CS7491 Class Notes: 01/17/06

Initial Project Questions/Comments:

Histogram Colors:

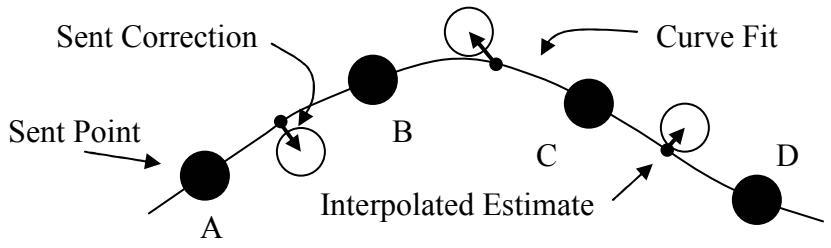
- 2 corrective vectors (x and y directions) per data point
- sort them largest to smallest by magnitude
- plot the magnitudes on the y-axis, sorted indices on the x-axis
- color positive values green, negative values red
    - used to display that the data's sign distribution is random with zero bias
    - shows that 1 bit per compressed value should be designated and fixed for sign
        - because it will be positive 50% of the time, negative the other 50%
        - no gain by variable length encoding

Loss of Accuracy through Linear Estimation:

- Consider:
    - quantized values from 0 to 4095
    - A sequence of data points {0, 4095, 0}
    - A linear estimate of the 3rd point is 8190
    - The corrective value is -8190
    - requires 1 more than the original number of quantized bits (plus sign)
        - Many times more than the expected compressed amount of bits
        - Greater than not compressing with linear estimation and correction
        - Or, Maybe not.
            - Allow both + and – values to wrap around integer range
            - → New worst case = Difference of half of quantized range

Interpolating Predictor

Motivation: Send a crude sampling of the data points first (perhaps every other point), then estimate the intermediate points through interpolation and only send the corrections.



Develop Cubic Interpolating Predictor:

Strategy: fit cubic to 4 points

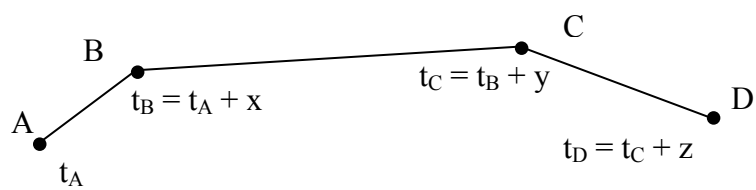
- Polynomial P(t) = constraint
- Interpolate Constraints:

  $P(t_A) = A \qquad P(t_B) = B$

  $P(t_C) = C \qquad P(t_D) = D$

How to choose t: Use arc length parameterization

- If not uniform: Use separation distance
  - However, this is difficult to solve



- Assume uniform separation
  - Can use any separation distance
  - Use $t_A = -3$, $t_B = -1$, $t_C = 1$, $t_D = 3$
  - Allows the center point, the interpolated point, to be evaluated as P(0)

Solve:

$$f(t) = at^3 + bt^2 + ct + d$$

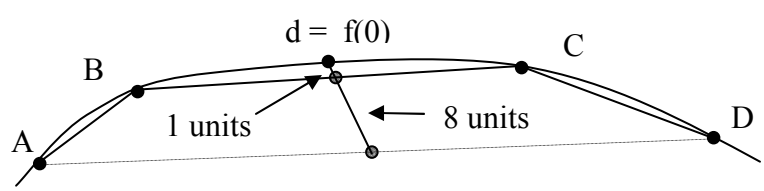
Find: $d = f(0)$

$$f(-1) = B = -a + b - c + d \qquad f(-3) = A = -27a + 9b - 3c + d$$

$$f(1) = C = a + b + c + d \qquad f(3) = D = 27a + 9b + 3c + d$$

$$\frac{B+C}{2} = b + d \quad \rightarrow \quad \frac{A+D}{2} = 9b + d \quad \rightarrow \quad 9\left(\frac{B+C}{2}\right) - \left(\frac{A+D}{2}\right) = 8d$$

$$d = \frac{9}{8}\left(\frac{B+C}{2}\right) - \frac{1}{8}\left(\frac{A+D}{2}\right) \quad \rightarrow \quad \boxed{d = \left(\frac{B+C}{2}\right) + \frac{1}{8}\left(\left(\frac{B+C}{2}\right) - \left(\frac{A+D}{2}\right)\right)}$$



