Honours Year Project Report

FEC in Distributed Streaming

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Abstract

This paper evaluates the use of Forward Error Correction (FEC) schemes in distributed media streaming. We propose three different FEC schemes applicable to distributed streaming, and we build mathematical model to analyze the usable rate as well as the expected number of usable packets of these schemes. Our model is then verified using simulations. The schemes' performance in packet loss burst length distribution is also studied using simulation. Our results show that to distribute packets in an FEC block across multiple senders is effective in mitigating the adverse effect caused by high loss rate and burstiness, hence improving the streaming quality. We also find that usually the best result is obtained by further distributing FEC packets across senders.

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FEC, Distributed Streaming, Peer to Peer

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Chapter 1

Introduction

Media streaming over best-effort networks has always been a research problem due to various factors present in the network such as bursty loss and delay, which can have adverse effect on the streaming quality. Traditionally a client-server model is used, where a connection is set up between a server containing audio/video files and a client requesting the media file. This approach has the disadvantage of not being scalable as too many clients may overwhelm the server, although some solutions including multicast (Deering, 1992), (Deering, Estrin, Farinacci, Jacobson, Liu, & Wei, 1996) has been proposed. Furthermore, such a model often assume a fixed path between the sender and the receiver throughout the streaming session, hence if the network condition along this path deteriorates by experiencing a congestion, streaming quality can not be guaranteed. Even if there is no congestion along the path, if the round-trip time between the sender and the receiver increases, the TCP-friendly sending rate of media applications has to be reduced (Sisalem & Schulzrinne, 1998), hence causing quality degrading.

Distributed streaming has emerged recently as a more robust and scalable alternative for traditional single server model for streaming. Instead of streaming media content from one single server to the clients, distributed streaming enables multiple senders which store the same media file to collaboratively stream the media data to a client. The receiver can choose more capable senders, and if the required bit rate cannot be provided by a single sender, multiple senders can cooperate to ensure a satisfactory level of throughput. Moreover, the negative effect due to a deterioration along the path between one particular sender and the receiver can be mitigated by adapting the sending rates of the other senders. Compare to the traditional client-server model, distributed streaming has the following advantages: (i) it is scalable, (ii) the aggregated bandwidth of all the senders means higher throughput and lower latency, (iii) the problem of a single point of failure can be resolved. As a result, the received media quality can be improved substantially.

With distributed streaming, error recovery remains a major issue. As most of the continuous media streaming applications require real-time media delivery, TCP may not be a suitable underlying protocol as it hides the lossy nature of the underlying IP network to achieve reliable delivery at the expense of timeliness. Instead, UDP, which is more time sensitive, is more appropriate. However its lack of reliability implies that packet loss can occur. If the loss is great, the streaming quality can be adversely affected. As a result, different techniques including retransmission and Forward Error Correction (FEC) need to be devised to deal with packet loss and error.

In this paper we evaluate the use of FEC schemes in distributed media streaming. We propose three different FEC schemes applicable to distributed streaming, and we build mathematical model to analyze the usable rate as well as the expected number of usable packets of these schemes. Our model is then verified using simulations. The schemes' performance in packet loss burst length distribution is also studied using simulation. Our results show that to distribute packets in an FEC block across multiple senders is effective in mitigating the adverse effect caused by high loss rate and burstiness, hence improving the streaming quality. We also find that usually the best result is obtained by further distributing FEC packets across senders.

1.1 Related Work

1.1.1 Sender Selection and Packet Allocation

With the advantages distributed streaming also presents many research challenges. First of all, given a set of peers which contain the same audio/video file, how to select, monitor and possibly switch sending peers to achieve best streaming quality is a major issue. PROMISE (Hefeeda, Habib, Botev, Xu, & Bhargava, 2003) proposes a solution where a topology-aware selection is performed.

This topology-aware technique takes the shared segments among network paths into consideration, hence is able to avoid peers whose paths are sharing a low-bandwidth segment. To begin with, an inferred topology of the network which interconnects the candidate peers and the receiver is built, assuming that the routes do not change during streaming process. Each peer has a maximum sending rate that it can contribute, denoted as R_p. Then the "segment goodness" of each segment of the network topology is computed as a function of the loss rate and available bandwidth. G_p, the peer goodness of each sender s (which is a leaf on the network topology) is computed using its availability (fraction of time the sender is available for serving) and the goodness of the network segments from s to the receiver. Finally, from all the peer sets (each is a subset of available peers) which satisfy the constraint that the sum of R_p of these peers is close to the playback rate of the media file within a reasonable range, one set of peers is selected as the senders so that the expected aggregate rate ($\sum G_p * R_p$) is maximized (Hefeeda et al., 2003).

With selected senders, it is crucial to allocate packets across these senders so that a higher throughput and lower loss rate and delay can be maintained. A receiver-driven transport protocol which employs a rate allocation algorithm and a packet partition algorithm has been proposed (Nguyen & Zakhor, 2002a). Given the following parameters:

- n: total number of senders
- $L(i, t)$: estimated loss rate of sender i over a time interval $(t, t + \Delta)$
- $B(i, t)$: estimated TCP-friendly bandwidth for sender i over a time interval $(t, t + \Delta)$
- $S(i, t)$: estimated sending rate of sender i over a time interval $(t, t + \Delta)$
- $S_{req}(t)$: required bit rate for the encoded video during time interval $(t, t + \Delta)$

the rate allocation algorithm at the receiver side tries to find $S(i, t)$ for $i = 1...n$ so that total packet loss is minimized for time interval $(t, t+\Delta)$ under the constraints that $0 \leq S(i, t) \leq B(i, t)$

and $\sum_{i=1}^{n} S(i,t) = S_{req}(t)$. To do that, senders are first sorted according to $L(i,t)$ in ascending order. Starting from the first sender, its sending rate is assigned to be its TCP-friendly estimated bandwidth. The process then continues to the next sender, and stops when sum of allocated sending rate reaches $S_{req}(t)$. Thus the total packet loss is given by $\sum_{i=1}^{n} L(i,t)S(i,t)$.

All senders then receive the sending rate information from the receiver. Using the packet partition algorithm, each sender i calculates the time difference $A(i,k)$ between the estimated arrival and playback time for kth packet for itself and all other senders. Only if a sender find out $A(i,k)$ is maximized for itself, it sends the kth packet.

1.1.2 Error Recovery and FEC

Extensive work has been conducted on error recovery techniques in the traditional client-server model. Perkins et al. examined various error recovery techniques and recommended that for non-interactive applications such as broadcasting, a media independent Forward Error Correction (FEC) scheme performs better than retransmission based scheme because it allows for exact repairing (Perkins, Hodson, & Hardman, 1998).

FEC techniques are among the effective tools to protect media data and minimize the adverse effect caused by packet loss. The idea is to send extra packets generated from data packets so that they can be used to recover lost data packets. One major drawback of FEC is that it results in bandwidth overhead, but this may be a less serious issue considering the aggregated bandwidth in distributed streaming. Parity Codes and Reed Solomon Codes are two of the most widely used media independent FEC schemes using block or algebraic codes. In these schemes K additional check packets are generated from $(N-K)$ data packets for the transmission of a total of N packets over the network which constitute one FEC block. For each FEC block, provided that no more than K packets are lost and at least $(N-K)$ packets are successfully delivered to the receiver, the loss is recoverable. In this paper, we will refer to an FEC block as (N,K) which consists $N-K$ data packets and K FEC packets.

There is few existing work on error recovery schemes in distributed streaming, especially in examining the performance of FEC schemes in the context of multiple senders. The rate allocation algorithm presented in Section 1.1.1 has been used with FEC (Nguyen & Zakhor, 2002b), but the emphasis is on the rate allocation algorithm, the effectiveness of FEC is not looked into. On the other hand, retransmission techniques has been explored in media streaming (Papadopoulos & Parulkar, 1996) (Ma & Ooi, 2005), and our work can be seen as a comparable research on FEC techniques.

1.2 About This Paper

We believe that evaluating media independent FEC schemes in distributed streaming is a novel area and has not been explored by others. In this paper we formulate mathematical models that can be used to compute usable rate and number of usable packets analytically, and are proved to be quite accurate by the simulations results. We also arrive at a definite conclusion on the effectiveness of the FEC schemes.

In Chapter 2 we present necessary background knowledge to facilitate the our explanation and analysis. We give an overview of the Gilbert Model, which is widely used to model the state of the IP network. Based on the model we introduce mathematical functions derived in an earlier paper to calculate the probability of having m packets lost out of n packets. Then we talk about existing findings that are relevant to our work.

Chapter 3 proposes two general FEC schemes, FEC-O and FEC-Dstr, that can be used in distributed streaming. By further distinguishing between data packets and FEC packets, we introduce two variants of FEC-Dstr. We also state our assumptions and methodology of studying these FEC schemes.

In Chapter 4, FEC schemes are evaluated in terms of three metrics respectively: usable rate of a block, expected number of usable packets and packet loss burst length distribution. For the first two, we use both mathematical analysis and simulations for evaluation; the burst length distribution is mainly measured using simulations. In addition, we also examine how each of the four factors (the number of senders, the level of protection, the network loss rate and the network burstiness) influence the performance of the three FEC schemes.

