9-12-08 Homework 2
CS 1050 A Due: 9-18-08

Every assignment will be due at the beginning of class. Recall that you can collaborate in groups and/or use external references, but you must acknowledge the group/references that you used, and you must *always write your solutions alone.* Remember that for 90% of the people, more than 50% of the understanding happens during writing/implementation/etc. (And this is not true only for CS 1050. It is true for mostly everything, at least technical).

Please read the entire homework before starting to work on it. This homework refers to material covered in Lectures II though VII. You may want to refer to the readings assigned during these Lectures, and to to the Lecture Outlines. All are posted on the class website. You should definitely refer to the posted solutions for Homework 1 to see what is "appropriate" writing style for answering problems and questions. We were lenient in the grading of Homework 1, but as you are getting feedback and instructions, we will be increasingly raising the expectation of clear, accurate and comprehensible written answers. Just as you have to write "code" that others should be able to understand relatively easily (if you are software engineers), and just as you have to write "reports", or "essays", or "opinions" that a reader should be able to easily "get the point" (if you are managers, scientists, writers, bloggers etc), similarly you have to eventually be able to write "technical arguments" so that convey to the reader relatively easily your sequence of thoughts.

Please stop by for questions during office hours of instructor or TAs and send email to mihail@cc.gatech.edu with title 1050 at any time. This helps you, but it also helps us! Sometimes it helps us understand where the class stands and where we should put more or less emphasis. And sometimes, you give us presentational and technical ideas that we would have not thought of otherwise. So keep all communication links open!

Please print this document, and write your solutions on the printout.
Please hand-in the completed printout.

PRINT YOUR NAME HERE:..... .....

WRITE YOUR EMAIL HERE:..... .....

Problem 1: (20 points)

(a) Argue that the following system specifications are consistent. "Whenever the system software is being upgraded, users cannot access the file system. If users can access the file system, then they can save new files. If users cannot save new files, then the system software is not being upgraded".

(b) Realize that "consistency" simply means that there are no inherent "contradictions" in the specifications. It is a minimal requirement that a set of system specifications has to pass. It simply means that, there exists at least one state of the system that satisfies the specifications. List all states of the above system that satisfy the specifications.

(c) Now this is a question of engineering judgement. Do you think that the above set of system specifications make sense from an engineering point of view? For example, it would make sense that, form an engineering point of view, one may want to say that "Users can access the file system if and only if the system software is not being upgraded. Users can save new files if and only if they can access the file system". (Recall "if and only if", denoted by $\Leftrightarrow$, is the logical operator defined as: $p \Leftrightarrow q$ is true if, both p and q are true, or both p and q are false. $p \Leftrightarrow q$ is false if, either p is true and q is false, or q is true and p is false.) Suppose you add these new specifications to the ones of part (a). Is the system still consistent?

(d) Now suppose that you consider the system specifications consisting only of: "Users can access the file system if and only if the system software is not being upgraded. Users can save new files if and only if they can access the file system". Argue that the above specifications are consistent. List all states that satisfy these specifications.

Problem 2: (10 points)

Let $P(x)$, $Q(x)$ and $R(x)$ be the statements "x is a clear explanation", "x is satisfactory", and "x is an excuse", respectively. Suppose thyt the domain of x consist of all English text. Express each of the following statements using quantifiers, logical connectives, and $P(x)$, $Q(x)$ and $R(x)$:

(a) All clear explanations are satisfactory.

(b) Some excuses are unsatisfactory.

(c) Some excuses are not clear explanations.

(d) Does (c) follow from (a) and (b)? Explain.

Problem 3: (20 points)

Prove or disprove, and state clearly what proof method you are using (direct proof and/or case analysis and/or contradiction and/or counter-example, etc):
(a) The sum of any three consecutive integers is always divisible by 3.
(b) The sum of any four consecutive integers is always divisible by 4.
(c) The sum of any four consecutive integers is never divisible by 4.

Problem 4: (15 points)

Prove that $\sqrt[3]{2}$ is irrational. State clearly what proof method you are using.

Problem 5: (15 points)

(a) It is not known whether there are infinitely many prime pairs, i.e., odd prime numbers whose difference is 2. It is actually a big open problem in number theory. Examples of prime pairs are (3,5), (5,7), (11,13), and (71,73). Give three more examples of prime pairs.
(b) Prove that (3,5,7) is the only prime triple. Hint: Given $2k+1$, $2k+3$, $2k+5$, where k is any integer other than 1 (in which case the triple would be (3,5,7)), show that one of these must be divisible by 3.

Problem 6: (20 points)

Suppose that m and n are integers such that $n^2 + 1 = 2m$. Prove that m is the sum of the squares of two integers. Hint: Realize that n must be an odd number.

Problem 7, Extra Credit: (20 points)

Suppose that on an island there are three types of people, knights, knaves and normals. Knights always tell the truth, knaves always lie, and normals sometimes tell the truth and sometimes they lie. Detectives questioned three inhabitants of the island, Amy, Brenda and Claire, as part of the investigation of a crime. The detectives know that one of the three committed the crime, but not which one. They also knew that the criminal was a knight, and the other two were not knights. Additionally, the detectives recorded the following statements: Amy: " I am innocent". Brenda: "What Amy says is true". Claire: "Brenda is not a normal". After analyzing the information, the detectives positively identified the guilty party. Who was it? Explain your reasoning.

Problem 8: Extra Credit (20 points)

Read Rosen pages 99 and 100 on polyominoes (extensions of dominoes) and triominoes, including Figure 7. Now consider the 8×8 checkboard. We will try to tile it with triominoes.
(a) Is there a square of the 8×8 checkboard that, if removed, the remaining squares on the board (63 in number) can be tiled using straight triominoes? Can you characterize all such squares?
(b) What can you say about tiling the 8×8 checkboard with one square removed, if you allow both straight and right triominoes?
(c) Can you generalize any of the above arguments?