Instructions

1. This homework has two parts: questions 1–5 are theory questions, and Q6 is a programming assignment with some parts requiring a written answer.

We will be using Gradescope to collect your assignments. Please read the following instructions for submitting to Gradescope carefully!

(a) Each subproblem must be submitted on a separate page. When submitting to Gradescope (under HW1 Writing), make sure to mark which page(s) correspond to each problem or subproblem. For instance, Q2 has 3 subproblems, so the solution to each must start on a new page. Similarly, Q3 and Q5 each have 3 subproblems, so the writeup for each subproblem should start on a new page.

(b) For the coding problem (Q6), please upload all codes under the HW1 Code assignment on Gradescope. In addition, include the writeup for each of Q6’s subproblems in your solution PDF.

(c) Note: This is a large class and Gradescope’s assignment segmentation features are essential. Failure to follow these instructions may result in parts of your assignment not being graded. We will not entertain regrading requests for failure to follow instructions.

2. LaTeX solutions are strongly encouraged (solution template available on the class website), but scanned handwritten copies are also acceptable. Hard copies are not accepted.

3. Question 1 is meant for undergraduate students only while question 4 is meant for graduate students only. Both these questions carry 5 points each. Each of the other questions is mandatory for all students and carries 10 points. The coding problem carries 50 points and a bonus. There is no extra credit for answering additional questions than what is required.

4. We generally encourage collaboration with other students. You may discuss the questions and potential directions for solving them with another student. However, you need to write your own solutions and code separately, and not as a group activity. Please list the students you collaborated with on the submission site.
Questions

1. [CS 4650 Students Only]
   Provide answers to the following operations or write “invalid” if not possible (all answers can be placed in a single page): [5 points]

   (a) \[
   \begin{bmatrix}
   2 & 4 \\
   4 & 7
   \end{bmatrix}
   \begin{bmatrix}
   2 \\
   3
   \end{bmatrix}
   \]

   (b) \[
   \begin{bmatrix}
   -1 & 3 & 6 \\
   5 & 0 & 3
   \end{bmatrix}
   \begin{bmatrix}
   2 & 1 & 3
   \end{bmatrix}^T
   \begin{bmatrix}
   -1 & -2 & 1
   \end{bmatrix}
   \]

   (c) \[
   \begin{bmatrix}
   5 & 2 \\
   1 & 7
   \end{bmatrix}
   \begin{bmatrix}
   9 & 1
   \end{bmatrix}
   \]

   (d) \[
   \begin{bmatrix}
   4 & 2 & 8
   \end{bmatrix}^T
   \begin{bmatrix}
   4 & 1
   \end{bmatrix}^T
   \]

   (e) \[
   \begin{bmatrix}
   4 & 2 & 8
   \end{bmatrix}^T
   \begin{bmatrix}
   4 & 1
   \end{bmatrix}
   \]
2. The entropy of a discrete random variable $X$ is defined as following (use base $e$ for all log operations unless specified otherwise):

$$H(X) = - \sum_{x \in \text{supp}(X)} P(x) \log P(x),$$

where the support of $X$, $\text{supp}(X)$, is composed of all values which have probability greater than 0.

(a) Compute the entropy of the distribution $P(x) = \text{Multinoulli}([0.15, 0.4, 0.45])$. [3 points]

(b) Compute the entropy of the uniform distribution $P(x) = \frac{1}{m} \forall x \in [1, m]$. [3 points]

(c) Consider the entropy of the joint distribution $P(X,Y)$:

$$H(X,Y) = - \sum_{x \in X} \sum_{y \in Y} P(x,y) \log P(x,y)$$

How does this entropy relate to $H(X)$ and $H(Y)$, (i.e. the entropies of the marginal distributions) when $X$ and $Y$ are independent? [4 points]
3. You've been hired as a consultant in a firm working to distinguish between *fake* and *real* news. Though there are many publishers, the firm has identified that the problem is hardest when news articles are from two publishers: New York Times (NYT) and Buzzfeed (BF). Before building a classifier, researchers at the firm want to run some numbers and they come to you. It's time to show off your knowledge!

*Note:* Assume for the following questions that there are only two types of news articles. Write your answer using up to 3 significant figures.