Finally in Chapter 5 we summarize our findings and highlight possible future research directions. In general we find that to distribute the packets in an FEC block over multiple senders is more effective in combating packet loss and burstiness than to send one complete block over a single channel.

Chapter 2

Background

2.1 The Gilbert Model

Many researchers use a simple two state Markov chain (sometimes referred to as the Gilbert Model) (Gilbert, 1960) to model bursty packet loss both for its analytical tractability and its reasonable agreement with experimental data. The network condition of each channel is represented by two states: a good state where a packet is delivered(denoted as 0) and a bad state where a packet is lost(denoted as 1). Parameter p represents the probability of transition from good state to bad state and q represents the probability of transition from bad state to good state. Based on this model, some parameters representing the network condition of the channel can also be computed: the average packet loss rate is given by $\frac{p}{p+q}$ and the expected burst length is $\frac{1}{q}$.

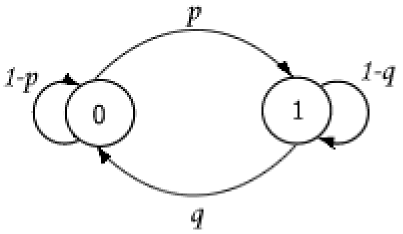

Figure 2.1: The Gilbert Model

2.2 Renewal Error Process

The Gilbert Model is based on the assumption that the lengths of gaps (intervals having no packet loss) before and after an error are independently distributed. (Elliott, 1965) thus deduced functions to calculate the probability $P(m,n)$ that m packets are lost in a block of n consecutive packets:

Let $g(i)$ denote the probability that a gap length is $i-1$, i.e., $g(i) = Pr(0^{i-1}1|1)$, where 1 denotes a lost packet and 0^{i-1} denotes $i-1$ consecutive successfully received packets. Also let $G(i)$ denote the probability that the gap length is at least $i-1$, i.e., $G(i) = Pr(0^{i-1}|1)$. Then we can calculate $R(m,n)$, the probability that $m-1$ packets are lost in the next $n-1$ packets following a packet loss, in a recurrent way (Elliott, 1965):

$$R(m,n) = \begin{cases} G(n), & m = 1; \\ \sum_{i=1}^{n-m+1} g(i)R(m-1, n-i), & 2 \leq m \leq n; \end{cases} \tag{2.1}$$

Given the average packet loss rate L of the channel, consequently we can obtain $P(m,n)$ from $R(m,n)$ using the following function (Elliott, 1965):

$$P(m,n) = \begin{cases} \sum_{i=1}^{n-m+1} LG(i)R(m, n-i+1), & 1 \leq m \leq n; \\ 1 - \sum_{i=1}^{n} P(i,n), & m = 0; \end{cases} \tag{2.2}$$

In our context where multiple channels are involved, each channel c_i is modeled using a Gilbert Model with parameters p_i and q_i. We define $P_i(m,n)$ to be $P(m,n)$ on channel i.

The above derivation will be useful in our mathematical analysis later.

Chapter 3

FEC Schemes, Assumptions and Methodology

3.1 FEC-O vs. FEC-Dstr

As distributed streaming involves multiple senders to send packets, generally speaking there are two ways of sending an FEC block. The first way is that each complete FEC block is sent by one sender only, we call this scheme FEC-O[1]. The second approach is called FEC-Dstr[2], where the packets in an FEC block are distributed to all the senders. These two schemes are illustrated in Figure 3.1, where 3 FEC blocks each with 6 packets (i.e., $N = 6$) are sent by three senders.

Note that here we are not differentiating data packets from FEC packets, what matters is only the shape of the block.

3.2 Variants of FEC-Dstr: FEC-Ded and FEC-RR

To evaluate the performance of the FEC schemes, we are most interested in their performance in terms of the number of data packets which are either successfully delivered or lost but recovered. FEC-Dstr does not distinguish between data packets and FEC packets. If the total packet loss

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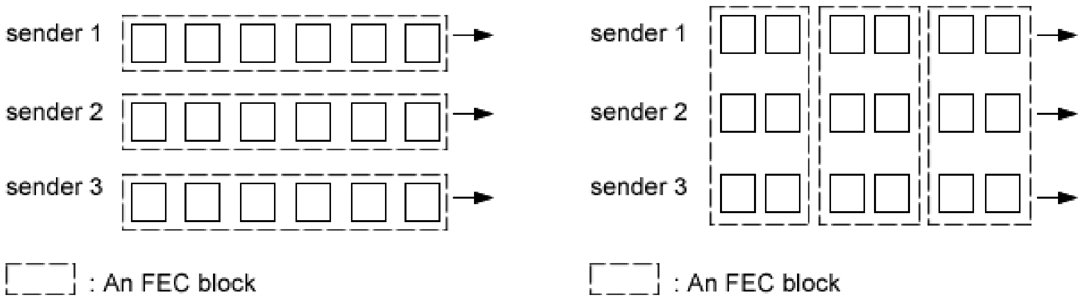
[1] O for "original"

[2] Dstr for "distributed"



(a) FEC-O

(b) FEC-Dstr

Figure 3.1: FEC-O and FEC-Dstr

in a block is less than K, this is fine since we can recover all lost data packets regardless of how many of the lost packets are data packets and how many are FEC packets. However, once the total packet loss exceeds K, we need to know exactly how many data packets are lost. Considering this we introduce two variants of the FEC-Dstr scheme: FEC-Ded[3] and FEC-RR[4]. Both of them distribute the packets within a block over all the senders, but each handles the distribution of FEC packets differently.

FEC-Ded: Under the FEC-Ded scheme, one or more channels are dedicated to send FEC packets only. Given $(N-K)$ data packets and K FEC packets to be sent over s channels, each sender is responsible for θ packets where $\theta = \frac{N}{s}$. There will be α senders dedicated to data packets and β to FEC packets, where $\alpha = \lfloor \frac{N-K}{\theta} \rfloor$ and $\beta = \lfloor \frac{K}{\theta} \rfloor$, and there will be one channel sending $((N-K) \bmod \theta)$ data packets and $(K \bmod \theta)$ FEC packets. In the special case where K is a multiple of θ, there will be $\frac{N-K}{\theta}$ channels for data packets and $\frac{K}{\theta}$ channels for FEC packets.

FEC-RR: Under the FEC-RR scheme, FEC packets are distributed to the senders in a round robin manner.

FEC-Ded and FEC-RR are illustrated in Figure 3.2, where a block containing 12 data packets and 6 FEC packets is sent over 3 senders.

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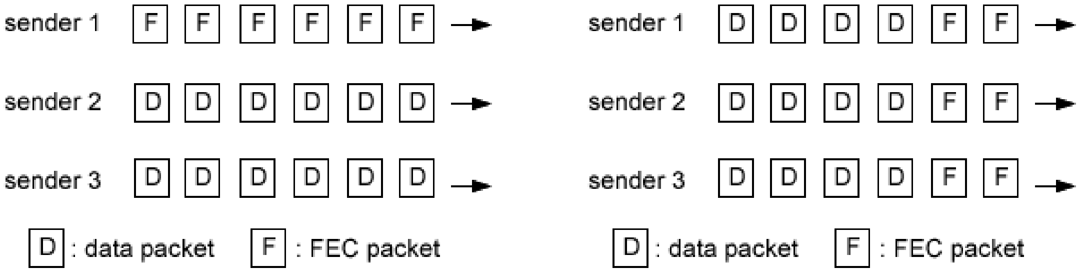
[3] Ded for ”dedicated”

[4] RR for ”Round Robin”



(a) FEC-Ded (b) FEC-RR

Figure 3.2: FEC-Ded and FEC-RR

3.3 General Assumptions

First of all we assume that the channels used to send media packets collaboratively are independent of each other. This is usually not true for the Internet, and we are looking at possible ways to handle the case where channels share common links.

We assume that packets are allocated to senders in a round robin manner under the FEC-Dstr scheme. Although not self-adapting to the changing network condition, it prevents bursty length when the channel condition of one of the senders suddenly deteriorates.

For simplicity we also assume that the total number of packets in a block, N, is divisible by the number of senders. In this way each sender is allocated the same number of packets under the FEC-Dstr scheme.

3.4 Methodology

The evaluation of the performance of these FEC schemes is based on three criteria: usable rate of a block, expected number of usable packets and burst length. The first two reflect the effectiveness of forward error correction against packet loss, and the third shows an FEC scheme's resilience against network burstiness.

In addition, there are many factors which may influence an FEC scheme's performance. In this paper, four relevant factors are identified and examined: the number of senders, the level of protection (the number of FEC packets used to protect a certain number of data packets), the network loss rate and the network burstiness. We have also considered the size of an FEC block (total number of packets in the block, both data and FEC) as one factor, but after numerous analysis and simulations we find that there is no evidence strong enough to convince us that it is a determining factor. We try to find out how each of these factors play a part in determining the performance of the FEC schemes based on the three criteria mentioned above. In order to study the effect of a particular factor, all other factors are held constant, i.e, under the *ceteris paribus* assumption.

Mathematical analysis and simulation are the main tools for evaluation. The usable rate of a block and the expected number of usable packets can be calculated analytically under valid assumptions, and the results are compared with simulation results which may help us strengthen our conclusion. The burst length is studied mainly using simulation, since mathematical analysis is too complicated to be realized.

