1050 Quiz **QL16**, Mar 2, 2006.   NAME:________________________.   Grade:_____ (____/10)
Closed books. Individual test. Do not look at other people's work. Please write legibly.

---

1) Prove that for every integer n larger than 1 P(n) is true, where P(n) is "n can be written as a prime or a product of primes". State precisely which kind of proof this is.

Use strong induction. Basis step: P(2) is true since 2 is prime. If n is prime then P(n) is obviously true. If n is composite, there are two integers, a and b, such that $2 \le a \le b < n$ and $n=ab$. Assuming that P(a) and P(b) are true, by strong induction, P(n) is true.

---

2) Provide a recursive complete definition for the Fibonacci numbers f_n and compute f_7.

$f_0=0$, $f_1=1$, $f_n= f_{n-1} + f_{n-2}$. $f_7=13$.

---

3) Set S_n has n different elements. We want to compute how many ways there is to make a set T_r of r elements taken from S_n. Provide a formula (using exponentials and factorials) for each one of the following cases.

T_r is ordered and has repetition: n^r

T_r is ordered and has no repetition: $P(n,r)=n!/(n-r)!$, *r-permutations*

T_r is not ordered and has no repetition: $C(n,r)=n!/(r!(n-r)!)$, *r-combinations*

T_r is not ordered and has repetition: $C(n+r-1,r)= (n+r-1)!/(r!(n-1)!)$

---

4) How many 6 bit-strings are there that either start with 1 or end with 0?

There are $A=2^5$ strings that start with 1, since there are 5 independent binary choices. There are $B=2^5$ strings that end with 0, since there are 5 independent binary choices. There are $C=2^4$ strings that start with 1 and end with 0, since there are 4 independent binary choices. Since the strings counted in C are counted in both A and B, the answer is $A+B-C=32+32-16=48$.

---

5) In a drawer, you have 10 red socks, 6 blue ones, and 4 green ones (please forgive the poor taste). How many must you pick without looking to ensure a matching pair? What principle are you using?

I must pick 4. Pigeon hole principle: I have 3 holes (colors), hence I need 3+1=4 socks to ensure that at least one color has 2 socks.