1050 Midterm 1, Feb 7, 2006. NAME:___________________________. Grade:____ (______/100)

Closed books. Individual test. Do not look at other people's work. Please write legibly. Use the back of sheet if needed.

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1) Let P(x,y) and Q(x,y) be two propositions with variables x and y. For example, Q(x,y) could be $x \vee y$. Explain the difference between the implication $P(x,y) \rightarrow Q(x,y)$, the bi-conditional $P(x,y) \leftrightarrow Q(x,y)$, and the logical equivalence $P(x,y) \Leftrightarrow Q(x,y)$.

The implication $P(x,y) \rightarrow Q(x,y)$ is: *a proposition that is true when Q(x,y) is true or P(x,y) is false for given values of the variables x and y.*

The biconditional $P(x,y) \leftrightarrow Q(x,y)$ is: *a proposition that is true when the truth values of P(x,y) and Q(x,y) are identical for given truth values of the variables x and y.*

The logical equivalence $P(x,y) \Leftrightarrow Q(x,y)$ is: *not a proposition. It states that $P(x,y) \leftrightarrow Q(x,y)$ is a tautology, i.e., P(x,y) and Q(x,y) have identical truth tables for the variables x and y.*

: points out of 9

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2) Express each system specification below using the propositions: p="Access is granted", q="The user has entered a valid password", r="The user has paid the subscription fee".

"If the user has entered a valid password or paid the subscription fee, then access is granted.": ____ $(q \vee r) \rightarrow p$____

"Access is granted if the user has not entered a valid password but has paid the subscription fee.": ____ $(\neg q \wedge r) \rightarrow p$____

"If the user paid the subscription fee, then it is not necessary to enter the password to gain access.": ____$r \rightarrow p$____

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3) Use terms such as "conjunction" to label the following propositions:

$(p \rightarrow q)$ is ____*an implication*____. **$(p \vee q)$ is** ____*a disjunction*_____

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4) Mark with a ✓ all correct equivalences:

$(p \rightarrow q) \Leftrightarrow (q \rightarrow p)$: ____. **$(p \rightarrow q) \Leftrightarrow (\neg q \rightarrow \neg p)$:** _✓_. **$(p \rightarrow q) \Leftrightarrow (q \rightarrow \neg p)$:** ____. **$(p \rightarrow q) \Leftrightarrow (p \vee \neg q)$:** ____.
$(p \rightarrow q) \Leftrightarrow (\neg p \vee q)$: _✓_. **$(p \rightarrow q) \Leftrightarrow (\neg p \wedge q)$:** ____. **$(p \rightarrow q) \Leftrightarrow \neg(p \wedge \neg q)$:** _✓_. **$(p \rightarrow q) \Leftrightarrow (p \wedge \neg q)$:** ____.
$(p \rightarrow q) \Leftrightarrow (p)$: ____. **$(p \rightarrow q) \Leftrightarrow (q)$:** ____. **$(p \rightarrow q) \Leftrightarrow (\neg p)$:** ____. **$(p \rightarrow q) \Leftrightarrow (\neg q)$:** ____.

: points out of 9

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5) Translate "$\forall x \forall z\, (x<z \rightarrow \exists y\; x<y<z)$" into English.

"If z is greater than x, there exist a y greater than x and smaller than z."

: points out of 5

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6) Let P(x) be the statement "will fail this class", where the universe of discourse consists of all students. Use quantifiers to write "If a student fails the class, he/she will not be the only one."

$\forall x\; P(x) \rightarrow \exists y\; y \neq x \wedge P(y)$

: points out of 5

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7) Let P(x) denote "x is my friend". Let Q(x) denote "x is perfect". Translate into a logical expression using quantifiers and connectives.

"Not all my friends are perfect": $\exists x\; P(x) \wedge \neg Q(x)$ *, or equivalently* $\neg(\forall x\; P(x) \rightarrow Q(x))$

"My friends are all perfect": $\forall x\; P(x) \rightarrow Q(x)$

: points out of 6

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8) Prove that $((p \oplus q) \oplus r) \Leftrightarrow (p=(q=r))$.

You can show that they have the same truth tables. Both expressions are true when exactly one or 3 of the variables are true.

: points out of 9

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9) T(x,y) means "x has emailed y ". Use quantifiers to express:
"Each student has received email from at least two different students. "

$\forall z\, \exists x \exists y\; x \neq y \wedge (z \neq x) \wedge (z \neq y) \wedge (T(x,z) \wedge T(y,z)$

: points out of 5

10) The pseudo-code below has a bug. Indicate how to fix it. *(Be precise.)*

```
for (i=0; i<n-1; i++) {
        for (j=0; j<n-1-i; j++) {
                if (T[j+1] < T[j]) {        # insert " temp=T [ j+1 ] ; "
                        T[j+1]=T[j];
                        T[j]=T[j+1];        # replace by " T [ j ] = temp ; '
                        }; }; };
```

: points out of 5

11) Write in pseudocode, an algorithm for reporting the number of occurrences of the most frequent value in the table T[n][n].
What is the complexity of your algorithm? _______ $\Theta(n^4)$ _______

```
int max=1;
for (i=0; i<n; i++) {  for (j=0; j<n; j++) {
    int count=0;
    for (m=0; m<n; m++) {  for (k=0; k<n; k++) {
        if (T[i][j] == T[m][k]) { count++; };  }; };
    if (count>max) {max=count};
    }; };
```

: points out of 12

12) Provide an example of an NP complete problem and a definition of NP.

To find an assignment of truth values that satisfies a Boolean expression of n variables (makes it true).
This is an NP (Nondeterministic Polynomial time) problem because no polynomial time algorithm is known to solve it, yet a solution may be checked in polynomial time.

: points out of 6

13) Let A and B be two sets. What can you conclude about A and/or B form each one of the following statements? Provide a proof for each one of your answers.

A–B = B–A : A=B (Assume $p \in A-B$. Then $p \in A \wedge p \notin B$, by this equality, $p \in B-A$ and thus $p \in B$.

$A \oplus B = (A \cup B) - (A \cap B)$: Nothing. This is a tautology.

: points out of 10

14) Compute the following quotient and reminder and justify your answer.

–23 div 7 = –4

–23 mod 7 = 5

Justification: $-23=-4 \bullet 7+5$

: points out of 9