CS 4540, Fall 2014
Homework 4
due: Wednesday, October 1, 2014.

Problem 1: [DPV] Problem 7.4 (Duff beer)
Problem 2: [DPV] Problem 7.5 (Canine Products)
Problem 3: [DPV] Problem 7.6 (Infinite feasible region, but bounded optimum)

Problem 4: There are $n$ bottles which contain different mixtures of three chemicals called $A ; B ; C$. The $i$-th bottle contains the chemicals in ratio $a_{i}: b_{i}: c_{i}$ (thus, $a_{i} /\left(a_{i}+b_{i}+c_{i}\right)$ fraction of the i-th bottle is chemical $A$, $b_{i} /\left(a_{i}+b_{i}+c_{i}\right)$ fraction of the $i$-th bottle is chemical $B$, and $c_{i} /\left(a_{i}+b_{i}+c_{i}\right)$ fraction of the $i$-th bottle is chemical $C$ ). We want to know whether it is possible to obtain a mixture containing the chemicals $A ; B ; C$ in ratio $a: b: c$ by mixing various amounts from the bottles.

In general, the input is the following: $\left(a_{1}, b_{1}, c_{1}\right), \ldots,\left(a_{n}, b_{n}, c_{n}\right)$ and a target $(a, b, c)$. You can assume all of these input numbers are non-negative integers. The output is $\alpha_{1}, \ldots, \alpha_{n}$ (which are non-negative real numbers) specifying the combinations of the $n$ bottles that yields the target mixture, and if no such combination exists we simply output NO. Give an efficient algorithm for this problem using linear programming.

For example, suppose the input has $n=2$ and the ratios in the bottles are $1: 1: 2$ and $3: 3: 1$, and we want to obtain a mixture with ratio $1: 1: 1$. Then the answer is YES we can take $\alpha_{1}=2$ copies of the first bottle and $\alpha_{2}=1$ copies of the second bottle.