3.5 Notations

To facilitate our mathematical analysis and explanation later, we introduce some notations in Table 3.1:

Table 3.1: Notations

$P_i(m,n)$	probability that exactly m packets are lost out of n packets on channel i
$U(m,n,x)$	probability that no more than m packets are lost out of n packets which are sent by x senders
$P_u(x)$	probability that x packets are usable out of one block
$P_{ext}(m,n,s)$	probability that exactly m packets are lost out of n packets over s channels
$P_{data}(m)$	probability that m data packets are lost out of one block
$P_{fec}(m)$	probability that m FEC packets are lost out of one block
B_s	number of bursts with length smaller or equal to 2
B_l	number of bursts with length greater than 2

Chapter 4

Analysis and Simulation

4.1 Usable Rate of a Block

First of all we try to evaluate the performance of FEC-O and FEC-Dstr under various conditions. Under these two schemes data packets and FEC packets are treated indifferently, mathematical analysis is only possible at the block level. Given an FEC block (N, K), we say that the block is *usable* if not more than K packets are lost during the transmission. The usable rate is therefore defined as in the probability that an arbitrary FEC block is usable. Based on this definition we propose mathematical models to calculate the usable rates for FEC-O and FEC-Dstr, plot graphs based on the calculation and compare the results with experimental simulations.

4.1.1 Mathematical Analysis

The analytical usable rates of FEC-O and FEC-Dstr are based on Equation (2.2).

FEC-O: An FEC block is allocated to one sender only, hence the usable rate of one block sent on channel i is

$$U_i = \sum_{j=0}^{K} P_i(j, N)$$

The average usable rate of all s channels is

$$U_{fec-o} = \frac{1}{s} \sum_{i=1}^{s} U_i$$

FEC-Dstr: Given that an FEC block contains a total of N packets, with K FEC packets and $N-K$ data packets, and that there are s senders, from our assumption in Section 3.3, each sender is allocated $\frac{N}{s}$ packets. Now define $U(m,n,x)$ to be the probability that no more than m packets are lost out of n packets which are sent by x senders, $U(K,N,s)$ then gives us the usable rate for a particular block under FEC-Dstr.

To calculate $U(K,N,s)$, we begin with a simpler case where $s=2$, i.e.,

$$U(m,n,2) = \sum_{i=0}^{m}[P_1(i,\frac{n}{2})\sum_{j=0}^{m-i}P_2(j,\frac{n}{2})]$$

where $P_i(m,n)$ is the probability that exactly m packets are lost out of n packets for sender i, which can be obtained using Equation (2.2).

Following this,

$$U(K,N,3) = \sum_{i=0}^{K}[P_1(i,\frac{N}{3})U(K-i,\frac{2N}{3},2)]$$

Hence in general,

$$U_{fec-dstr} = U(K,N,s) = \sum_{i=0}^{K}[P_1(i,\frac{N}{s})U(K-i,\frac{(s-1)N}{s},N-1)] \tag{4.1}$$

4.1.2 Analytical Results

Using the functions derived above, we can plot the graphs of block usable rate against a chosen parameter and examine the performance of FEC-O and FEC-Dstr.

To study the effect of number of senders on block usable rate, we plot graphs using $(12,3)$, $(12,4)$, $(24,5)$, $(24,8)$, $(36,7)$ and $(36,12)$ as the FEC block parameters and $\{0.05,0.45\}$, $\{0.03,0.27\}$, $\{0.1,0.4\}$, $\{0.15,0.3\}$ for parameter $\{p,q\}$ of the Gilbert Model. The results are very consistent. Figure 4.1 is a representative one showing that the usable rates of FEC-O and FEC-Dstr hardly changes with the number of senders, other factors being constant. This indicates that the number of senders may not be a determining factor in influencing the usable rates of FEC schemes.

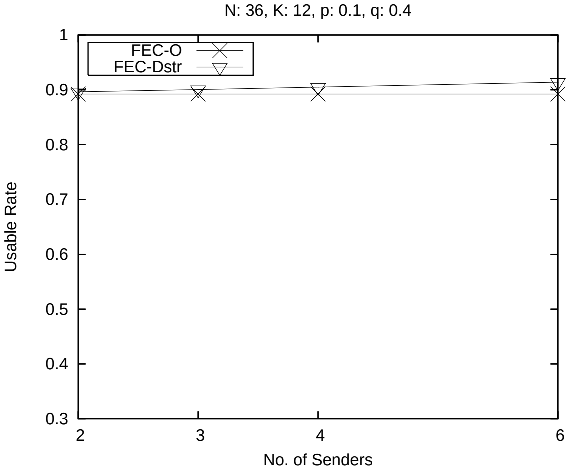


Figure 4.1: No. of Senders vs. Usable Rate

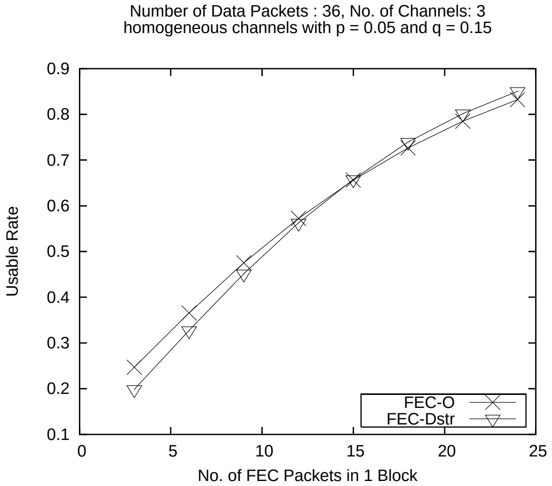
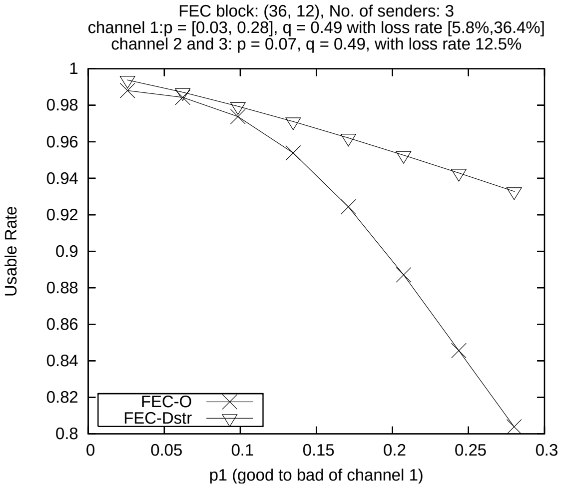


Figure 4.2: Protection Level vs. Usable Rate

Similarly for the protection level, graphs plotted using different sets of parameters show great consistency. In Figure 4.2, we can see that as the same number of data packets is protected by more FEC packets, the usable rates for both schemes will increase. It is also interesting to note that when the protection level is low, FEC-O performs better; however FEC-Dstr catches up once there are sufficient FEC packets.



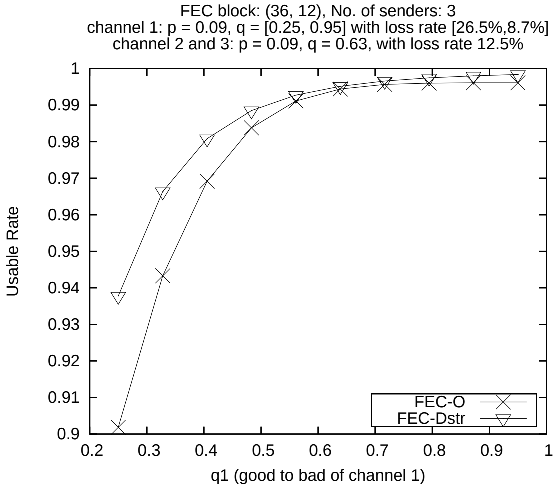


Figure 4.3: Effect of p on Usable Rate

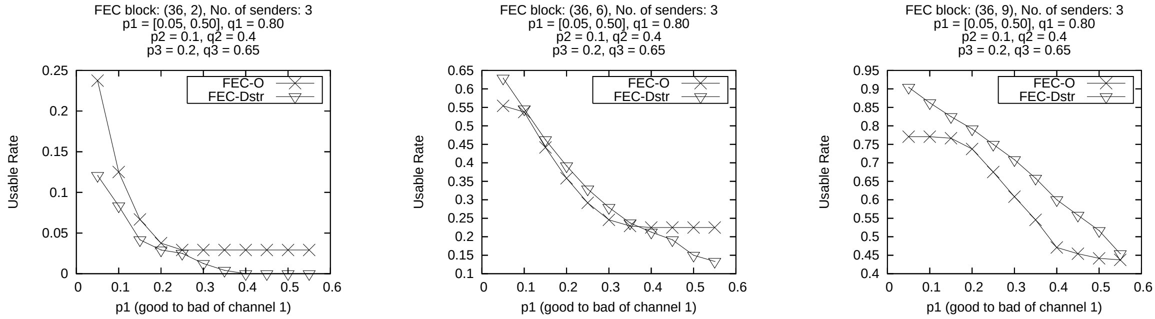
Figure 4.4: Effect of q on Usable Rate

3 channels are used when we study the effect of p and q. Two channels are given fixed

p and q while the third channel's p or q is varied. It does not matter which channel is the varying channel as we do not differentiate data packets from FEC packets here. We also choose a reasonable protection level that is neither too high nor too low, i.e., $(27, 9)$ and $(36, 12)$ are used as the FEC block parameter (N, K); $\{0.03, 0.21\}$, $\{0.05, 0.35\}$, $\{0.07, 0.49\}$, $\{0.09, 0.63\}$, $\{0.11, 0.77\}$ and $\{0.13, 0.91\}$ are used as parameter $\{p, q\}$ of the Gilbert Model. The typical results are shown in Figure 4.3 and Figure 4.4. An increasing p in the varying channel results in greater loss rate, hence lower usable rates; also we can see that a smaller q results in lower usable rates because of the high burstiness. In both figures FEC-Dstr performs better than FEC-O.

