Problem 1: [DPV] Problem 7.19 (verify max-flow)

Problem 2: You are given an input to max-flow: a directed graph $G = (V, E)$ with capacities $c_e > 0$ for each edge $e \in E$, a designated source $s \in V$ and a sink $t \in V$. Let $n = |V|$ and $m = |E|$.

You are also given a maximum flow $f$ specified by $f_e \geq 0$ for each edge $e \in E$.

Assume all of the capacities are integers.

(a) For a specific edge $e^* \in E$ we increase its capacity $c_e$ by 1 unit. Give an $O(m + n)$ time algorithm to compute a max flow for this new input.

(b) Assume the flow $f$ is acyclic – there is no cycle $C$ in $G$ which has positive flow along $C$. Now for a specific edge $e^* \in E$ we decrease its capacity $c_e$ by 1 unit. Give an $O(m + n)$ time algorithm to compute a max flow for this new input.

Problem 3:

There are $n$ vacation days $D = \{1, \ldots, n\}$. There are $m$ workers. Worker $i$ is available to work on the subset of days $S_i \subseteq D$, they cannot be assigned to days not in $S_i$. There is an additional parameter $C$: each worker can be assigned to work at most $C$ days in total.

Your goal is to give a polynomial-time algorithm to determine whether there is an assignment of a single worker to each vacation day while satisfying the other constraints. Your algorithm needs to reduce it to a max-flow computation.

Part (b):

The vacation days are partitioned into $j$ holidays: $D = D_1 \cup D_2 \cup \cdots D_j$. For example, New Years day is days $D_1 = \{1\}$, Thanksgiving is days $D_2 = \{2,3,4\}$, etc. We want to solve the above problem with the additional constraint that a worker is assigned to at most 1 day for each holiday. For example, in the above example of holidays, worker 1 can be assigned to days 1 and 4, but worker 2 cannot be assigned to both days 2 and 3 since that would be $> 1$ day during holiday 2. Once again show how this problem can be reduced to a max-flow computation.
Problem 4:

[DPV] Problem 7.14 (value of game for pizza business)

Make sure to show your work — that includes stating the LPs for Joey and Tony and showing how you solved the LPs.

Hint: you can simplify one of the LPs so that it has only 2 variables, and then use the solution to that to find a matching solution to the other LP.