1) What is $3+4+5+6\ldots+397$?

\[
(3+397)(397-3+1)/2 = (400)(395)/2 = (200)(395) = 79000
\]

2) Explain the difference between Induction and **Strong Induction**

*Induction uses $P(0)=true$ and $P(n)\rightarrow P(n+1)$ to prove $\forall n P(n)$.*

*Strong induction uses $P(0)=true$ and $P(1) \land P(2) \land \ldots P(n) \rightarrow P(n+1)$ to prove $\forall n P(n)$.*

3) Let $C(n)$ denote the number of regions of the plane delimited by $n$ lines in general position. Provide a **recursive** definition for $C(n)$ and a **justification**. *(Make sure that the definition is complete.)*

$C(0)=1$. $C(n)=C(n-1)+n$, *because the $n^{th}$ line is split into $n$ segments by the $n-1$ previous lines and each segment splits a region.*

4) Provide an explicit polynomial expression for $C(n)$ and show its derivation.

$C(n)=1+n(n+1)/2$.

*Derivation: $C(n)=C(n-1)+n=C(n-2)+(n-1)+n=C(0)+1+2+3\ldots+n=1+n(n+1)/2$*

5) Prove the above formula for $C(n)$ using mathematical induction. *(State $P(n)$ and the induction steps.)*

$P(n)$ stands for "$C(n)=1+n(n+1)/2$".

*Basis step: $P(0)$ is true since $C(0)=1$ (no line: 1 region) and $1+0(0+1)/2=1$.*

*Derivation: Assume $P(n)$. Hence $C(n)=1+n(n+1)/2$.*

$C(n+1)=C(n)+(n+1)$, *as explained in question 2. Substituting $C(n)$ yields $C(n+1)=1+n(n+1)/2+(n+1)=1+(n(n+1)+2(n+1))/2=1+(n+2)(n+1)/2$, hence $P(n+1)=true$.***