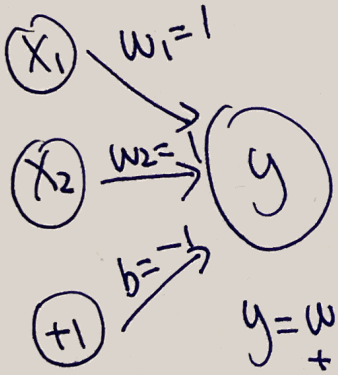
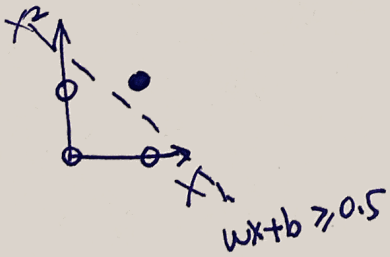
Logical AND

X_1	X_2	y
0	0	0
0	1	0
1	0	0
1	1	1

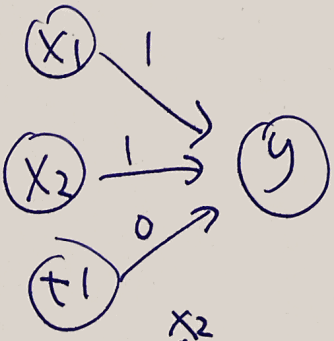


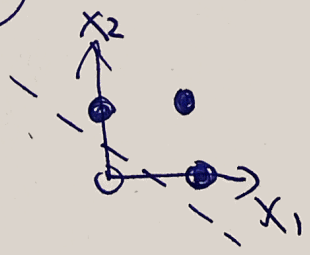
$$y = w_1 x_1 + w_2 x_2 + b$$
$$+1 \quad +1 \quad -1$$

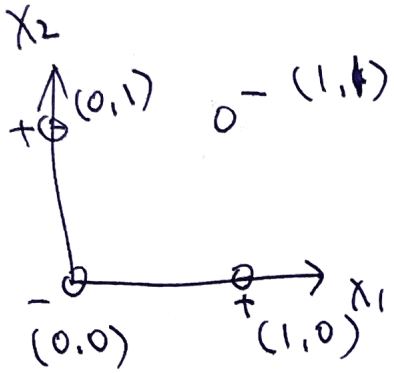


OR

X_1	X_2	y
0	0	0
0	1	1
1	0	1
1	1	1







$\tanh(0+0) = 0$

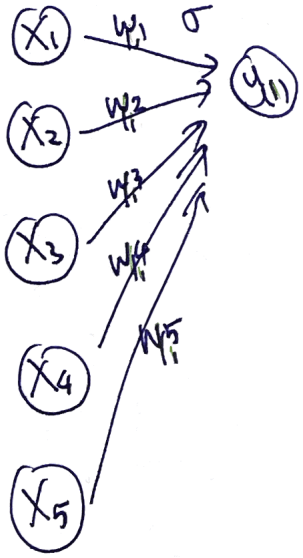
$\tanh(0+1) = \tanh(1+0) \approx 0.76$

$\tanh(1+1) \approx 0.96$

x		y
(0,0)	$\longrightarrow$	0
(1,0)	$\longrightarrow$	$-1+2\times0.76 = 0.52 > 0$
(0,1)	$\longrightarrow$	
(1,1)	$\longrightarrow$	$-2+2\times0.96 = -0.08 < 0$

---

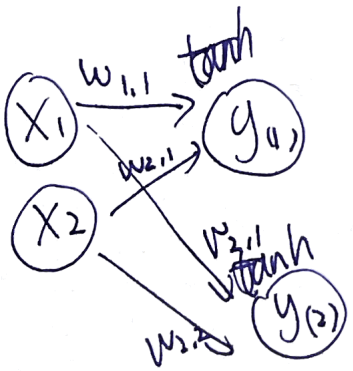
sigmoid.