4.1.3 Simulations

Simulations are conducted in order to examine whether our analysis is valid. The simulation is first run using the same set of parameters used in obtaining analytical results. The curves that we get are almost identical to the one presented in Section 4.1.2, showing that our earlier analysis is reasonable.



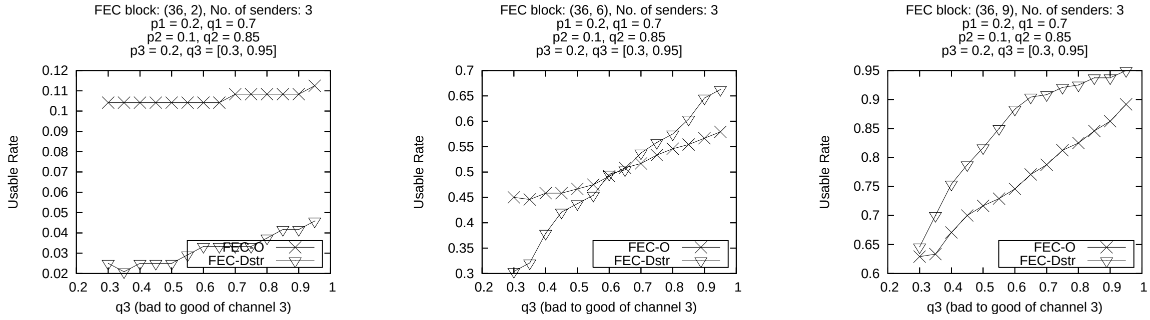
(a) FEC block (36, 2)

(b) FEC block (36, 6)

(c) FEC block (36, 9)

Figure 4.5: Simulation results on p for Usable Rate

We use simulations to model more complex cases where the channel conditions are arbitrary and heterogeneous, which are more representative of real-life situation. The results are presented in Figure 4.5 and Figure 4.6. We can clearly see that when the protection level is very low, FEC-O outperforms FEC-Dstr. However as the protection level gets higher FEC-Dstr begins



(a) FEC block (36, 2) (b) FEC block (36, 6) (c) FEC block (36, 9)

Figure 4.6: Simulation results on q for Usable Rate

to catch up and eventually surpasses FEC-O. These results are consistent with our findings in Section (4.1.2).

4.1.4 Summary

To sum up, the protection level is an important determinant of the usable rates of FEC schemes. When the protection level is low, FEC-O is a better choice in terms of usable rate of a block despite the fact that the usable rates for both schemes are low. When the protection level is high enough, FEC-Dstr is clearly the better choice.

4.2 Expected Number of Usable Packets

The usable rate of a block measures the "completeness" of the block, i.e., no data packets are affected by loss. However we are also interested in the number of usable data packets. A data packet is *usable* when it is successfully delivered to the receiver, or when it is recovered by FEC packets under the condition that at least $(N-K)$ packets out of the whole block are successfully delivered. If less than $(N-K)$ packets are received, any data packet that is lost can not be used. Number of usable data packets is an important metric that directly reflects the overall quality of the streamed media content.

Computing the number of usable packets is not possible under the FEC-Dstr scheme, hence in this section we compare FEC-O with the two variants of FEC-Dstr scheme, FEC-Ded and FEC-RR.

4.2.1 Mathematical Analysis

We denote the number of usable packets in a block by x, its expectation $E(x)$ can be expressed as:

$$E(x) = \sum_{x=0}^{N-K} x P_u(x)$$

where $P_u(x)$ is the probability that x packets are usable out of one block.

Obviously $P_u(N-K)$ is the usable rate of the block, which is computed in Section 4.1.1. To compute $P_u(x)$ with $0 \leq x \leq N-K-1$, consider that if there are x usable packets, $(N-K-x)$ data packets would be lost. Furthermore the total number of lost packets in the block must be greater than K. As a result,

$$P_u(x) = P_{data}(N-K-x) \sum_{i=(2K+x+1-N)|0}^{K} P_{fec}(i)$$

where $P_{data}(N-K-x)$ is the probability that $(N-K-x)$ data packets are lost out of one block and $P_{fec}(i)$ is the probability that i FEC packets are lost out of one block (See Table 3.1). The summation part of the equation is explained as follows: the upper bound of the number of lost FEC packets is K, and the lower bound of that is $(2K+x+1-N)$, because to have $(N-K-x)$ lost *and* unusable data packets, the minimum number of lost FEC packets is $(K+1)-(N-K-x)$. In case where $(2K+x+1-N)$ is negative, 0 is used as the lower bound. It can also proven that the lower bound will always be smaller than the upper bound by contradiction: to have the lower bound exceed the upper bound, we have $2K+x+1-N > K$, from which we get $x > N-K-1$, however the possible range of x is $[0, N-K-1]$.

So now the problem is simplified to computing $P_{data}(m)$ and $P_{fec}(m)$, given m. Before we proceed to calculate them, we compute $P_{ext}(m,n,s)$, the probability that exactly m packets are lost out of n packets over s channels, to facilitate our later derivation. Similar to Equation (4.1),

$$P_{ext}(m,n,s) = \sum_{i=0}^{m} [P_1(i, \frac{n}{s}) P_{ext}(n-i, \frac{(s-1)n}{s}, n-1)]$$

$P_{data}(m)$ and $P_{fec}(m)$ are calculated differently under different FEC schemes. While it is important to differentiate data packets from FEC packets, it is difficult to do so in mathematical analysis. Hence we assume that under FEC-O and FEC-RR, all the data packets are sent before FEC packets are sent. In addition we assume that the probability of FEC packet loss is independent from the previously sent data packets, i.e., we are treating all the data packets as one group which is independent from the group of all the FEC packets. The assumption here may be invalid when channels are bursty, however we will examine its validity later in Section 4.2.3 by comparing the analytical results with the simulation results, and we find the assumption, although may be not accurate, is still reasonable in general.

FEC-O: Based on our assumption, sender i sends $(N-K)$ data packets and then K FEC packets. Hence $P_{data}(m)$ is just $P_i(m, N-K)$ and $P_{fec}(m)$ is $P_i(m, K)$, where $P_i(m, n)$ is the probability that exactly m packets are lost out of n packets for sender i.

FEC-Ded: If there are α senders sending $(N-K)$ data packets and β senders sending K FEC packets and if $\frac{N-K}{\alpha} \equiv \frac{K}{\beta}$, it would be the ideal case. $P_{data}(m)$ would be $P_{ext}(m, N-K, \alpha)$ and $P_{fec}(m)$ is $P_{ext}(m, K, \beta)$, where $P_{ext}(m, n, s)$ is the probability that exactly m packets are lost out of n packets over s channels.

More generally, given $(N-K)$ data packets and K FEC packets to be sent over s channels, each sender is responsible for θ packets, where $\theta = \frac{N}{s}$. There will be α senders dedicated to data packets and β to FEC packets, where $\alpha = \lfloor \frac{N-K}{\theta} \rfloor$ and $\beta = \lfloor \frac{K}{\theta} \rfloor$, and there will be one channel sending $((N-K) \bmod \theta)$ data packets and $(K \bmod \theta)$ FEC packets. In this case $P_{data}(m)$ would be $P_{ext}(m, N-K, \alpha+1)$ and $P_{fec}(m)$ is $P_{ext}(m, K, \beta+1)$, where $P_{ext}(m, n, s)$ needs to be modified to handle the channel sending mixed packets.

FEC-RR: $(N-K)$ data packets and K FEC packets are both distributed across all s senders. Under the assumptions that K is also a multiple of s, calculating $P_{data}(m)$ and $P_{fec}(m)$ is relatively simple: $P_{data}(m)$ is $P_{ext}(m, N-K, s)$ and $P_{fec}(m)$ is $P_{ext}(m, K, s)$.

4.2.2 Analytical Results

We have derived the functions to calculate the expected number of usable packets given the necessary parameters. By varying these parameters we can plot these functions

Firstly the number of senders s is examined as a variable in determining the expected number of usable packets, with other factors held constant. Figure 4.7 shows that the expected number of usable packets for each of three FEC schemes hardly changes as s increases, suggesting that the effect of the number of senders seems to be insignificant. Figure 4.8 indicates that with more FEC packets protecting the same number of data packets, the number of usable packets will increase. This is expected since higher protection level is able to recover more lost packets.

