1050 Quiz **QL13**, Feb 21, 2006. NAME:_____. Grade:_____ (____/10)
Closed books. Individual test. Do not look at other people's work. Please write legibly.

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1) What is 3+4+5+6….+397?

$(3+397)(397-3+1)/2 = (400)(395)/2 = (200)(395)=79000$

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2) Explain the difference between Induction and **Strong** Induction

Induction uses $P(0)$=true and $P(n)\rightarrow P(n+1)$ to prove $\forall n\ P(n)$.

Strong induction uses $P(0)$=true and $P(1)\wedge P(2)\wedge..P(n)\rightarrow P(n+1)$ to prove $\forall n\ P(n)$.

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3) Let C(n) denote the number of regions of the plane delimited by n lines in general position. Provide a **recursive** definition for C(n) and a **justification**. (*Make sure that the definition is complete.*)

$C(0)=1$. $C(n)=C(n-1)+n$, because the nth line is split into n segments by the n-1 previous lines and each segment splits a region.

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4) Provide an explicit polynomial expression for C(n) and show its derivation.

$C(n)=1+n(n+1)/2$.

Derivation: $C(n)=C(n-1)+n=C(n-2)+(n-1)+n=C(0)+1+2+3...+n=1+n(n+1)/2$

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5) Prove the above formula for C(n) using mathematical induction. (*State P(n) and the induction steps.*)

$P(n)$ stands for "$C(n)=1+n(n+1)/2$".

Basis step: $P(0)$ is true since $C(0)=1$ (no line: 1 region) and $1+0(0+1)/2=1$.

Derivation: Assume $P(n)$. Hence $C(n)=1+n(n+1)/2$.

$C(n+1)=C(n)+(n+1)$, as explained in question 2. Substituting $C(n)$ yields

$C(n+1)=1+n(n+1)/2+(n+1)=1+(n(n+1)+2(n+1))/2=1+(n+2)(n+1)/2$, hence $P(n+1)$=true.