1) Prove that for every integer \( n \) larger than 1 \( P(n) \) is true, where \( P(n) \) is “\( n \) can be written as a prime or a product of primes”. State precisely which kind of proof this is.

Use **strong** induction. Basis step: \( P(2) \) is true since 2 is prime. If \( n \) is prime then \( P(n) \) is obviously true. If \( n \) is composite, there are two integers, \( a \) and \( b \), such that \( 2 \leq a \leq b < n \) and \( n=ab \). Assuming that \( P(a) \) and \( P(b) \) are true, by strong induction, \( P(n) \) is true.

2) Provide a recursive complete definition for the Fibonacci numbers \( f_n \) and compute \( f_7 \).

\[
f_0=0, f_1=1, f_n=f_{n-1} + f_{n-2}, f_7=13.
\]

3) Set \( S_n \) has \( n \) different elements. We want to compute how many ways there is to make a set \( T_r \) of \( r \) elements taken from \( S_n \). Provide a formula (using exponentials and factorials) for each one of the following cases.

\( T_r \) is ordered and has repetition: \( n^r \)

\( T_r \) is ordered and has no repetition: \( P(n,r) = n!/(n-r)! \), \( r \)-permutations

\( T_r \) is not ordered and has no repetition: \( C(n,r) = n!/(r!(n-r)!) \), \( r \)-combinations

\( T_r \) is not ordered and has repetition: \( C(n+r-1,r) = (n+r-1)!/(r!(n-1)!) \)

4) How many 6 bit-strings are there that either start with 1 or end with 0?

There are \( A=2^5 \) strings that start with 1, since there are 5 independent binary choices. There are \( B=2^5 \) strings that end with 0, since there are 5 independent binary choices. There are \( C=2^4 \) strings that start with 1 and end with 0, since there are 4 independent binary choices. Since the strings counted in \( C \) are counted in both \( A \) and \( B \), the answer is \( A+B-C=32+32-16=48 \).

5) In a drawer, you have 10 red socks, 6 blue ones, and 4 green ones (please forgive the poor taste). How many must you pick without looking to ensure a matching pair? What principle are you using?

**I must pick 4. Pigeon hole principle: I have 3 holes (colors), hence I need \( 3+1=4 \) socks to ensure that at least one color has 2 socks.**