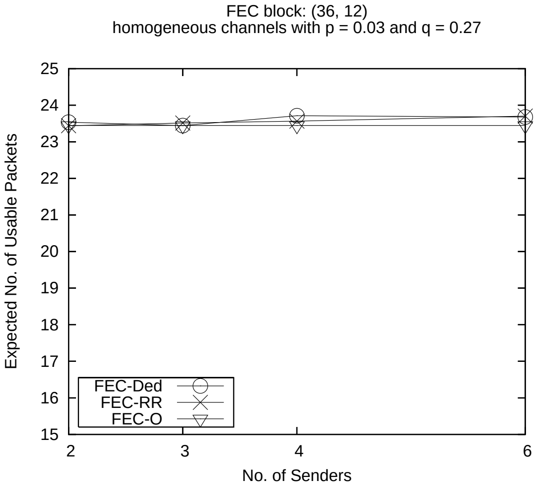


Figure 4.7: Number of Senders vs. Number of Usable Packets

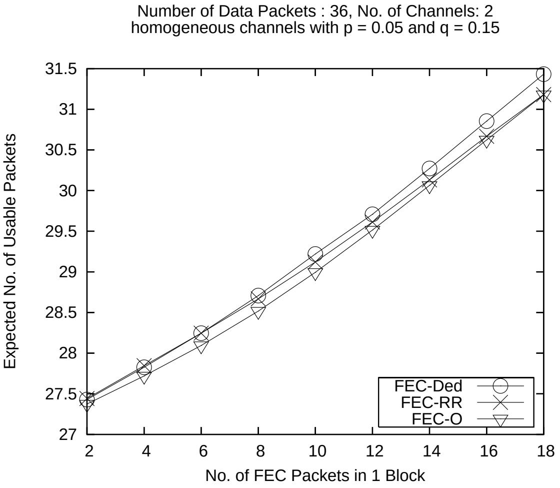
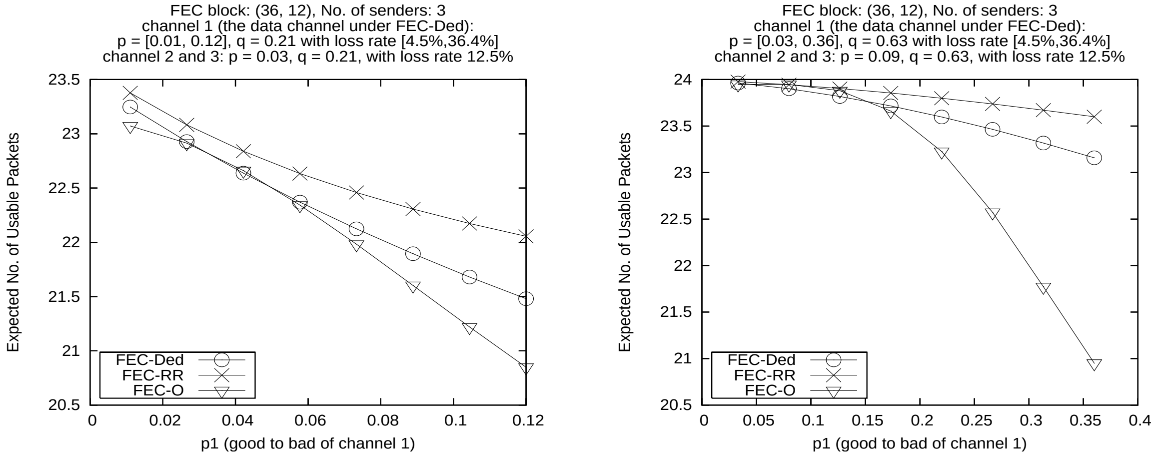


Figure 4.8: Protection Level vs. Number of Usable Packets

Three channels are used when we study the effect of p and q. Two channels are given fixed p and q while the third channel's p or q is varied. Again, different sets of parameters are used to make sure that the results are consistent. $(18, 6)$, $(27, 9)$, $(36, 12)$ are used for the FEC block parameter (N, K); $\{0.03, 0.21\}$, $\{0.05, 0.35\}$, $\{0.07, 0.49\}$, $\{0.09, 0.63\}$, $\{0.11, 0.77\}$ and $\{0.13, 0.91\}$ are used as parameter $\{p, q\}$ of the Gilbert Model.

Under FEC-RR and FEC-O it does not matter which channel is the varying channel; however under FEC-Ded a dedicated data channel is differentiated from a dedicated FEC channel. Hence we plot the functions by varying both the data channel and the FEC channel's condition

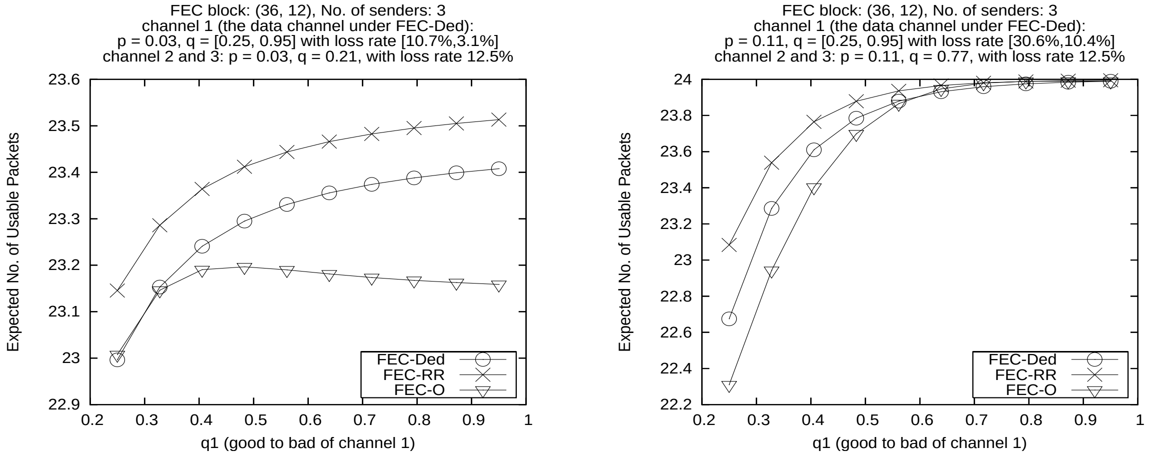


(a) 2 Fixed channels are bursty

(b) 2 Fixed channels are less bursty

Figure 4.9: Effect of p on Number of Usable Packets

respectively, and the difference is negligible. Thus we only present one set of results here.



(a) 2 Fixed channels are bursty

(b) 2 Fixed channels are less bursty

Figure 4.10: Effect of q on Number of Usable Packets

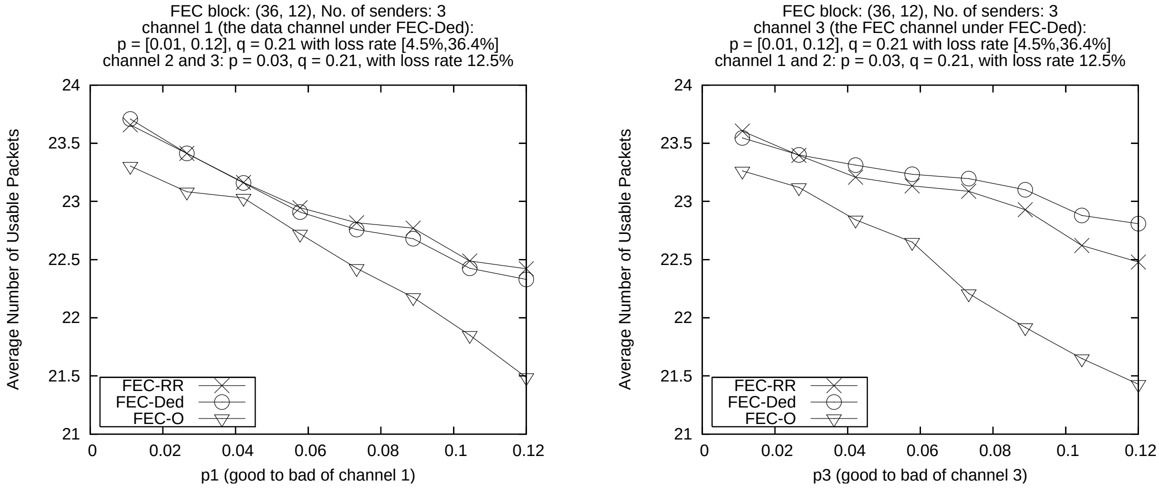
Figure 4.9 shows that an increasing p in the varying channel results in greater loss rate, hence smaller expected number of usable packets. In general FEC-O performs the worst, while FEC-RR seems to be the best. Also when the 2 fixed channels are less bursty, FEC-Dstr schemes are less affected by an augmenting p than FEC-O.

On the other hand, in Figure 4.10 we see that a smaller q implies greater burstiness causes the expected number of usable packets to drop in all 3 FEC schemes. Comparatively FEC-RR outperforms the other two and FEC-O has the lowest number of usable packets. When the 2 fixed channels are less bursty, the discrepancies between the 3 schemes are minimized, and FEC-O's performance may be close to that of FEC-Ded.

4.2.3 Simulations

We conduct simulation experiments to further strengthen the conclusions obtained from the analytical results since the mathematical analysis is based on certain assumptions.