(a) John, the head of the firm, thinks that only 5% of all articles in all newspapers are fake. But he has seen that 70% of all fake news comes from BF. BF also accounts for 25% of all news articles. What is the probability that a randomly selected article from BF is fake, according to John's estimates? [3 points]

(b) Christina, another employee, has multiple news provider subscriptions. She has counted that for every 1000 articles that she sees on her feed, she ends up reading the whole article 40 times. If she reads an article it ends up being fake news half of the times. Furthermore, the fake articles she read were only 20% of the times from NYT. For a random article that appears on her feed, what is the probability that she ends up reading it, it turns out to be fake, and is from NYT? [3 points]

(c) Gary thinks the fake news problem is much worse. He estimates that 15% of all articles in all newspapers are fake. He further finds that 20% of all fake news is from NYT and 60% of all real articles comes from NYT. What is the probability that a randomly chosen NYT article is fake news, according to Gary's estimate? [4 points]
4. [CS 7650 Only]

You are training two logistic regression models on a dataset (same one for both models). \( \hat{\theta} \) is the optimal weight for an unregularized logistic regression model, and \( \theta^* \) is the optimal weight for a logistic regression model with \( L_2 \) regularization. Prove that \( ||\theta^*||_2^2 \leq ||\hat{\theta}||_2^2 \). [5 points]

Note: you may find it useful to look at the likelihood equations for regularized and unregularized logistic regression.
5. A collection of movie reviews (data $\mathcal{D}$) contains the following keywords and binary labels for whether each review was positive (+) or negative (−). The data is shown below: for example, the cell at the intersection of “Review 1” and “epic” indicates that the text of Review 1 contains 2 tokens of the word “epic”.

Answer the following questions, reporting all scores as log-probabilities, to three significant figures.

<table>
<thead>
<tr>
<th>Review</th>
<th>great</th>
<th>amazing</th>
<th>epic</th>
<th>boring</th>
<th>terrible</th>
<th>disappointing</th>
<th>Y</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>4</td>
<td>0</td>
<td>2</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>+</td>
</tr>
<tr>
<td>2</td>
<td>1</td>
<td>2</td>
<td>2</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>+</td>
</tr>
<tr>
<td>3</td>
<td>2</td>
<td>1</td>
<td>2</td>
<td>0</td>
<td>1</td>
<td>1</td>
<td>+</td>
</tr>
<tr>
<td>4</td>
<td>1</td>
<td>0</td>
<td>1</td>
<td>2</td>
<td>0</td>
<td>1</td>
<td>−</td>
</tr>
<tr>
<td>5</td>
<td>1</td>
<td>0</td>
<td>1</td>
<td>2</td>
<td>2</td>
<td>0</td>
<td>−</td>
</tr>
<tr>
<td>6</td>
<td>1</td>
<td>1</td>
<td>0</td>
<td>2</td>
<td>2</td>
<td>2</td>
<td>−</td>
</tr>
</tbody>
</table>

(a) Assume that you have trained a Naive Bayes model on data $\mathcal{D}$ to detect positive vs. negative movie reviews. Compute the model’s predicted scores for both positive and negative classes for the following sentence $S$ (i.e. $P(\text{+} | S)$ and $P(\text{−} | S)$), and determine which label the model will apply to $S$. [4 points]

$S$: The film was great, the plot was simply amazing! Makes other superhero movies look terrible, this was not disappointing.

(b) The counts in the original data are sparse and may lead to overfitting, e.g. a strong prior on assigning the negative label to reviews that contain “terrible”. What would happen if you applied smoothing?

Apply add-1 smoothing and recompute the Naive Bayes model’s predicted scores for $S$. Did the predicted label change? [4 points]

(c) What is an additional feature that you could extract from text to improve the classification of sentences like $S$ (not necessarily in NB), and how would it help improve the classification? [2 points]
6. In this problem, you will do text classifications for Hate Speech. You need to both answer the questions and submit your code.

Hate speech is a

(a) deliberate attack,
(b) directed towards a specific group of people,
(c) motivated by aspects of the group’s identity.

The three premises must be true for a sentence to be categorized as HATE. Here are two examples:

(a) “Poor white kids being forced to treat apes and parasites as their equals.”
(b) “Islam is a false religion however unlike some other false religions it is crude and appeals to crude people such as Arabs.”

In (a), the speaker uses “apes” and “parasites” to refer to children of dark skin and implies they are not equal to “white kids”. That is, it is an attack to the group composed of children of dark skin based on an identifying characteristic, namely, their skin colour. Thus, all the premises are true and (a) is a valid example of HATE.

Example (b) brands all people of Arab origin as crude. That is, it attacks the group composed of Arab people based on their origin. Thus, all the premises are true and (b) is a valid example of HATE.

This problem will require programming in Python 3. The goal is to build a Naive Bayes model and a logistic regression model that you learnt from the class on a real-world hate speech classification dataset. Finally, you will explore how to design better features and improve the accuracy of your models for this task.

