Probability Review

Note Title 9/12/2006

Event - set of possible outcomes of an experiment

A, B, C

dice roll = 6
flipped coin lands on heads

Discrete Probability (Pr[ ])

[ ]

$$0 \le Pr[A] \le 1$$

- likelihood
- Percent outcome

Side Note:

assume representative

Determine if a light will be red at 11:34 am

only happens ONCE!

Ergodic → doing a test multiple times will yield same results as sampling over time

Important Sets

S - Universal Set

$\emptyset$ - NULL set

$$Pr[S] = 1$$

$$Pr[\emptyset] = 0$$

Events are treated as sets

- all sets are included in the Universal set

$$Pr[\bar{A}] = 1 - Pr[A]$$

compliment

A = flip of a coin is Heads

More Interesting sets:

Mutually Exclusive (ME)

- at most, one outcome can occur

Eg:

- a coin cannot land on both HEADs and Tails
- a die cannot land on both side 1 and 6

Collectively Exhaustive (CE)

- at least one of the events must occur

dice 1, 2, 3, 4, 5, 6

Six sided die roll is both ME + CE

$A = [1, 2, 3, \ldots m]$

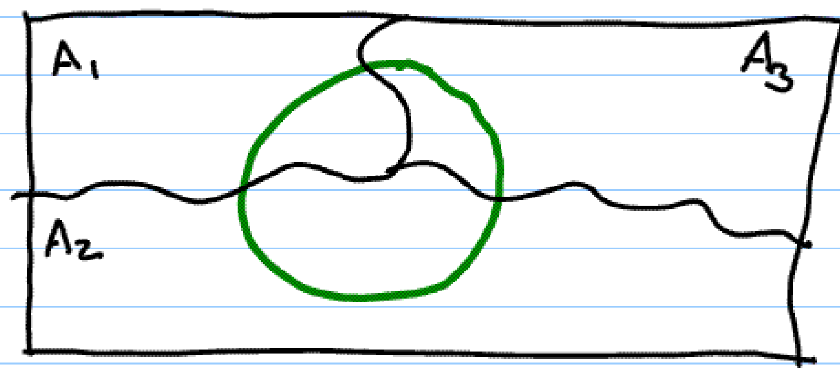
A is both ME + CE iff

$$\sum_{i=1}^{m} \Pr[A_i] = 1 \quad \text{and}$$

$$\Pr[B] = \sum_{i=1}^{m} \Pr[A_i \cap B]$$

A

B



outcomes

if A_i are ME then

$$\Pr[A_j \cup A_i] = \Pr[A_j] + \Pr[A_i]$$

Conditional Probability

What is the Probability of A given that we know B

if B is in the set of possible outcomes is A also in this set?

$$Pr[A|B] = \frac{Pr[A \cap B]}{Pr[B]}$$

$$Pr[A \cap B] = Pr[A|B] \cdot Pr[B]$$

What if A and B are Independent

$$Pr[A \cap B] = Pr[A] \cdot Pr[B]$$

$$Pr[A] = Pr[A|B]$$

telling me about B tells me nothing about A

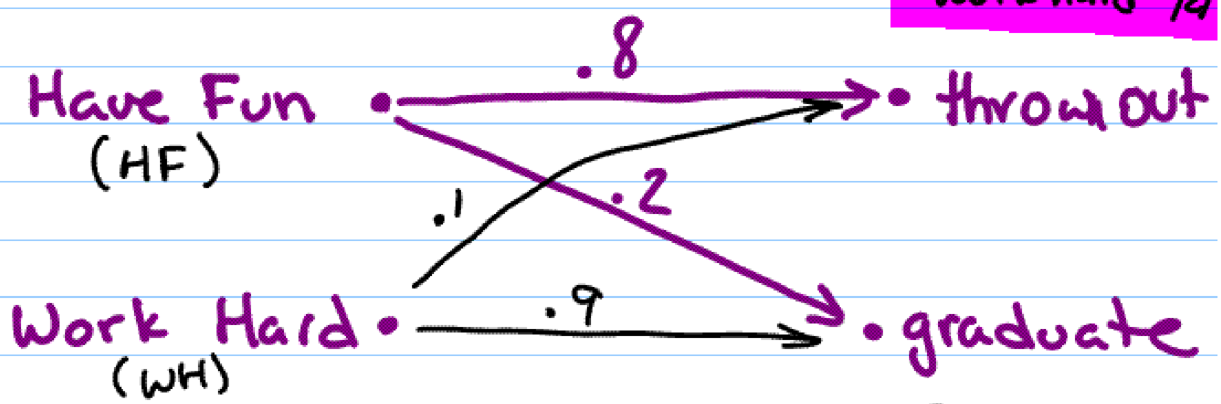
An Example?

$$\Pr\Big[\overbrace{\text{Graduate GT}}^{A} \;\Big|\; \overbrace{\text{we work REALLY Hard}}^{B}\Big]$$

2 ME class

		Prior
B	Have Fun?	1/4
	Work Hard	3/4
A	Thrown out of Tech	
	Graduate	

Prior have fun 1/4
work hard 3/4



$$\Pr[\text{Error}] = \Pr[\text{HF} \cap \text{ERROR}] + \Pr[\text{WH} \cap \text{error}]$$

E

$$= \Pr[E \mid HF] \cdot \Pr[HF] +$$
$$\Pr[E \mid WH] \cdot \Pr[WH]$$
$$= (.2 \cdot .25) + (.1 \cdot .75)$$
$$= .05 + .075 = .125$$
$$= 1/8$$

Priors impact everything

BAD
- where do they come from?
- how do we pick them

→ counting

→ fiddle with them to get results you want

$$P(A \cap B) = P(B \cap A)$$

Bayes Rule

$$P[A|B] = \frac{Pr[A \cap B]}{Pr[B]}$$

$$= \frac{Pr[B|A] \cdot Pr[A]}{Pr[B]}$$

How do we use this for Pattern Rec?

salmon

all types of fish

$A \approx$ class $\omega_i \in \Omega$

$B \approx$ data d

ω_i = Salmon

$$Pr[\omega_i \mid d]$$

What is the probability of class ω_i given that we see data d?

Bayes Rule

$$Pr[\omega_i \mid d] = \frac{\overbrace{Pr[d \mid \omega_i]}^{\text{likelihood}} \cdot \overbrace{Pr[\omega_i]}^{\text{Prior}}}{\underbrace{Pr[d]}_{\text{evidence}}}$$

$$Pr[d \cap \omega_i] + Pr[d \cap \overline{\omega_i}]$$

who cares what the probability $Pr[d]$ is? it is your data and You are stuck with it

You can compute $Pr[\omega_i \mid d]$ without $Pr[d]$

- Normalize?

Normalize by $\sum_i Pr^*[\omega_i | d]$

$$Pr^*[\omega_i | d] = Pr[\omega_i | d]\, Pr[d]$$

Next class

- Continuous Probability
- Gaussians
- Distance Metrics
- Baysean Classifiers