The simulation is first run using the same set of parameters used in obtaining analytical results. Most of the curves that we get are almost identical to the ones presented in Section 4.2.2, an example is given in Figure 4.11 (a). However there is one exception: when the varying channel is the FEC channel under FEC-Ded, FEC-Ded performs better than FEC-RR and FEC-O. This is not observable from the analytical results, where FEC-RR always performs the best. This new observation, shown in Figure 4.11 (b), is highly logical considering that a deteriorating condition in the FEC channel will have less significant impact on FEC-Ded than on FEC-RR.

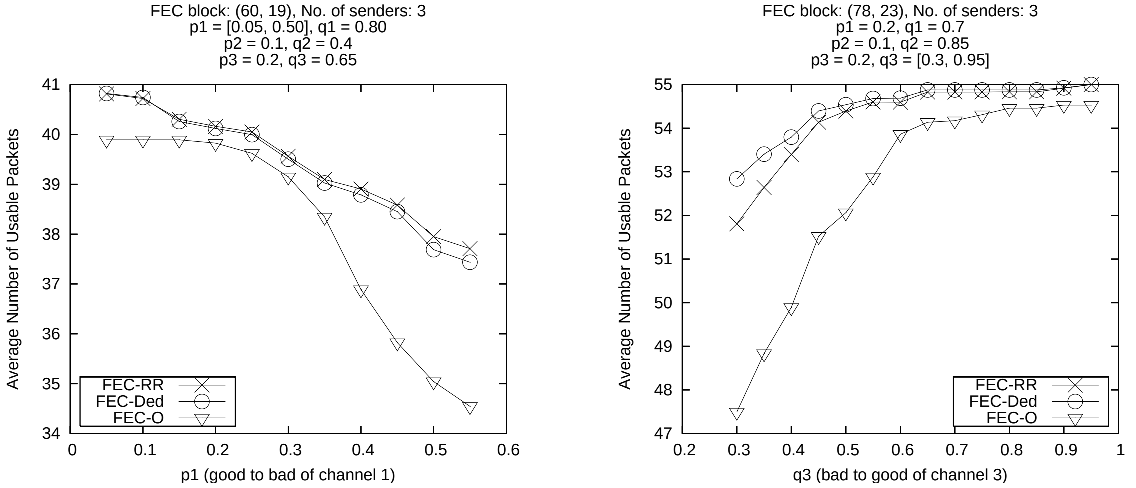


(a) Average No. of Usable Packets vs. p1

(b) Average No. of Usable Packets vs. p3

Figure 4.11: Redo the analytical experiments in simulation

We are more interested in cases where the channel conditions are arbitrary and heterogeneous, which are more representative of real-life situation. We purposely choose arbitrary parameters such as (78, 23) for the FEC block parameter (N, K), and the chosen channel condition parameters p and q also differ greatly from each other. Nevertheless, the simulation results show great concordance with our previous findings and are presented in Figure 4.12.



(a) Average No. of Usable Packets vs. p1

(b) Average No. of Usable Packets vs. q3

Figure 4.12: Simulation on Number of Usable Packets

Figure 4.12 shows that even under heterogeneous channel conditions, it is obvious that FEC-O has the lowest number of usable packets, which confirms our earlier findings from the analytical approach. Again we can see that FEC-Ded performs better than FEC-RR when the deteriorating channel is dedicated to FEC packets. Hence our assumptions used in mathematical analysis may be distorting the analytical results, in the sense that the advantage of using FEC-Ded by sending FEC packets on the worst channel is not reflected. Other than that, the mathematical analysis is still quite accurate.

4.2.4 Summary

Using both mathematical analysis and simulation experiments, we are able to conclude that in terms of number of usable packets, FEC-O is no match for the 2 FEC-Dstr schemes. FEC-Ded and FEC-RR have consistently to be proven to perform better and FEC-RR usually has the highest number of usable packets unless one of the channels exhibit high burstiness or loss rate and FEC-Ded happens to dedicate this channel for FEC packets.

4.3 Packet Loss Burst Length

Besides usable rate, burst length is also a very important metric in evaluating an FEC scheme's performance. It is very complex to do a mathematical analysis on the burst length distribution, hence we choose to study it based on the results obtained from simulations.

4.3.1 Counting the Burst Length

A burst of packet loss is usually defined as consecutive loss of packets bounded by received packets at the start and the end. In our case, we are interested in looking at the *burst of consecutive unusable data packets* at the ADU level. Hence the following rules for counting bursts apply:

- If no more than K packets are lost out of one block, every data packet is then usable and there is no unusable packets.
- If more than K packets are lost out of one block, any data packet that is lost is unusable. Any lost FEC packet should be ignored and not counted.
- As a result, unusable data packets occurred consecutively (they are allowed to be interleaved with FEC packets, since FEC packets are not counted) form a burst
- A burst can encompass data packets from multiple FEC blocks, since at the ADU level the concept of a block does not apply

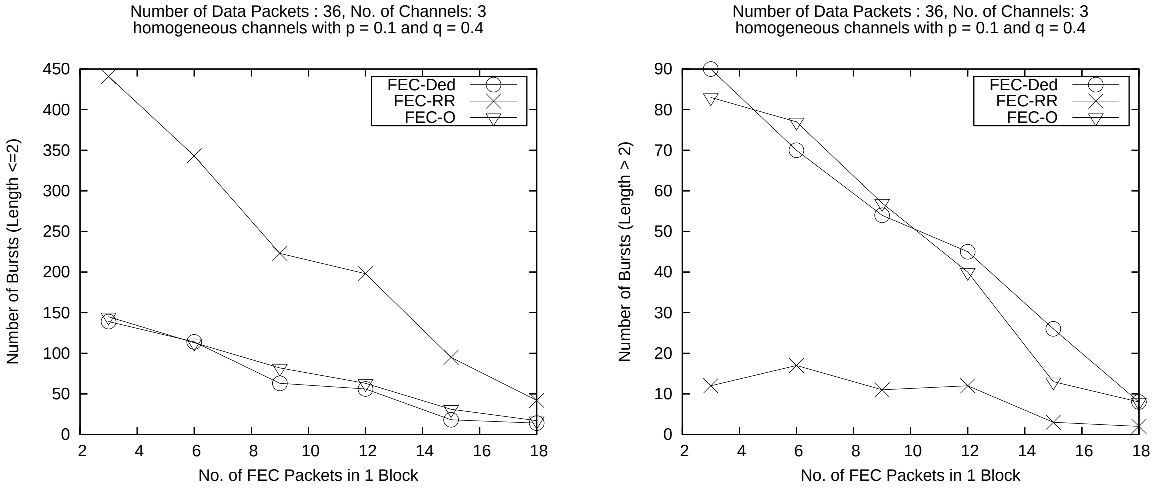
We choose to measure the number of bursts with length smaller than 2 (denoted by B_s) and the number of bursts with length greater than 2 (denoted by B_l) instead of the average burst length, because we think that the average burst length can not accurately reflect the quality of playback. A decrease in the average burst length can be due to a drop in B_l, which is good; however it can also be attributed to an increasing B_s with B_l unchanged, in this case the effect to the playback quality is adverse. Hence we can not correlate the behavior of the average burst length with the final playback quality. 2 is chosen as the watershed as we deem that two consecutive lost packets are still acceptable and can be tolerated.

We ran the simulations under the 3 schemes respectively. FEC-Ded and FEC-RR are exactly as described in Section 3.2. For FEC-O, within a block the FEC packets are interleaved with data packets to reduce the negative impact of bursty loss.

4.3.2 Simulation Results

Protection Level

We are concerned with whether the level of FEC protection will have any effect on the burst length. With a fixed number of data packets (i.e., $N-K$ is fixed), we raise the level of protection by increasing the number of FEC packets. For each protection level, we run the simulation to obtain B_s and B_l for each of the three schemes.



(a) B_s: Bursts with Length ≤ 2

(b) B_l: Bursts with Length > 2

Figure 4.13: Effect of protection level on Burst Length

The simulation was run using 36, 54 and 60 as the parameter $(N-K)$. For each of them,

K, the number of FEC packets is increased from 0 to 1/2 of $(N - K)$. A total of 84 blocks are sent using simulation.

The final results obtained are very consistent. From Figure 4.13 it is clear that as the level of protection gets higher, the number of bursts decreases. This is expected as more protection is positively correlated with greater resilience to burstiness. It can be also observed that FEC-RR performs much better than the other two schemes in terms of B_l. Even when there is minimal protection, FEC-RR still manages to keep B_l at a very low level. On the other hand, FEC-RR has a higher B_s compared to the other two schemes, showing that while it is effective in curbing long bursts, it still cannot eliminate packet loss bursts.

