Problem 2: Independence

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(a) Let \( x, y, z \) be binary random variables. For each of the following joint distributions, determine whether \( x \) and \( z \) are dependent, showing your analysis:

i. \( p(x, z) = \begin{array}{cc}
  x = 0 & x = 1 \\
  z = 0 & 0.02 & 0.08 \\
  z = 1 & 0.18 & 0.72 \\
\end{array} \)

ii. \( p(x, z) = \begin{array}{cc}
  x = 0 & x = 1 \\
  z = 0 & 0.025 & 0.125 \\
  z = 1 & 0.125 & 0.725 \\
\end{array} \)

(b) Let \( x, y, z \) be binary random variables. Suppose \( p(x, y, z) \) factorizes as \( p(x, y, z) = p(x)p(y|x)p(z|y) \). Find a specific numerical parameterization for this model with the properties:

i. \( z \perp x \mid y \)

ii. \( x = z \).

By \( x = z \) we mean that when a sample \((x, y, z)\) is drawn according to \( p(x, y, z) \), it will always be the case that \( x \) and \( z \) have the same value.