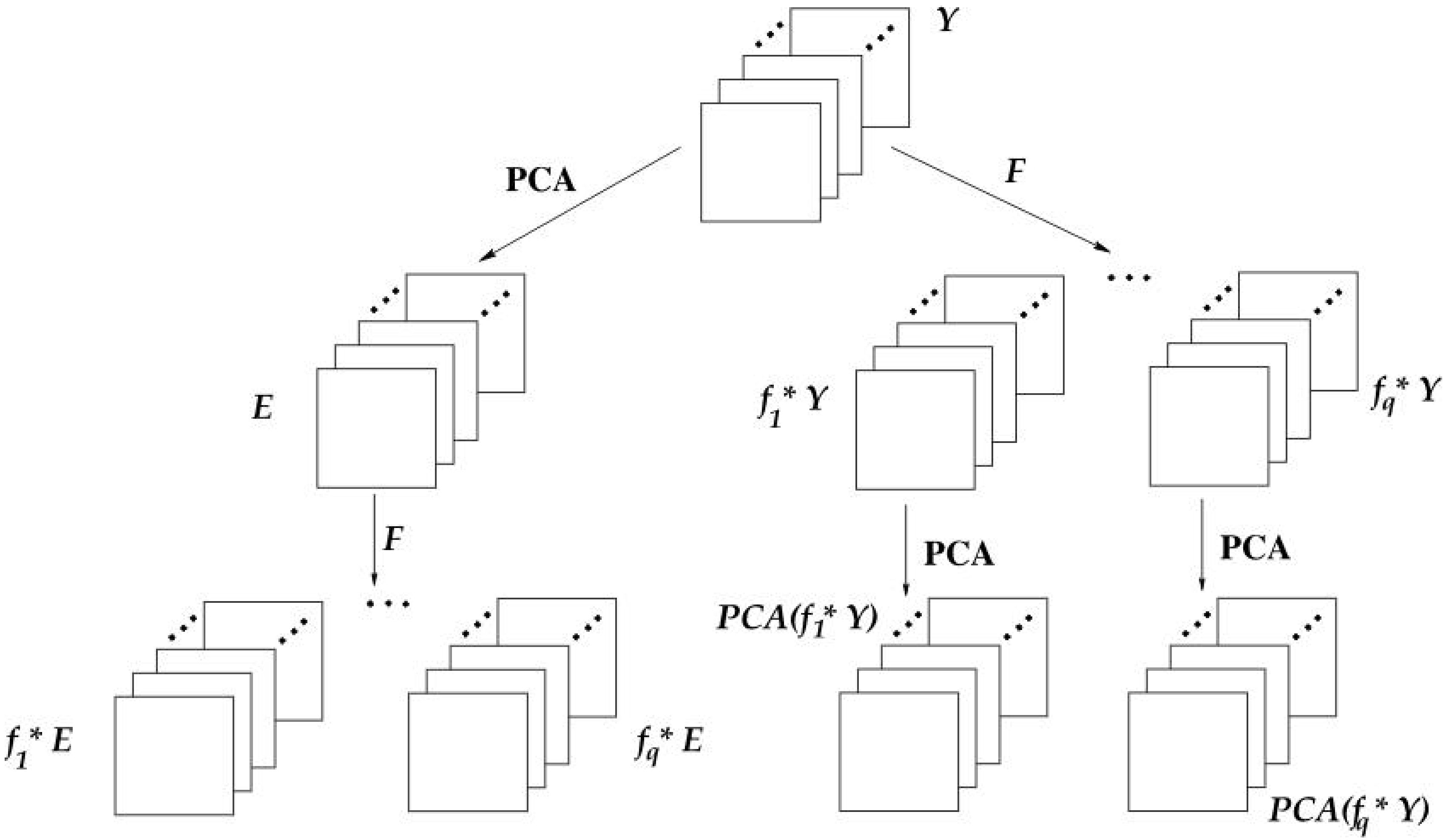
- Recognition of objects under varying illumination
  - **global illumination changes**
  - **highlights**
  - **shadows**
- Dramatic effects of illumination on objects appearance
- Training set under a single ambient illumination