Effect of p

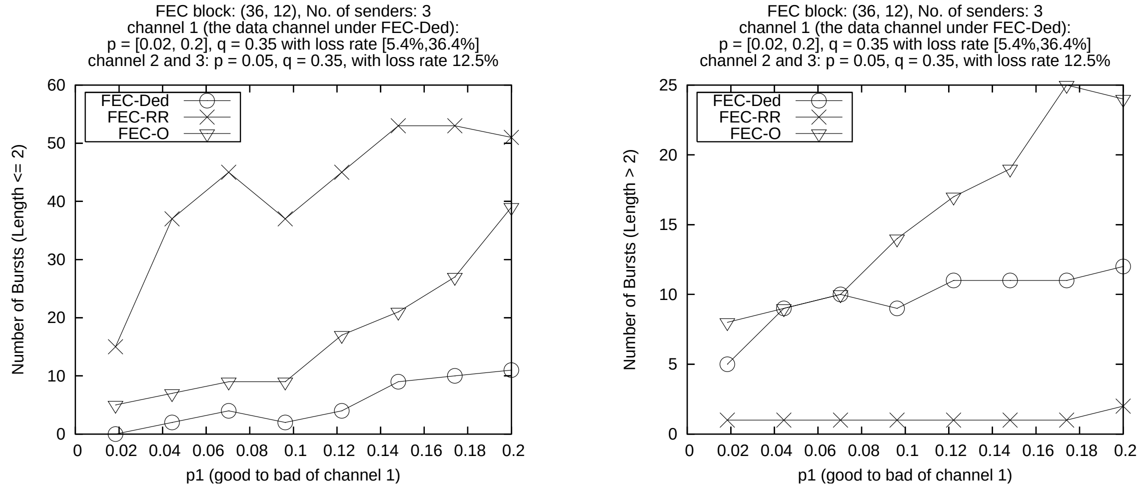
The parameter p (the probability of transition from good state to bad state) directly determines a channel's loss rate. We conduct the simulation using three channels, two of which having the same fixed channel condition while the third one having variable p. Under FEC-Ded, it makes a difference whether it is the channel dedicated to data packets or to FEC packets (or mostly FEC packets). Hence our simulations handle the data channel and the FEC channel differently.

We use $(18, 6)$, $(24, 8)$ and $(36, 12)$ for the FEC block parameter (N, K); $\{0.03, 0.21\}$, $\{0.05, 0.35\}$, $\{0.07, 0.49\}$, $\{0.09, 0.63\}$, $\{0.11, 0.77\}$ and $\{0.13, 0.91\}$ are used as parameter $\{p, q\}$ of the Gilbert Model for the fixed channels. 84 blocks are sent in the simulation.

Figure 4.14 shows how p of the data channel under FEC-Ded influences the burst length distribution. When all channels are bursty, a higher p induces greater B_s and B_l except for FEC-RR, which keeps B_l low all the time. When the channels are less bursty, only FEC-D is significantly influenced by p, FEC-RR and FEC-Ded are more resilient to higher loss rate.

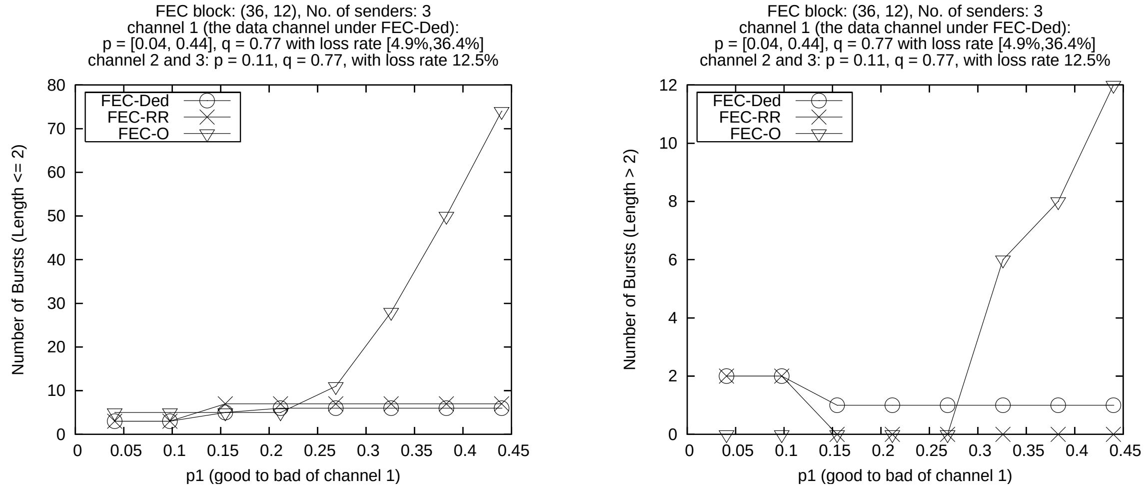
In all cases, FEC-RR manages to perform the best in terms of B_l. When the channels are bursty, it has the highest B_s, but once the channels experience less burstiness, FEC-O's B_s surges and surpasses the FEC-Dstr schemes.

We also explore how p of the FEC channel under FEC-Ded influences the burst length. We find that there is no significant difference in FEC-Ded's burst length behavior, whether the variable channel is the FEC channel or the data channel. This is perhaps because that p does not determine the burstiness of the network channel, and hence has less obvious effect on the burst length distribution. This can be contrasted with our findings in the following section, on the effect of q.



(a) B_s on Bursty Channels

(b) B_l on Bursty Channels



(c) B_s on Less Bursty Channels

(d) B_l on Less Bursty Channels

Figure 4.14: Effect of p (data channel) on Burst Length

Effect of q

Parameter q is the determining factor of a channel's burstiness in the Gilbert Model. A smaller q implies greater expected burst length, hence greater burstiness. The simulations are similar to those of p.

We use $(18, 6)$, $(24, 8)$ and $(36, 12)$ for the FEC block parameter (N, K); $\{0.03, 0.21\}$, $\{0.05, 0.35\}$, $\{0.07, 0.49\}$, $\{0.09, 0.63\}$, $\{0.11, 0.77\}$ and $\{0.13, 0.91\}$ are used as parameter $\{p, q\}$ of the Gilbert Model for the fixed channels. 84 blocks are sent in the simulation.

Figure 4.15 shows that in general, as q increases, B_s and B_l decrease. The adverse effect on FEC-RR by a small q is only reflected in a greater B_s, B_l is almost unaffected by q, showing that FEC-RR is indeed resistant to bursty loss.

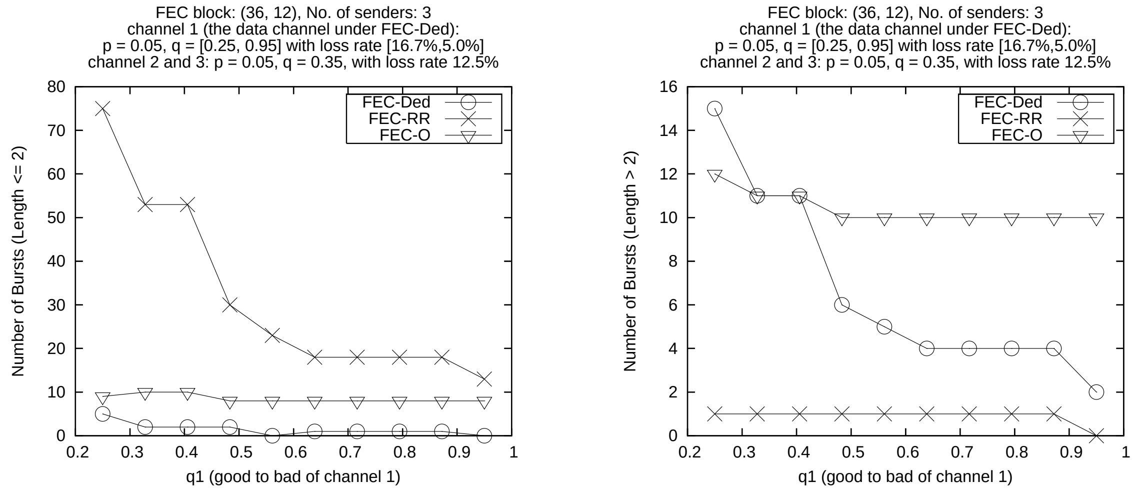
When the variable channel is the FEC channel under FEC-Ded and the other 2 fixed channels are not bursty, we see that FEC-Ded has a higher B_s and lower B_l in Figure 4.16 than in Figure 4.15, this may be explained by the fact that burstiness mainly affects the FEC channel, hence causing less damage to the data packets.

Effect of Number of Senders

We did simulations using different number of senders as well. Just like the usable rate and the expected number of usable packets, no trend can be spotted in the burst length distribution as the number of senders varies, suggesting that the number of senders may not be a determining factor of the burst length distribution.

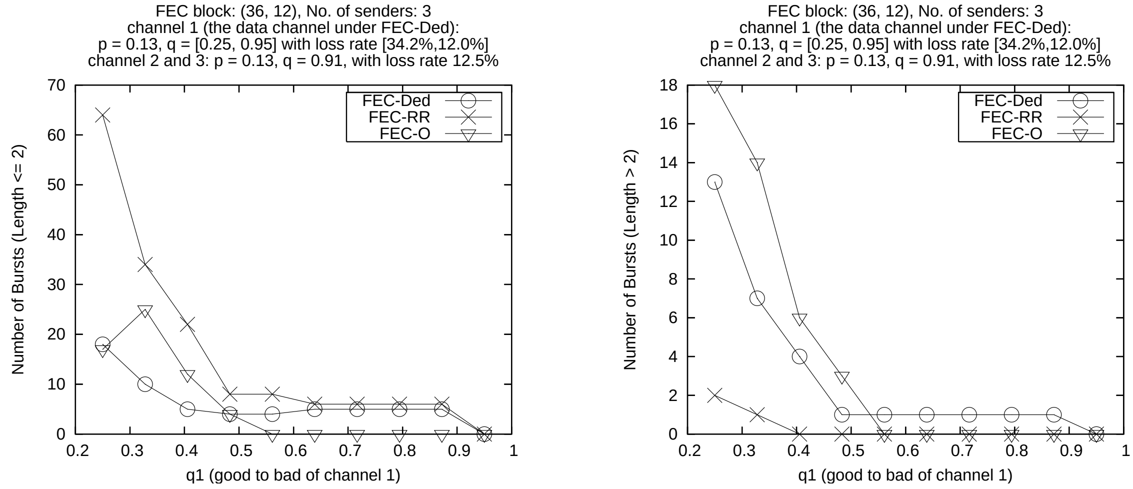
4.3.3 Summary

FEC-RR is very effective in keeping the burst length short even under highly bursty channel condition, although it has greater B_s. For FEC-Ded, if there is only one bursty channel and it is dedicated to FEC packets, its performance in B_l is comparable to that of FEC-RR. FEC-O results in a higher B_l in general.