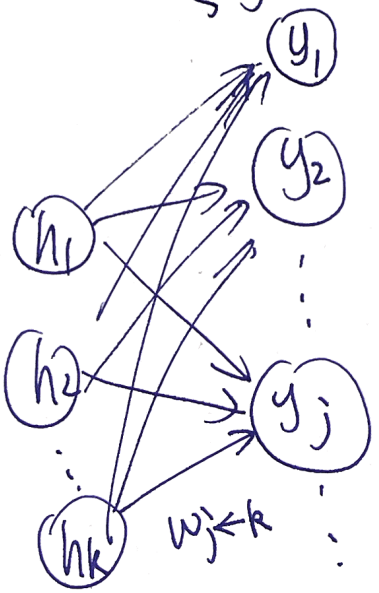
$y_{(1)} = \sigma(w_{(1)}x + b_{(1)})$

$y_{(1)} = \tanh(w_{(1)}x + b_{(1)})$

$y_{(2)} = \tanh(w_{(2)}x + b_{(2)})$



sigmoid.



$$y_j = \sum_K w_{j \leftarrow k} h_k$$

$$\Delta w_{j \leftarrow k} = \mu \delta_j h_k$$

learning rate — μ

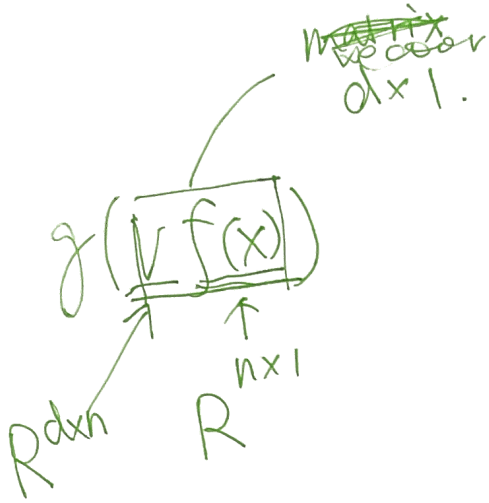
error term — δ_j

hidden node value — h_k

$$= \mu (t_j - y_j) \underline{y_j'} h_k$$

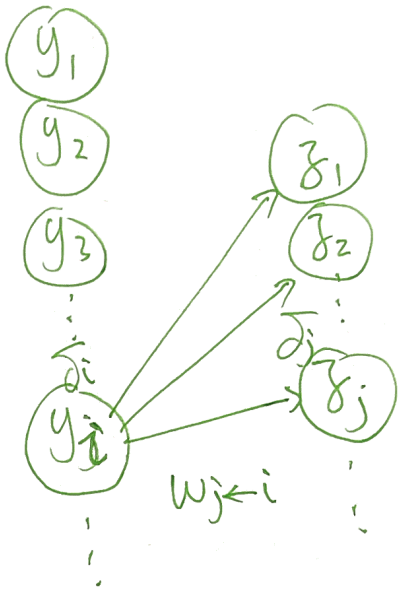
derivative of sigmoid.

$$y' = y(1-y)$$



$$z = \begin{bmatrix} g(V_1 f(x)) \\ g(V_2 f(x)) \\ \vdots \\ g(V_d f(x)) \end{bmatrix}$$

z: $d \times 1$ vector.



$$\delta_i = \left(\sum_j w_{j \leftarrow i} \cdot \delta_j \right) \cdot y_i'$$

$$\begin{bmatrix} 17 & 23 & 4 & 10 & 11 \\ 24 & 5 & 6 & 12 & 18 \\ 1 & 7 & 13 & 19 & 25 \end{bmatrix}_{3 \times 5} \times \begin{bmatrix} e_1 = 0 \\ e_2 = 0 \\ e_3 = 0 \\ e_4 = 1 \\ e_5 = 0 \end{bmatrix}_{5 \times 1} \quad i^* = 4 \qquad = \begin{bmatrix} 10 \\ 12 \\ 19 \end{bmatrix}_{3 \times 1}$$

$z = g(Vf(x))$ — $z \in \mathbb{R}^{d \times 1}$, $V \in \mathbb{R}^{d \times n}$, $f(x) \in \mathbb{R}^{n \times 1}$

$$\frac{\partial L}{\partial V_{ij}} = \frac{\partial L}{\partial z} \cdot \frac{\partial z}{\partial V_{ij}}$$

$\frac{\partial L}{\partial V_{ij}}$: $[1 \dots d]\,[1 \dots n]$

$\frac{\partial L}{\partial z} \in \mathbb{R}^{1 \times d}$, $\frac{\partial z}{\partial V_{ij}} \in \mathbb{R}^{d \times 1}$

$$\frac{\partial z}{\partial V_{ij}} = \frac{\partial g(a)}{\partial a} \frac{\partial a}{\partial V_{ij}}$$

$a = Vf(x)$

$$\frac{\partial z}{\partial V} \in \mathbb{R}^{d \times n}$$