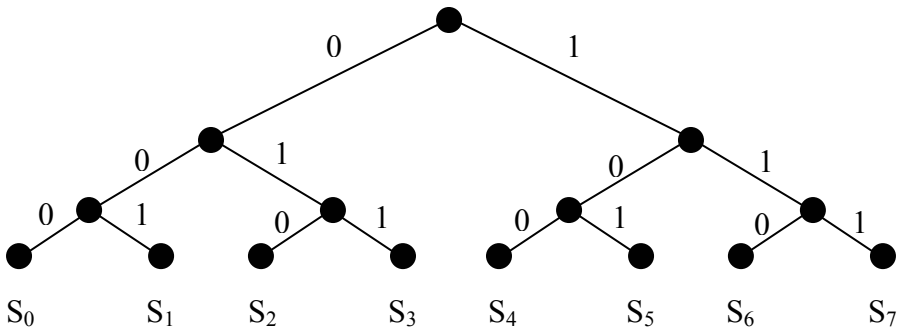
Exercise: Use a cubic to extrapolate a curve, just as before using 0th, 1st, 2nd, etc, order predictors, and compare the two.

Encoding Data

We have symbols from Set $\{s_0, s_1, s_2, \ldots\}$, how to encode them?

To send a LONG string

- *Assume same probability of symbol occurrence*
- Assume 2^n symbols
- Encode the symbols by their bit reference
- Cost per symbol? → cost = bits per symbol = n bps



Message = $S_1S_2S_3$ = 001010011

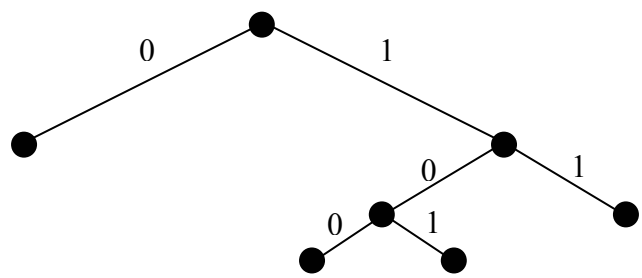
Cost:

- Need to know n
- Dictionary (the hidden cost)
    - S_i represent integers
    - Only useful if sending the same integer many times
    - If only send once, why encode the data?; encoded bits are extra bits to send

Don't Assume Same Probability:

- Merge Symbols, allowing a smaller bit length to reach more frequent symbols.
- ➔ Variable Length Encoding

Variable Length Encoding



Theoretical idea:

- $h = height = \#bits$
- $reference\# \approx \dfrac{1}{probability(symbol)}$

  A symbol with larger probability should be given a smaller reference number.

- $\log_2(reference\#) = \#bits = h$
- $h = \log_2\left(\dfrac{1}{p}\right)$

  A symbol with larger probability should have a smaller cost.