(a) B_s with Fixed Bursty Channels

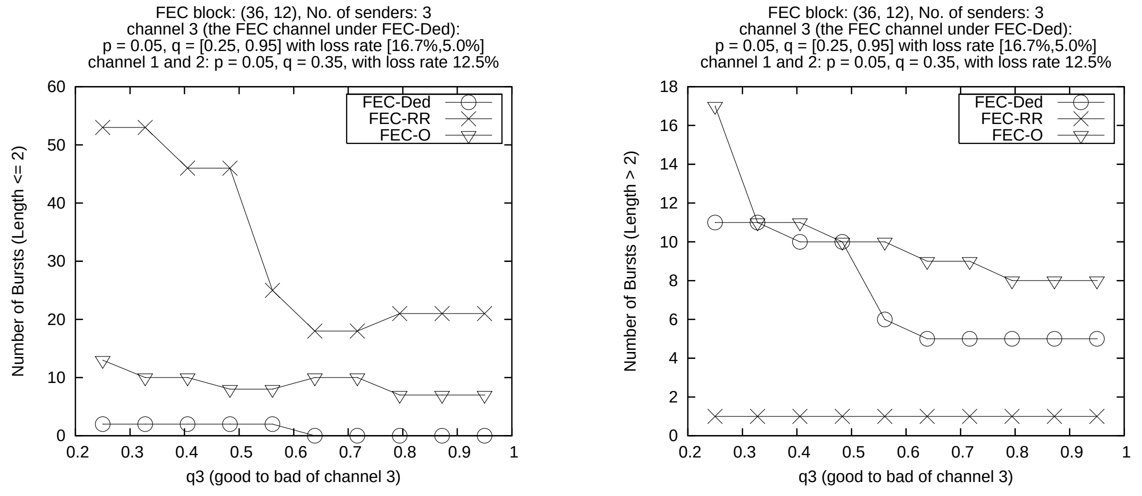
(b) B_l with Fixed Bursty Channels



(c) B_s with Fixed Less Bursty Channels

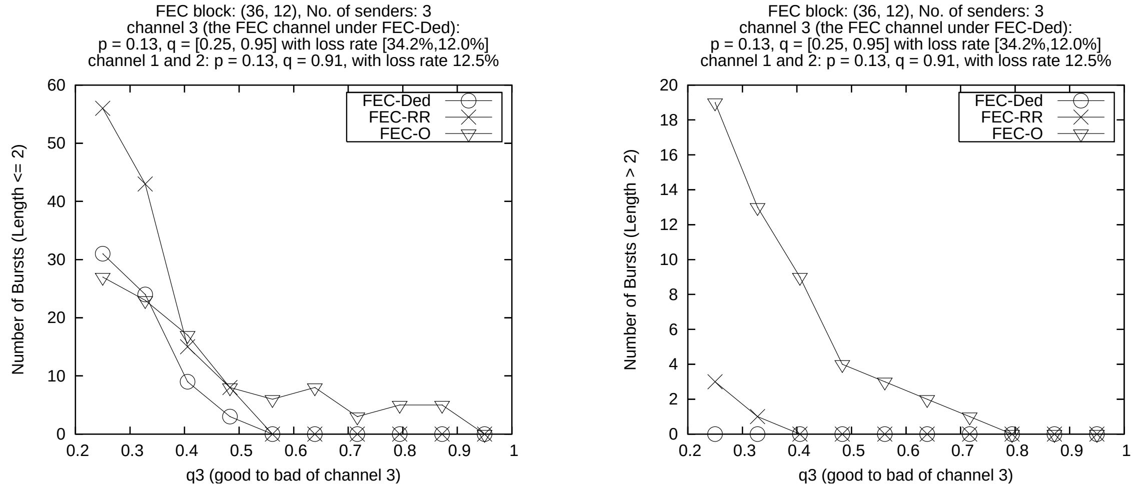
(d) B_l with Fixed Less Bursty Channels

Figure 4.15: Effect of q (data channel) on Burst Length



(a) B_s with Fixed Bursty Channels

(b) B_l with Fixed Bursty Channels



(c) B_s with Fixed Less Bursty Channels

(d) B_l with Fixed Less Bursty Channels

Figure 4.16: Effect of q (FEC channel) on Burst Length

Chapter 5

Conclusion

5.1 Summary of Findings

In this paper, we propose three FEC schemes, FEC-O and two variants of FEC-Dstr, FEC-Ded and FEC-RR, that can be used in distributed streaming for error correction. Their performance is evaluated in terms of (i)usable rate of a block, (ii)expected number of usable packets and (iii)burst length distribution using mathematical analysis and simulations.

We find that given a reasonable level of protection, the following conclusions can be reached:

- FEC-Dstr scheme outperforms FEC-O in terms of block usable rate in all aspects
- FEC-O also does poorly compared to FEC-Ded and FEC-RR in terms of number of usable packets
- FEC-RR usually has the highest number of usable packets out of all 3 schemes
- For burst length, FEC-RR clearly demonstrates that it is the most effective scheme in keeping the burst length short
- FEC-Ded is a suitable replacement for FEC-RR to achieve high number of usable packets and short burst length when only one channel is bursty and that channel can be dedicated for FEC packets only

5.2 Contributions

This project is an extension of a Ph.D student's work in the School of Computing. His original work is restricted to a special case where the FEC block is fixed at (3,1) and the number of channels is held constant at three. Since there is only one FEC packet in each block, only FEC-Ded and FEC-O were studied. In this paper I broaden the scope so that more complex cases involving arbitrary number of multiple senders with arbitrary FEC block size and protection level can be studied. More specifically, some of my more important contributions in shaping and developing this project include:

- Constructed mathematical functions to compute the usable rate of a block in general (Section 4.1.1)
- Came up with the idea of using expected number of usable packets as another important evaluation metric
- Introduced FEC-RR as a novel FEC scheme to be studied
- Found analytical means to compute expected number of usable packets (Section 4.2.1)

5.3 Future Work

We are considering the possibilities of adapting the FEC schemes under changing network conditions to increase both the usable rate of a block and the number of usable packets and to reduce burst length. We are also exploring if the assumption of independent channels can be removed.

References

Deering, S., Estrin, D. L., Farinacci, D., Jacobson, V., Liu, C.-G., & Wei, L. (1996). The PIM architecture for wide-area multicast routing. *IEEE/ACM Transactions on Networking*, *4*(2), , 1996, 153–162.

Deering, S. E. (1992). *Multicast routing in a datagram internetwork.* PhD thesis, Stanford, CA, USA.

Elliott, E. O. (1965). A model of the switched telephone network for data communications. *Bell Syst. Tech. J.*, *44*(1), January, 1965, 89–109.

Gilbert, E. N. (1960). Capacity of a burst-noise channel. *Bell Syst. Tech. J.*, *39*, September, 1960, 1253–1265.

Hefeeda, M., Habib, A., Botev, B., Xu, D., & Bhargava, B. (2003). Promise: Peer-to-peer media streaming using collectcast. *Proceedings of ACM International Conference on Multimedia (MM'03)*, Berkeley, California, USA, Novemeber, 2003.

Ma, L., & Ooi, W. T. (2005). Retransmission in distributed media streaming. *NOSSDAV '05: Proceedings of the international workshop on Network and operating systems support for digital audio and video* (pp. 117–122), New York, NY, USA, June, 2005: ACM Press.

Nguyen, T., & Zakhor, A. (2002a). Distributed video streaming over internet. *Proceedings of SPIE Conference on Multimedia Computing and Networking*, San Jose, California, January, 2002.

Nguyen, T., & Zakhor, A. (2002b). Distributed video streaming with forward error correction. *Proc. Packet Video Workshop*, Pittsburgh, USA, April, 2002.

Papadopoulos, C., & Parulkar, G. M. (1996). Retransmission-based error control for continuous media applications. *Proc. 6th Intl. Workshop on Network and Operating System Support for Digital Audio and Video (NOSSDAV)*, Zushi, Japan, April, 1996.

Perkins, C., Hodson, O., & Hardman, V. (1998). A survey of packet loss recovery techniques for streaming audio. *IEEE Network*, *12*, September/October, 1998, 40–48.

Sisalem, D., & Schulzrinne, H. (1998). The loss-delay based adjustment algorithm: A TCP-friendly adaptation scheme. *Proceedings of NOSSDAV*, Cambridge, UK., July, 1998.