Graphics Qualifier, Fall 2019

Please point out any **ambiguities** that you see in the questions. For each ambiguity, pick an interesting **interpretation** that is reasonable for a qualifying exam, and explain it precisely. In your answers, please try to be **clear**, **concise**, **concrete**, and reasonably complete (when appropriate), but without wasting time on details that we know that you know. We do realize that you may not be able to provide a complete answer to each chosen question in the time given, but please make sure that what you say is **correct** and **convincing** and clearly differentiate between what you believe to be correct and what you only propose as a **conjecture**. We prefer that you formulate your solution in terms of points, vectors, dot and cross products, rather than coordinates.

1 General (Answer 4 of 6)

1.1 Point closest to lines in 3D

We are given two lines, L_1=Line(Q_1, $\vec{T}_1$) and L_2=Line(Q_2, $\vec{T}_2$), in 3D, each represented by a point and a unit tangent vector. Explain how to define and compute the point, X, that is "closest" to them in some sense. Explain an application of this construction for computing a point X from a pair of pixels, P_1 and P_1, each on a picture taken by two registered cameras with viewpoints at points E_1 and E_2. Assume that all points and vectors mentioned above (except for X) are given by their coordinates in a global frame.

1.2 Shortest interpolant in 3D

You are given 3 points, A, B, C, in 3D and want to compute an interpolating quadratic, polynomial motion, P(t), such that P(0)=A, P(1)=C, and, for some b in [0,1], that P(b)=B. You want to choose b so as to obtain the motion that traces the shortest path. Explain how you represent the motion, how you compute the optimal value of b, and how you evaluate P(t) for any time t.

1.3 Genus of a triangle mesh in 3D

You are given triangle mesh M that is a manifold with a single manifold border-loop. It has T triangles, V vertices, and E border-edges. Explain how to compute its genus G of M. Provide an example of an application or algorithm that might benefit (be simpler to implement or faster) if we know that G=0.

1.4 Occlusion in 3D

You are given two spheres, S_1 and S_2, of different radii and a planar quad Q with vertices A, B, C, and D. Explain what it means to say that Q "hides" S_1 from S_2. Outline an algorithm for testing whether it does. You do not need to provide details of how to perform standard geometric queries or constructions.

1.5 Using swing-rings in a triangle mesh

You are given a triangle mesh that is a manifold with manifold borders loops. It is represented by a Corner Table that, for each corner c, stores the *swing-corner-around-vertex*, c.s, in array S[c] and the ID, c.v, of the corresponding *vertex-at-corner* in V[c]. The *next-corner-in-triangle*, c.n, is not stored, but computed using the assumption that the 3 corners of a triangle have consecutive IDs. Assume that both swing and next operators are clockwise wrt a consistent orientation of the triangles. At a border vertex, v, the swings define a sorted sequence of corners. The c_0 and c_k be respectively the first and last corners of such a sequence. We set S[c_k]=c_o, so that the swings form a cycle, skipping once over two border edges incident upon v. We will say that the swing from c_k is a *super-swing*, even though it is not identified as such in the data structure. Given a corner c, provide a precise expression for testing whether c.s is a *super-swing*. Assuming that it is, provide an algorithm for visiting, in order, all the vertices along the border loop that contains c.v.

1.6 Central curve through ball-tube in 3D

The free-space for a motion is represented by a *clean* cycle of balls, such that each ball intersects the previous and next balls in the cycle, but is disjoint from all other balls. The boundary of the union of these balls defines a tube T. We want to move the center G(t) of a small ball of fixed radius through that tube in a periodic motion, so that the small ball remains inside the tube. We assume that the radius of the small ball is sufficiently small. Propose a simple construction of a velocity-continuous motion inside T. Provide the details of evaluating G(t).

2 Animation (Answer 2 of 4)

2.1 Planar velocity field in 2D

You are given 3 arrows, A_1, A_2, and A_3, in the plane. Each arrow, A_i, is represented by a point, P_i and by a planar velocity vector $\vec{V}_i$. These define an interpolating velocity field, which associates, with each point P, a vector $\vec{V}$(P). Explain in details how to compute $\vec{V}$(P) at any given point P and how to animate a particle as it is advected by this static field.