The dataset you will be using is collected from Twitter online. Each example is labeled as 1 (hatespeech) or 0 (Non-hatespeech). To get started, you should first download the data and starter code from the following location:


Try to run:

```
python main.py --model AlwaysPredictZero
```

This will load the data and run a default classifier AlwaysPredictZero which always predicts label 0 (non-hatespeech). You should be able to see the reported train accuracy = 0.4997. That says, always predicting non-hatespeech isn’t that good. Let’s try to build better classifiers!

Note that you need to implement models without use of any machine learning packages such as sklearn. Your solution should use simple Python lists or numpy arrays. (numpy arrays are recommended since they are much faster than Python lists. Your program should finish running within an acceptable time limit in Gradescope.) Note the Gradescope machine will not have any external packages installed so make sure
to also delete/comment any statements importing external packages. We will only provide a training set, and evaluate your code based on our test set.

To have a quick check with your implementations, you can randomly split the dataset we give you into train and test set at a ratio of 8:2, compare the accuracy between the models you have implemented and related models in sklearn packages. You are encouraged to try sklearn in non-submitted code, to get an idea of what the expected outputs should and how much training time your models should require. None of the models you implement for this question should take longer than a few minutes to train.

Once you are done, please only submit your utils.py and classifiers.py to Gradescope.

(a) **(Naive Bayes)** In this part, you should implement a Naive Bayes model with add-1 smoothing, as taught in the class. You are required to implement the NaiveBayesClassifier class in classifiers.py. You would probably want to take a look at the UnigramFeature class in utils.py that we have implemented for you already. After you finish coding, run python main.py --model NaiveBayes to check the performance.

We will test your implementation on our test set. [15 points if accuracy ≥ 0.65, 9 pts if accuracy ≥ 0.60]

**List** the 10 words that, under your model, have the highest ratio of \( \frac{P(w|1)}{P(w|0)} \) (the most distinctly hatespeech words). List the 10 words with the lowest ratio. What trends do you see? [10 points]

(b) **(Logistic Regression)** In this part, you will implement a Logistic Regression model. You are required to implement the LogisticRegressionClassifier class in classifiers.py. First, implement a logistic regression model without regularization and run python main.py --model LogisticRegression, compare the performance with your Naive Bayes approach. Describe how both of these models differ from the perceptron classifier we talked about in class. Next, we would like to experiment with \( L_2 \) regularization. Add \( L_2 \) regularization with different weights, such as \( \lambda = \{0.0001, 0.001, 0.01, 0.1, 1, 10\} \).

**Describe** what you observed. (Again, you may want to split the training set we give you into your own training and test sets to observe the performance.) [10 points]

For the code submission, hard-code the best \( \lambda \) you observed. Your model accuracy on our test set must be greater than 0.68 to receive full credit. Similar to part (a), partial credit will be awarded for accuracy above 0.60. Please make sure your model doesn’t train for too long (more than a few minutes) locally, otherwise gradescope will time out while grading your assignment and you will receive no credit. [15 points]

(c) **(Bonus)** In the last part, you’ll explore and implement a more sophisticated set of features. You need to implement the class BigramFeature or modify the class CustomFeature in utils.py. Here are some common strategies (you are welcome to implement some of them but try to come up with more!):
i. Remove stopwords (e.g. a, the, in),
ii. Use a mixture of unigrams, bigrams or trigrams, (It may take exponentially more time to run bigram features than unigram features. Make sure total running time is within the acceptable time limit in Gradescope. For example, you can choose to implement bigrams based on Naive Bayes model, since it takes much less time than Logistic Regression.)
iii. Use TF-IDF (refer to http://www.tfidf.com/) features.
You may also implement the PerceptronClassifier for this part. You can refer to Eisenstein Ch 2 and lecture slides for more details on implementing this.
Use your creativity for this problem and try to obtain an accuracy as high as possible on your test set! We will call the BonusClassifier, which should simply specify which classifier to use, and the CustomFeature class which should contain any custom features you'd like to be used for this part. Note: if these two classes are unimplemented, Gradescope will not be able to run your code and you will not receive any bonus.
Describe the features you implemented, the model you decided to use for this part and your reasoning for this. [Bonus: 10 points]
You will receive up to 10 bonus points: up to 3 points based on novel features you try, 2 points based on whether you implemented perceptron, and the rest based on how well your model performs compared to other submissions:

$$Bonus = 5 + 5 \cdot \frac{1}{\text{rank}}$$

e.g. if you rank first in the class, you will receive the full bonus points! or if you implemented new features but did not implement perceptron and ranked first in class, you will receive 8 points.