\( \text{NP} \) stands for nondeterministic Polynomial time = problems that can be solved in poly-time on a non-deterministic machine.

allowed to guess at each step (there exists a path to the accepting state)

\( \text{NP} = \) all search problems

\( \text{P} = \) search problems that can be solved in poly-time.

Hence, \( \text{P} \subset \text{NP} \).

Eg., \( \text{MST} \in \text{P}, \) shortest paths \( \in \text{P} \).

if \( \text{P} = \text{NP} \): if we can verify solutions efficiently
then we can construct solutions efficiently.

(e.g., is verifying a proof for a theorem as hard as constructing the proof?)

if \( \text{P} \neq \text{NP} \): then there are some search problems that
can't be solved efficiently.

What are these problems?
Recall search problem:

- Given input $I$, find a solution $S$ if one exists & output NO if none exists.
- Moreover, given $S$, in time polynomial in $|I|$, we can verify that $S$ is in fact a solution to $I$.

$NP =$ all search problems = Problems where solutions can be verified efficiently

- Colorings $\in NP$
- SAT $\in NP$
- Knapsack-search $\in NP$
- MST $\in P$, hence MST $\in NP$.

$NP$-complete problems:

hardest problems in $NP$. 
Colorings is NP-complete, which means

a) Colorings ∈ NP.

b) if we can solve colorings in Poly-time then we can solve every problem in NP in Poly-time.

Thus, if P ≠ NP, then there is no Poly-time algorithm for colorings.

How to show (b)?

Reductions:

Problems A & B (Example A = MST, B = Colorings)

A → B or A ≤ B

means we can reduce A to B.
\[ A \rightarrow B \text{ means:} \]
if we can solve B in poly-time then we can use that algorithm to solve A in poly-time.

So we suppose there is an efficient algorithm for B, and we use that algorithm as a black-box for A.

So we take input I for A
create input \( f(I) \) for B
Run Black-box alg. for B
given solution S for \( f(I) \), convert to output \( h(s) \) for I.
given NO for \( f(I) \), output NO for I.
We need to define $f$ & $h$

Prove that if $S$ is a solution to $f(I)$
then $h(S)$ is a solution to $I$

If NO solution for $f(I)$
then NO solution for $I$.

To show colorings is NP-complete,
we need to show:

a) Colorings $\in$ NP

b) for all $A \in$ NP,
$A \rightarrow$ Colorings

How to do (b) for all $A \in$ NP?
Suppose we know SAT is NP-complete.
So we know that for every \( A \in \text{NP} \),
\[ A \rightarrow \text{SAT}. \]

Suppose we show \( \text{SAT} \rightarrow \text{Colorings} \).
Then,
\[ A \rightarrow \text{SAT} \rightarrow \text{Colorings} \]
so \( A \rightarrow \text{Colorings} \).

To show Colorings is NP-complete, need to show:
(a) Colorings \( \in \text{NP} \)
(b) for a known NP-complete problem \( A \),
Show \( A \rightarrow \text{Colorings} \).

We'll take for granted that
\( \text{SAT} \) is NP-complete.
Then we'll show a bunch of other problems
are NP-complete.