CS 4540, Fall 2014
Homework 6
due: Wednesday, October 22, 2014.

Problem 1: [DPV] Problem 7.19 (verify max-flow)

Problem 2: You are given an input to max-flow: a directed graph $G = (V, E)$ with capacities $c_e > 0$ for each edge $e \in E$, a designated source $s \in V$ and a sink $t \in V$. Let $n = |V|$ and $m = |E|$.

You are also given a maximum flow f specified by $f_e \geq 0$ for each edge $e \in E$.

Assume all of the capacities are integers.

(a) For a specific edge $e^* \in E$ we **increase** its capacity c_e by 1 unit. Give an $O(m+n)$ time algorithm to compute a max flow for this new input.

(b) Assume the flow f is acyclic – there is no cycle C in G which has positive flow along C. Now for a specific edge $e^* \in E$ we **decrease** its capacity c_e by 1 unit. Give an $O(m+n)$ time algorithm to compute a max flow for this new input.

Problem 3:

There are n vacation days $D = \{1, \dots, n\}$. There are m workers. Worker i is available to work on the subset of days $S_i \subseteq D$, they cannot be assigned to days not in S_i. There is an additional parameter C: each worker can be assigned to work at most C days in total.

Your goal is to give a polynomial-time algorithm to determine whether there is an assignment of a single worker to each vacation day while satisfying the other constraints. Your algorithm needs to reduce it to a max-flow computation.

Part (b):

The vacation days are partitioned into j holidays: $D = D_1 \cup D_2 \cup \cdots D_j$. For example, New Years day is days $D_1 = \{1\}$, Thanksgiving is days $D_2 = \{2, 3, 4\}$, etc. We want to solve the above problem with the additional constraint that a worker is assigned to at most 1 day for each holiday. For example, in the above example of holidays, worker 1 can be assigned to days 1 and 4, but worker 2 cannot be assigned to both days 2 and 3 since that would be > 1 day during holiday 2. Once again show how this problem can be reduced to a max-flow computation.

Problem 4:

[DPV] Problem 7.14 (value of game for pizza business)

Make sure to show your work – that includes stating the LPs for Joey and Tony and showing how you solved the LPs.

Hint: you can simplify one of the LPs so that it has only 2 variables, and then use the solution to that to find a matching solution to the other LP.