Graph coloring using eigenvalue decomposition

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MostNeg: An approximately 2-coloring algorithm

1. Calculate the eigenvector v corresponding to highest eigenvalue of L.
2. Map each vertex to corresponding component in v.
3. Partition the vertices into two groups according to the sign of mapped value.

Relation to other problems:

- Finding a maximum cut of an unweighted graph.
- Edwards formula [3] gives a lower bound on the number of edges in bipartition of graph.

RecCut: A correct coloring algorithm

1. Calculate the eigenvector v corresponding to the highest eigenvalue of L.
2. Partition the vertices based on the signs of the eigenvector v.
3. Recurse on the pieces induced by the partition.

Facts:

- Generalization of *MostNeg*.
- Similar to spectral clustering algorithm of [6].

Simulation results

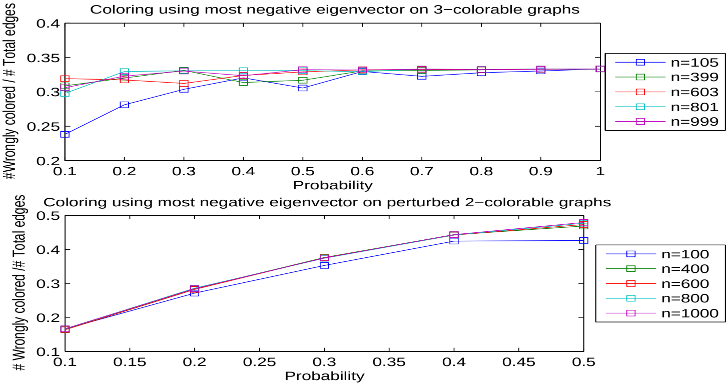

Figure: Performance of *MostNeg*

Simulation results - Contd

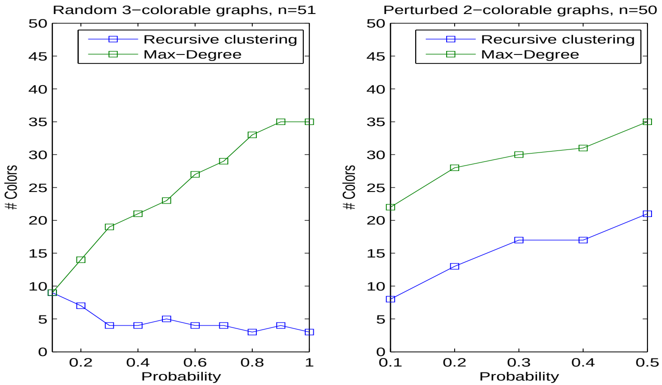

Figure: Performance of *RecCut* versus *MaxDeg*

Simulation results - Contd

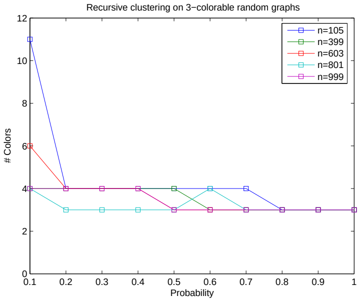

Figure: Performance of *RecCut* for random 3-colorable graphs

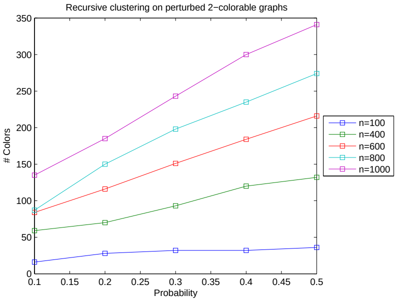

Figure: Performance of *RecCut* for nearly bipartite graphs

Observations

1. In figure 1, we observe that *MostNeg* algorithm performs independent of the number of vertices in graph. For random 3-colorable graphs, it is independent of both number of vertices and edges in graph. Moreover, fraction of correctly colored edges is always (for random 3-colorable graphs and nearly bipartite graphs) more than the lower bound provided by edwards formula [3].
2. *RecCut* performs better on dense graphs than on sparse graphs as seen in figure 6.
3. *RecCut* out-performs *MaxDeg* (figure 2), the trivial upper bound on coloring in both classes of graphs under consideration. However, *RecCut* uses more colors than used by *MaxDeg* for peterson graph and knesar graph with $n = 12, k = 5$ which means that graph has 792 vertices.

Conclusions and future work

- Our proposed algorithm recursive clustering performs well in practice.
- Closely related to the spectral graph partitioning algorithm mentioned in [6]. Here, number of partitions equals the number of colors used to correctly color the graph. Open question to get a theoretical bound on number of colors.
- Eigenvector corresponding to most negative eigenvalue partitions the graph into two partitions nicely with low intra-partition bounds.
- Instead of using just one eigenvector, devise algorithm which uses more eigenvectors to get a better coloring.
- The conjecture mentioned in [1], that a correct coloring can be obtained by considering all negative eigenvalues still remains open.

References

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