CS 4540, Fall 2014
Homework 2
due: Wednesday, September 3, 2014 (at the start of class).

We say a formula is an *E4-SAT* formula if it is a Boolean formula f in CNF with n variables and m clauses where each clause has *exactly* 4 variables. In the *MAX E4-SAT* problem we are given as input an E4-SAT formula f and want to maximize the number of clauses that are satisfied.

Problem 1:

Give a randomized polynomial-time algorithm that takes as input an E4-SAT formula f and outputs an assignment that **in expectation** satisfies at least αm clauses for a constant α. Try to maximize the α that you achieve (and state clearly in your proof what α you achieve). You need to explain your algorithm and then prove it achieves the claimed result.

Problem 2:

Prove that any E4-SAT formula with at most 15 clauses is satisfiable. Use the probabilistic method to solve it (i.e., use your solution to Problem 1), don't argue about what a formula with 15 clauses must look like.

For an undirected graph $G = (V, E)$, a **3-way cut** is a subset of edges $F \subset E$ such that there is a partition of the vertices V into three disjoint parts $S \cup T \cup U = V$ where every edge of F has its endpoints lying in different parts and every edge of $E \setminus F$ has both of its endpoints in the same part. Thus, removing the edges F from the graph disconnects these three parts from each other.

Problem 3:

Show that any graph with m edges has a 3-way cut with at least αm edges for constant $\alpha > 0$ – once again, try to maximize α.