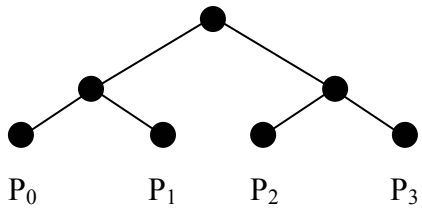
<u>*Expected Cost*</u>

- Expected Actual Cost = $\sum h_i p_i$ h based on actual, used, tree structure.
- Expected Theoretic Cost = $\boxed{Entropy = \sum p_i \log_2\left(\dfrac{1}{p_i}\right)}$
  - Theoretical Only
    - Could be more (dictionary cost, …)
    - Could be less (specific off-probability strings, …)

Example:

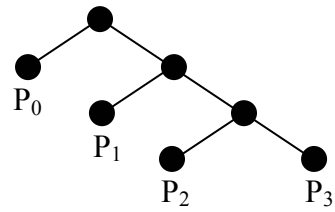
$$P_0 = 0.7,\ P_1 = 0.1,\ P_2 = 0.1,\ P_3 = 0.1$$

Type 1:



log(4) = 2 bps

Type 2:



1*0.7 + 2*0.1 + 3*0.1 + 3*0.1 = 1.5 bps

→ Structure/Probability affects the expected cost

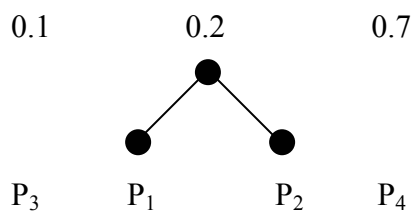
<u>*Huffman Tree Algorithm:*</u> One method for sorting the tree based on symbol probability

1) Keep all p_i sorted from smallest and up at each step

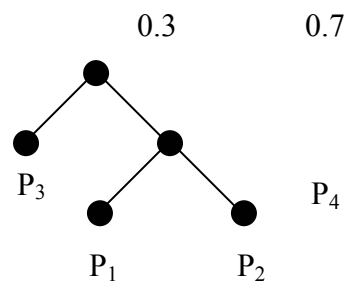
   $P_1 = 0.1$ $P_2 = 0.1$ $P_3 = 0.1$ $P_4 = 0.7$

2) Merge the left most two and resort, repeat until all are merged

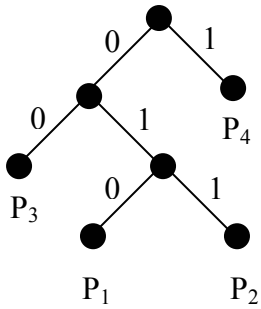
A)



B)



C)



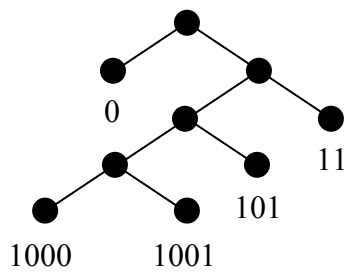
Note: Only good if symbols are sent 3, 4, 5, … times.

Probably not good for a set of floating point corrections, where there is minimal repetition.

How to Encode/Decode the binary tree?

Send path strings to be reconstructed directly

Tree



Sent Data

0
1000
1001
101
11

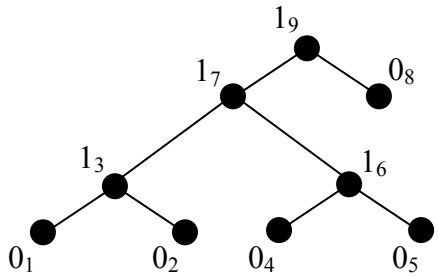
- Reconstruct directly based on the data, which is the directly represented tree
- How to discriminate the strings? → One method is to use padding
- **Cost** $\geq nh = n\log(n) + n(PaddingCost)$, for the best case, even, tree
- **High Cost** to directly send the tree structure

Encode/Decode using recursion

One method: (*Note: This may be slightly different than the actual method seen in class*)

- Start at the left bottom, then work up, taking the bottom left nodes first.
- A "0" is a final node (non-branching)
- A "1" binds the previous two nodes

Tree



Sent String

$0_1 0_2 1_3 0_4 0_5 1_6 1_7 0_8 1_9$

→ 001001101

- **Cost** = 2n – 1, Only 1 bit per node, 2n-1 nodes per tree