Homework 5, Wed 10-12-11
CS 2050, Intro Discrete Math for Computer Science
Due Fri 10-21-11, Note: This homework has 1 page and 6 problems.
Problem 6 is optional (extra credit).

Problem 1: 250 Points
(a) What is the largest $n$ for which one can solve in one second a problem using an algorithm that requires $f(n)$ bit operations, where each bit operation is carried out in $10^{-9}$ second, for these values of $f(n)$:
   (i) $f(n) = \log_2 n$, (ii) $f(n) = n$, (iii) $f(n) = n \log_2 n$, (iv) $f(n) = n^2$, (v) $f(n) = 2^n$.
(b) How much time does an algorithm take to solve a problem of size $n$ if this algorithm uses $2n^2 + 2^n$ bit operations, each requiring $10^{-9}$ second, with these values of $n$:
   (i) $n = 10$, (ii) $n = 20$, (iii) $n = 50$, (ii) $n = 100$.
(c) True or false? If true then give a proof, if false then give a counter-example:
   For function $f_1(x)$, $f_2(x)$ and $g(x)$ defined over the positive reals,
   if $f_1(x) = O(g(x))$ and $f_2 = O(g(x))$, then $f_1(x) = O(f_2(x))$.

Problem 2: 20 Points
Show, by induction that:
(a) $2n + 1 < 2^n$, for all integers $n \geq 3$.
(b) $n^2 < 2^n$, for all integers $n \geq 5$.

Problem 3: 20 Points
Show, by induction that:
(a) 3 divides $n^3 + 2n$, for all positive integers $n$.
(b) $\sum_{i=1}^{n} i^3 = \left( \frac{n(n+1)}{2} \right)^2$, for all integers $n \geq 1$.

Problem 4: 20 Points
Prove or disprove that all checkboards of these shapes can be completely covered using right triominoes whenever $n$ is a positive integer: (a) $3^n \times 3^n$ (b) $6^n \times 6^n$.

Problem 5: 20 Points
Prove that, for every integer $n \geq 8$, any postage of $n$ cents can be formed using just 3-cent stamps and 5-cent stamps.

Problem 6: 20 Points, Extra Credit
Assume that a chocolate bar consists of $n = p \times q$ squares arranges in a rectangular pattern. The bar and each smaller rectangular piece of the bar can be broken along a vertical or horizontal line separating the squares. Assuming that only one piece can be broken at a time, determine how many breaks you must successively make to break the bar into $n$ separate squares.