Inter-arrival Times for Random Way-point are Exponential!

Muhammad Mukarram Bin Tariq
College of Computing, Georgia Tech
mtariq@cc.gatech.edu
First Compiled: December 11, 2005

December 11, 2005

Recently while playing with mobility models, I noticed that the if a node follows the Random way-point model, then the inter-arrival to a circular sub-region seem to follow exponential distribution. I simulated a RWP node in a 1km x 1km area, and generated a very fine time-granularity trace of mobile node's location. Using this trace, I analysed the inter-arrival times to different sub-regions in the 1km x km region. This observation may have important implications for routing protocols that use RWP model. Here are some details.

1. Beginning at a random instance in time, the time before the next visit of the random way-point mobile node to a particular sub-region in the region of the mobility of the mobile node, is exponentially distributed! Put an other way, the arrivals to a sub-region are Poisson. This observation is backed by perfect fitting of the empirical distribution of waiting time before next arrival with a distribution of the form $p = 1 - exp(-t/\hat{\gamma})$ using the MATLAB curve-fitting toolbox. Table 1 shows the results for goodness-of-fit parameters for each of the some of the regions in figure 1. Note that estimated and empirical values of γ are close, the RMSE is small, and R^2 is close to 1 for all the regions. Also note from figure 1 that the value γ for the different sub-regions follows a clear trend; it is small for the regions in the center, and large for the sub-regions that are towards the edges. I have also observed that the expected wait-time before arrival is smaller for a large sub-region in a location than a smaller sub-region in the same location. I am not aware of any existing work that makes similar observation.

2. The fraction of time that a node spends in a particular sub-region of the larger region in which it moves, follows a distinct pattern. The node tends to spend more time in sub-region that are close to the center of the larger region, and lesser time in the regions that are towards the edges, essentially forming a *dome-shaped* distribution. We found that this behavior is consistent and steady, i.e., the fraction of time that the node spends in a region remains fairly constant over time. We also found that fraction of time that the node spends in a sub-region is proportional to the size of the region. In particular, the fraction

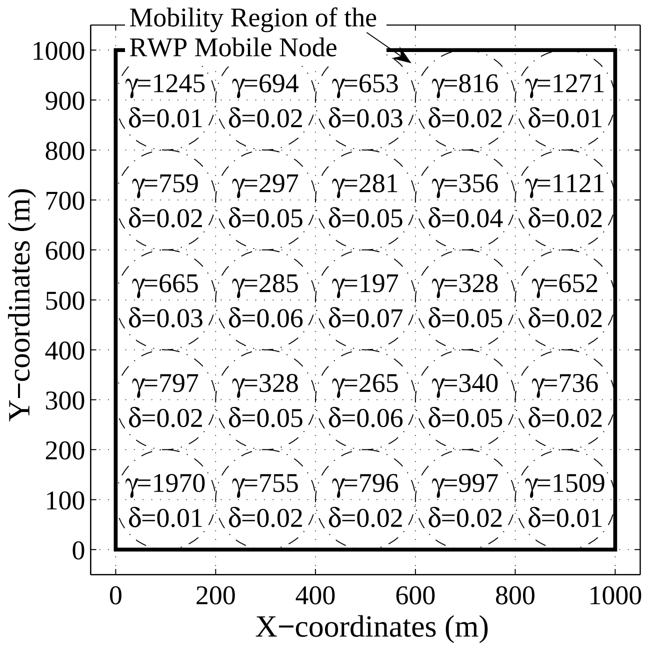

Figure 1: Properties of RWP Mobility Model: Each circle represents a circular region of 100m radius. δ is time-limiting probability of instantaneous contact with mobile node in that region, computed as the fraction of time spent in the region with $t \to \infty$. γ is empirically observed expected inter-arrival time in that region. The RWP parameters are: $v_{min} = 9m/s$, $v_{max} = 11m/s$, and exponentially distributed pause time with mean of 1s

of time that the node spends in a larger region in at a location would be larger than the fraction of time that the node would spend in a smaller region at the same location. In figure 1, the value of δ in each small circular region represents the fraction of time that the node spent in that region. This second observation has been made before, for example in by Jardosh et.al in their Mobicom 2003 paper on their obstacle based mobility models.

I believe that these properties can be proved to hold analytically with some effort; I am looking into that possibility.

Table 1: Goodness-of-fit results for curve fitting with $1 - exp(-t/\hat{\gamma})$

Location of Region (x,y) – meters	Estimated Mean (γ)	Mean with Curve Fitting ($\hat{\gamma}$)	Root Mean Square Error (RMSE)	R-square
(100, 100)	1970.41	1910.13	0.0113	0.9965
(300, 100)	754.58	765.56	0.0039	0.9991
(500, 100)	796.07	740.81	0.0086	0.9956
(700, 100)	997.02	945.12	0.0087	0.9962
(900, 100)	1508.83	1520.57	0.0041	0.9995
(100, 300)	797.27	816.97	0.0066	0.9978
(300, 300)	328.04	328.28	0.0013	0.9998
(500, 300)	264.92	270.87	0.0024	0.9991
(700, 300)	340.38	338.18	0.0021	0.9995
(900, 300)	735.56	722.16	0.0040	0.9990
(100, 500)	665.38	676.41	0.0028	0.9995
(300, 500)	285.10	290.54	0.0022	0.9993
(500, 500)	197.49	201.08	0.0031	0.9981
(700, 500)	327.55	324.75	0.0037	0.9982
(900, 500)	652.42	676.34	0.0084	0.9958
(100, 700)	758.95	777.82	0.0044	0.9989
(300, 700)	296.72	289.72	0.0035	0.9983
(500, 700)	281.09	277.88	0.0016	0.9996
(700, 700)	356.03	354.25	0.0031	0.9989
(900, 700)	1120.66	1081.59	0.0106	0.9951
(100, 900)	1245.08	1276.41	0.0116	0.9955
(300, 900)	694.31	698.57	0.0032	0.9994
(500, 900)	652.95	660.44	0.0034	0.9993
(700, 900)	815.99	787.74	0.0082	0.9961
(900, 900)	1271.40	1320.74	0.0106	0.9963