Illumination insensitive recognition

Our Approach

- *Global* eigenspace representation
- *Local* gradient based filters
- Efficient combination of global and local representations
- Robust coefficient recovery in eigenspaces

Eigenspaces and filtering



Filtered eigenspaces

$$y_{r_i} = \sum_{j=1}^{n} q_j e_{j r_i} \quad 1 \leq i \leq k$$

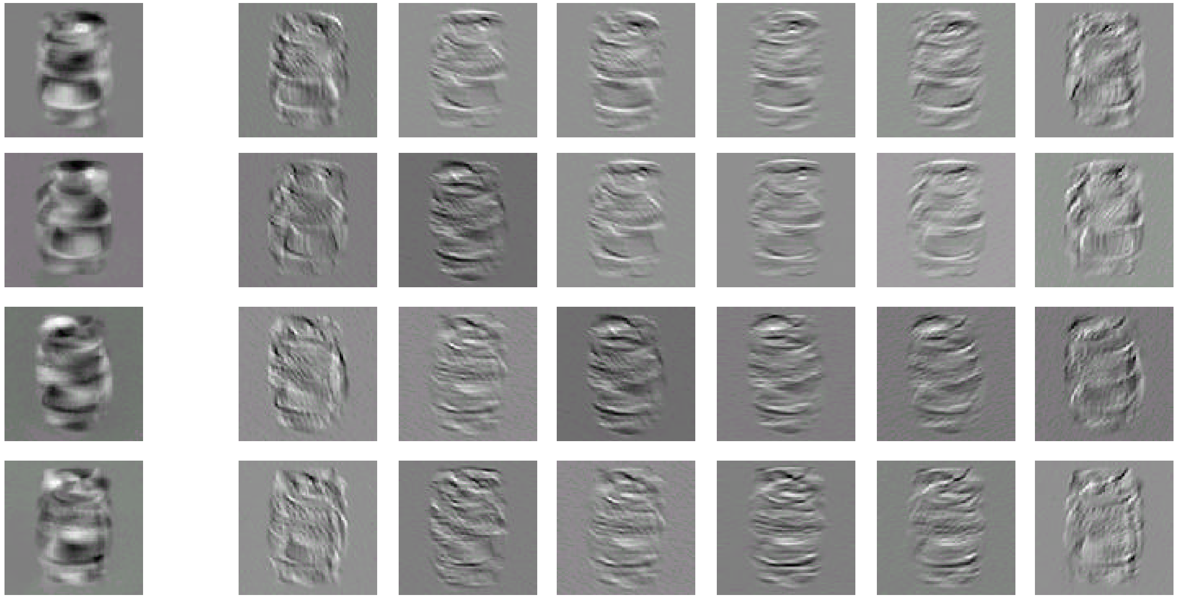
$$(f * x)(r) = \sum_{i=1}^{p} q_j (f * e_i)(r)$$

Global illumination

↓

Gradient-based filters

Steerable filters [Simoncelli]

