Graphics Qualifier 2014

General questions

G1: Motion Fitting

You want a particle P to be at location A at time t=a, at location B at time t=b, etc.

a) Provide a formulation of its parametric position P(t) when you are given only two such constraints: P(a)=A and P(b)=B. Let I(a,A,b,B,t) return such P(t).
b) Now assume that you are given 5 such interpolated points. Provide the code for implementing I(a,A,b,B,c,D,d,D,e,E,t) using combinations of I().
c) Provide an intuitive explanation that justifies your code.
d) Discuss the usefulness of that approach for interpolating more such constraints.

G2: Finger Motions

Assume that you have a large image, I, of a map of Atlanta shown on the screen and that, at each frame, you have access to the current positions, L and R, of the fingers of the user. Explain in details how you would implement an interactive system where the user can move, translated, and scale the map by moving her fingers (we want an effect that is similar to what happens when using Google map on an iPhone).

G3: Blob Design

The user has drawn a closed-loop curve C. Explain at a high level how you would compute a 3D triangle mesh T that bounds a blobby shape that has C as silhouette (assuming parallel projection). Provide a precise mathematical formulation of the shape and explain briefly how you would compute T. We expect you to make references to mathematical concepts that may help you explain the semantics of this “fleshing” procedure. If time permits, discuss the differences in the various approaches published so far.

G4: Locked Loops

You are given two polygonal curves P and Q in 3D. These need not be planar curves. Suggest a practical (and polynomial) algorithm for testing whether it is possible to separate them by or prove that such an algorithm is impossible. Provide an answer for two different situations:

a) P and Q remain rigid and should not collide
b) P and Q should not collide but can deform

G5: Stereoscopic Panorama

You place a camera horizontally on a horizontal turntable, but off center, so that the axis of the camera does not pass through the center of rotation of the turntable, but is tangent to the circle upon which the camera travels as the turntable rotates. You capture a high resolution video of a static scene for a whole 360 rotation. We want to use that video to produce an interactive environment where the viewer can look in any direction and see a stereoscopic image of a room in which the turntable is located. Make all necessary assumptions (on the position of the viewer’s head and on the size of the turn table) and explain how to support such a stereoscopic panorama.

G6: Mesh Slicing

You are given a watertight and manifold triangle mesh M represented by a corner table. You are also given a plane P. Provide an overview and details of a simple algorithm for computing the intersection loops of M with P. For simplicity, assume “general position” (explain exactly what is meant by this). The result of your algorithm should be a set of closed-loop polygonal curves (explain why).

Rendering

R1: Light Paths

Your first job at ILM is to write a renderer that uses photon mapping, and that accounts for all the various possible light paths in a scene. Your manager has asked you to make sure that certain light paths in particular are properly accounted for. She is specifying these different light paths using Paul Heckbert's notation, where L is a light source, D is a diffuse surface, S is a specular surface, and E is the eye. The asterisk symbol (+) refers to the light path striking one or more objects of a particular surface type. For each of the following light paths, describe in words what lighting effects this accounts for. Then describe how your photon mapping renderer will properly account for the effect.

a) LDS+E
b) LS+DE
c) LD+E

R2: Reconstruction Filters

Your manager at ILM has asked you to prepare a tutorial for other employees about *reconstruction filters* for computer graphics. She has suggested several contexts in which reconstruction filters are used, and would like you to write about each of them. For each context, specify which reconstruction filter is typically used (e.g. its shape or equation), why this particular filter is used, and what computation is necessary to perform the reconstruction.

a) Rendering a texture under magnification with a mipmap
b) Displaying an image on an LCD monitor
c) High quality pixel-level anti-aliasing in a ray tracer

R3: Cartoon Rendering

You have decided to quit ILM and work instead at a video game company. Your manager (who moved at the same time as you from ILM) asks you to create a real-time renderer for video games. She would like your renderer to incorporate several non-photorealistic effects. For each effect below, describe how you would produce these effects for rendering a human-like character in a real-time setting, say, using OpenGL. Pre-processing is allowed in order to prepare the character for eventual rendering.

a) Silhouettes
b) Cel shading (e.g. limited number of shades on a character)
c) Hatch marks in the shadow regions on a character. Be sure to specify how the directions of these hash marks are determined.

