Predicate Logic

0. Motivation

1. Predicates and Propositional Functions

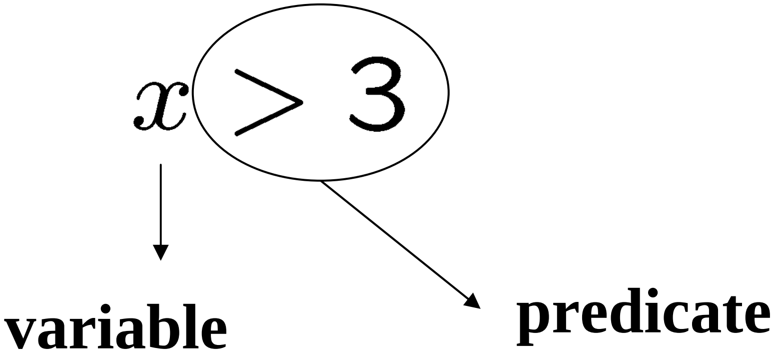
2. Quantifiers (and nested quantifiers)

3. Negating quantifiers

0. Motivation

Increase the expressiveness of propositional logic

1. Predicates and Propositional Functions



$$P(x) = x > 3$$

is a propositional function

A propositional function may involve several variables:

$$P(x, y) = x > (3 + y)$$

We can use $\neg, \wedge, \vee, \Rightarrow$ **to create very general propositional functions:** $[P_1(x, y) \vee P_2(z)] \Rightarrow \neg Q(x, z)$

2. Quantifiers (and nested quantifiers)

To create propositions from propositional functions, we "bind" every variable of a propositional function by a quantifier.

We will only use the "exists" and "for all" quantifiers:

$\exists$ $\forall$

$$\forall x \exists y \exists z \quad [\ [P_1(x,y) \vee P_2(z)] \Rightarrow \neg Q(x,z)\ ]$$

Realize that this entire statement is now a proposition.
It can be either true or false.

3. Quantifiers (and nested quantifiers) (continued)

Examples: (in all cases below, x and y are reals)

$$\forall x \exists y \ (x = 2y)$$

In English: For every real x, there exists a real y, such that x is 2 times y.

(Obviously true: No matter what x is, y will be half of x.)

$$\forall x \forall y \ (x = 2y)$$

In English: For every real x and for every real y, x is 2 times y.

(Obviously false: "Counter-Example":
For x=2 and y=3 hence 2y=6,
2 is not equal to 6.)

3. Negating quantifiers

$$\neg[\forall x\, P(x)] \iff [\exists x\, \neg P(x)]$$

$$\neg[\exists x\, P(x)] \iff [\forall x\, \neg P(x)]$$

3. Negating Quantifiers (continued)

Example (for nested quantifiers):

$\forall x \forall y\ P(x, y)$, where x, y are real numbers and $P(x, y) = [x - y = y - y]$.

In English:

For all real numbers x and y, x-y=y-x.

Negation:

$$\neg \forall x \forall y\ P(x, y) \Leftrightarrow$$
$$\neg \forall x \forall y\ (x - y = y - x) \Leftrightarrow$$
$$\exists x \neg [\forall y\ (x - y = y - x)] \Leftrightarrow$$
$$\exists x \exists y\ \neg (x - y = y - x) \Leftrightarrow$$
$$\exists x \exists y\ (x - y \neq y - x)$$