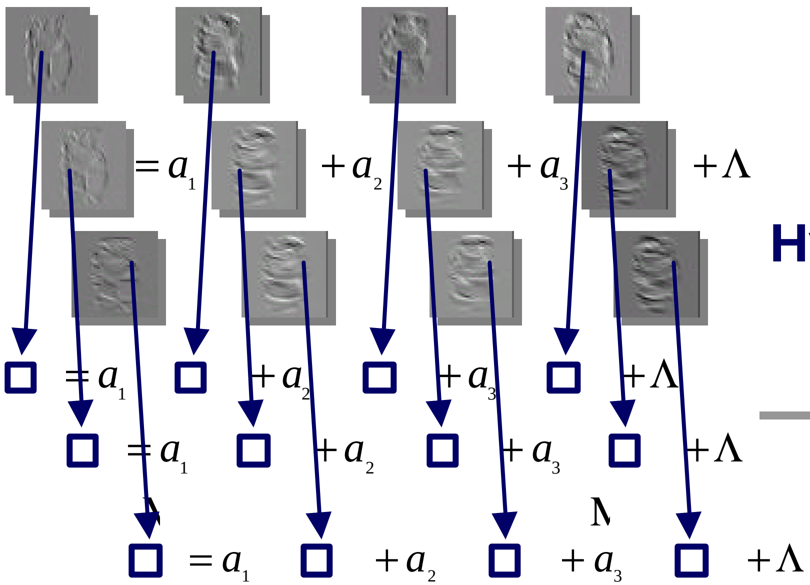


Robust coefficient recovery

Highlights and shadows

Robust coefficient recovery

Robust solution of linear equations

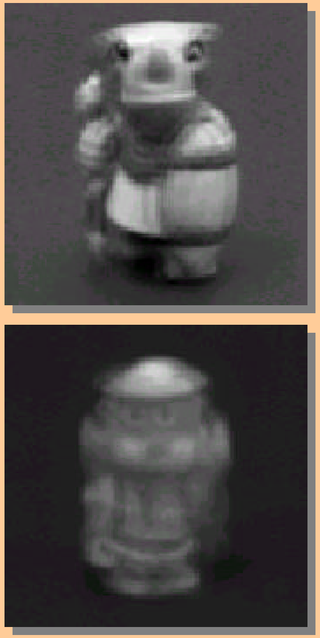


Hypothesize
&
Select

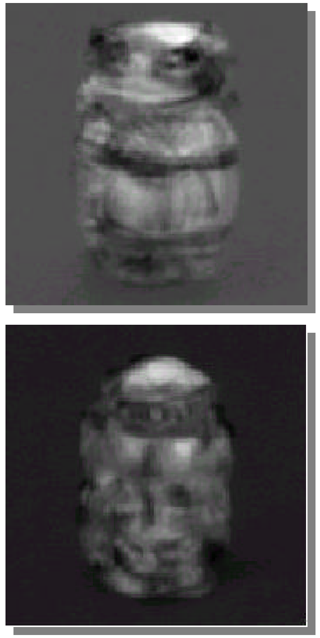
Experimental results

Test images

Our approach



Standard method



→ Demo

Experimental results

Robust filtered method - all eigenvectors used

obj.	1	2	3	4	5	%	ang.
1	360	0	0	0	0	100.0	5.25
2	0	308	16	0	0	95.1	10.55
3	0	0	504	0	0	100.0	1.05
4	19	4	3	332	2	92.2	3.37
5	15	2	17	0	578	94.4	3.34
avg.						96.4	4.19

Standard method - all eigenvectors used

obj.	1	2	3	4	5	%	ang.
1	141	0	14	26	179	39.2	10.50
2	0	254	62	5	3	78.4	18.90
3	0	4	317	0	183	62.9	3.47
4	23	6	38	249	44	69.2	7.11
5	3	1	51	0	557	91.0	6.82
avg.						70.3	8.53

Research issues

- Comparative studies (e.g., LDA versus PCA, PCA versus ICA)
- Robust learning of other representations (e.g. LDA, CCA)
- Integration of robust learning with modular eigenspaces
- Local versus Global subspace represenations
- Combination of subspace representations in a hierarchical framework

Further readings

- Recognizing objects by their appearance using eigenimages (SOFSEM 2000, LNCS 1963)
- Robust recognition using eigenimages (CVIU 2000, Special Issue on Robust Methods in CV)
- Illumination insensitive eigenspaces (ICCV 2001)
- Mobile robot localization under varying illumination (ICPR 2002)
- Eigenspace of spinning images (OMNI 2000, ICPR 2000, ICAR 2001)
- Incremental building of eigenspaces (ICRA 2002, ICPR 2002, CogVis 2002)
- Multiple eigenspaces (Pattern Recognition 2002)
- Robust building of eigenspaces (ECCV 2002)

- Special issue of Pattern Recognition on Kernel and Subspace Methods in Computer Vision (Guest Editors A. Leonardis and H. Bischof), to appear in 2003.