9-16-08 Homework 4

CS 1050 A <u>Due</u>: 9-23-08

Every assignment will be due at the beginning of class. Recall that you can collaborate in groups and/or use external references, but you must acknowledge the group/references that you used, and you must *always write your solutions alone.* Remember that for 90% of the people, more than 50% of the understanding happens during writing/implementation/etc. (And this is not true only for CS 1050. It is true for mostly everything, at least technical).

Please read the entire homework before starting to work on it. This homework refers to material mainly covers induction, structural induction and strong induction.

Please stop by for questions during office hours of instructor or TAs and send email to mihail@cc.gatech.edu with title 1050 at any time. This helps you, but it also helps us! Sometimes it helps us understand where the class stands and where we should put more or less emphasis. And sometimes, you give us presentational and technical ideas that we would have not thought of otherwise. So keep all communication links open!

Please print this document, and write your solutions on the printout.
Please hand-in the completed printout.

PRINT YOUR NAME HERE:... ....

WRITE YOUR EMAIL HERE:.. ....

Problem 1: (25 points)

(a1) Prove by induction that, for all postive integers n, 6 divides $n^3 - n$.
(a2) Can you also give a direct proof?

(b1) Give a direct proof that, whenever n is an odd integer, $n^2 - 1$ is divisible by 8.
(b2) Prove by induction that, whenever n is an odd integer, $n^2 - 1$ is divisible by 8.

(c) Prove by induction that, for all positive integers n, $1^3 + 2^3 + \ldots + n^3 = \left(\frac{n(n+1)}{2}\right)^2$.

Problem 2: (25 points)

(a1) Prove that a non-negative integer is divisible by 9 if and only if the sum of its digits is divisible by 9.
(b2) Argue that the following recursive algorithm computes the remainder of the division of a non-negative integer by 9: Let x be the initial number in decimal representation. Let y be the sum of all the digits of x. If $y < 10$ then output y, else $x := y$ and recurse.

(a1) Let xy be a two digit non-negative integer in decimal notation, that is, $xy = x10 + y$. Prove that, for any $0 \leq x, y \leq 9$, $xy + yx$ is a multiple of 11.

(a2) Let us generalize: Let $a_{n-1}a_{n-2}\dots a_1a_0$ be an n digit non-negative integer in decimal notation, where n is an even number. That is, $a_{n-1}a_{n-2}\dots a_1a_0 = a_{n-1}10^{n-1} + a_{n-2}10^{n-2} + \dots + a_1 10 + a_0$. Prove that, $a_{n-1}a_{n-2}\dots a_1a_0 + a_0a_1\dots a_{n-2}a_{n-1}$ is a multiple of 11. Hint: Realize that 1001 is a multiple of 11 (since 1001=990+11), 100001 is a multiple of 11 (since 100001=99990+11), and so on.

Problem 3: (25 points)

(a) Prove, by induction, that the recurrence $T(n) = 3T(\frac{n}{3}) + n$ with $T(1) = 0$ solves to $T(n) = n \log_3 n$. You may assume that n is a power of 3.

(b) Prove, by induction, that the recurrence $T(n) = 3T(n-1) + 1$ with $T(0) = 0$ solves to $T(n) = (3^n - 1)/2$.

Problem 4: (25 points)

(a) Suppose that have 2c coins and 5c coins. What amounts can you form using only such coins? (Give full proof/may need strong induction). Let x be an amount that can be formed using such coins. How can you combine 2c coins and 5c coins to form the number x, using the smallest possible number of coins? (Give full proof/may need strong induction).

(b) Rosen, page 292, problem 14. (May need strong induction)

Extra Credit: (25 points)

Rosen, page 283, number 74.