R4: Perlin Noise

Each of the following questions investigates some aspect of Perlin noise, as described in Ken Perlin's 1985 paper "An Image Synthsizer".

a) How can you generate an "infinite amount" of Perlin noise, without storing a large table of values?
b) What does it mean for Perlin noise to be *band limited*? How do we know it is band limited, or how could we verify whether or not this is the case?
c) Perlin uses a variant of his noise function to create what appear to be bumps on the surface of objects. How is this accomplished?

Animation

1. Given a linear ordinary differential equation and the initial state as follows,

$$\begin{bmatrix} \dot{x} \\ \dot{y} \end{bmatrix} = \begin{bmatrix} -1 & 0 \\ 0 & -10 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} \begin{bmatrix} x_0 \\ y_0 \end{bmatrix} = \begin{bmatrix} 10 \\ 10 \end{bmatrix}$$

a. Please compute the state at the next time step (h = 0.1) using the first-order implicit Euler:

$$\mathbf{x}_1 = \mathbf{x}_0 + hf(\mathbf{x}_1)$$

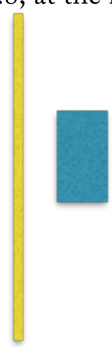
b. One student from CS4496 made a mistake on implementing implicit Euler by approximating the derivative, $f(\mathbf{x}_1)$, using the $\mathbf{x}_1$ computed from the explicit Euler. Please discuss the accuracy and plot the stability region for this formula.

$$\mathbf{x}_1' = \mathbf{x}_0 + hf(\mathbf{x}_0)$$
$$\mathbf{x}_1 = \mathbf{x}_0 + hf(\mathbf{x}_1')$$

2. Consider a 3D rigid body with the current position **x**, orientation **R**, linear velocity **v** and angular velocity w. The mass of the rigid body is 1kg and the inertia of the rigid body in the body space is $\mathbf{I}_b$. The width, length, and depth of the rigid body is 0.6, 1.0, and 0.2 respectively. If the only external force in the environment is only gravity (g = -10, for simplicity)

a. What are the position, orientation, linear momentum, and angular momentum at the next time step, assuming we use explicit Euler integrator with time step h = 0.1?

b. Does the rigid body intercept with the plane, x = -0.8, at the next time step?



$$\mathbf{x} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \quad \mathbf{R} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad \mathbf{v} = \begin{bmatrix} -2 \\ 2 \\ 0 \end{bmatrix} \quad \omega = \begin{bmatrix} 0 \\ 0 \\ 5 \end{bmatrix}$$

$$x = -0.8$$

$$\mathbf{I}_b = \begin{bmatrix} 5 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

width = 0.6
length = 1.0
depth = 0.2

3. Considering the following scenario. A 50kg human figure enters an elevator (with height = 2m) which cable is about to break. As the simulation starts, the elevator and the human figure start to free fall from x meter high. Now, the human figure is well prepared for situations like this because he always wears a jet pack when he rides an elevator. When used, this jet pack provides 500 Newtons of upward force for 0.5 second and is assumed to have no impact on the elevator (the real jet pack needs to follow Newton 3rd Law but let's simply things here). However, this jet pack can only be used once so you have to figure out the best time to use it such that you can reduce most of the landing impact and not bump your head at the ceiling of the elevator.

4. Consider two rigid bodies connected by a revolute joint (a hinge) as shown in the figure. Please derive the constraint force that enforces the revolute joint, assuming the axis of the hinge is perpendicular to the 2D plane. You can define any parameters for the rigid bodies necessary for your computation.