2.2 Collision of cones

At time t, you are given two **disjoint**, infinite, right-circular cones, K_0, and K_1, in 3D. Both move without rotating. K_0 moves with constant velocity $\vec{V}_0$. K_1 moves with constant velocity $\vec{V}_1$. How would you compute their collision time and place? Explain and justify your approach. You do not need to provide the detail of trivial geometric constructions.

2.3 Flocking in 3D

You are given a smooth, piecewise-circular, closed-loop curve C, with circular arcs C_i. You are given initial positions, B_j, of the centers-of-mass of a set of birds. You want to program a periodic (repeating) flocking motion of the birds that fly, as a group, along the curve C. You want the group behavior to appear natural: (1) each bird should fly roughly parallel to the curve, maintaining a constant distance from it, (2) as much as possible, the structure of the group should remain constant, even though distances between birds may change do to the curvature of the curve, and (3) the group does not appear to be swirling unnecessarily around C. Explain how to compute the motion of each bird.

2.4 Tetrabot crossing a river

A simplified tetrabot has a spherical body with center C and 4 telescopic legs connected to the body by ball-joints at the four vertices of a regular tetrahedron. It walks (or rather "rolls") by keeping at all times at least 3 of its feet on the ground. You must program the tetrabot to cross a river by walking only on points of a given, discrete set $\{P_i\}$ representing small stones that stick out of the water. Your input is the three points S_0, S_1, S_2 where initially the robot has three of its legs and the three points F_0, F_1, F_2 where the robot has three legs at the end of the crossing. You are told that a valid path exists. Provide the outline of an algorithm that computes a valid path C(s) for the center of the robot's body and of a strategy for moving the legs. Initially, ignore dynamics. Then, discuss how taking dynamics into account may change your solution.

3 Modeling (Answer 2 of 4)

3.1 Bijective map between two triangle meshes

We are given two manifold-without-border, zero-genus, triangle-meshes, T_1 and T_2, in 3D. They define roughly similar, quasi-parallel surfaces, but have different connectivity and different vertex and triangle counts. For example, one may be the result of simplifying or beautifying the other. Define a good bijective map M from T_1 to T_2. Given an arbitrary point P_1 of T_1 explain how to compute the corresponding point P_2=M(P_1). Discuss how you would use M to transfer texture from one mesh to the other. Your map may only work when the two meshes are "sufficiently close". That is fine. Suggest an approach for testing whether you did obtain a valid map.

3.2 BSP

A solid S is defined by a BSP (Binary-Space Partitioning) tree, T, for which each node n represents a linear halfspace H_n. Provide a CSG definition of S from the root node t of the tree. Assume that p(n), l(n), r(n) denote respectively the parent and the left/right children of n. We say that a point of the boundary of S is a true vertex of S when its *valence* (i.e., the number of incident edges of S) is more than two. Assume a general-position arrangement for all half-spaces (no 2 are parallel, not 4 pass through the same point). Explain how you would generate a sufficient set of candidate points that must include all the true vertices of S, and how you would test these candidates to establish which are the true vertices of S. Can the valence of a true vertex of S be more than 3? Justify your answer.

3.3 Best triangle-morph in 3D

We are given two similar triangles, T_0 and T_1, in 3D. The artist asked you to provide a "nice" morph that produces a time-parametrized triangle, T_t, which, as t changes smoothly from 0 to 1, starts as T_0, and evolves smoothly to become T_1. She told you that linear interpolation is ugly and that she wants a *steady* motion. Explain how you could use prior art to do this easily, whether that solution has any degrees of freedom, and, if so, how you would use them to produce the best morph or expose them through a nice user-interface.

3.4 Mesh cutting

We are given a water-tight, manifold, zero-genus triangle mesh, M. The artist wants an efficient tool for designing a "cut", C, that is a manifold loop of edge of M. She wants to do this by selecting only 3 or 4 vertices of M. Propose an "good" approach, justify why it is better than some "naïve" approach, and give details of the less trivial steps.