# Topics:

Convolutional Neural Networks

# **CS 4644-DL / 7643-A ZSOLT KIRA**

# Assignment 2

- Implement convolutional neural networks
- Resources (in addition to lectures):
  - DL book: Convolutional Networks
  - CNN notes <a href="https://www.cc.gatech.edu/classes/AY2022/cs7643">https://www.cc.gatech.edu/classes/AY2022/cs7643</a> spring/assets/L10 cnns notes.pdf
  - Backprop notes
     https://www.cc.gatech.edu/classes/AY2022/cs7643 spring/assets/L10 cnns backprop notes.pdf
  - HW2 Tutorial @113, Conv @116, Focal Loss @117
  - Slower OMSCS lectures on dropbox: Module 2 Lessons 5-6 (M2L5/M2L6) (<a href="https://www.dropbox.com/sh/iviro188gq0b4vs/AADdHxX\_Uy1TkpF\_yvlzX0nPa?dl=0">https://www.dropbox.com/sh/iviro188gq0b4vs/AADdHxX\_Uy1TkpF\_yvlzX0nPa?dl=0</a>)
- FB/Meta Office hours Friday 3pm EST!
  - Pytorch & scalable training
  - Module 2, Lesson 8 (M2L8), on dropbox

$$y(r,c) = (x*k)(r,c) = \sum_{a=-\frac{H-1}{2}}^{\frac{H-1}{2}} \sum_{b=-\frac{W-1}{2}}^{\frac{W-1}{2}} x(a,b) k(r-a,c-b)$$

$$(0,0)$$

$$k_1 = 3$$

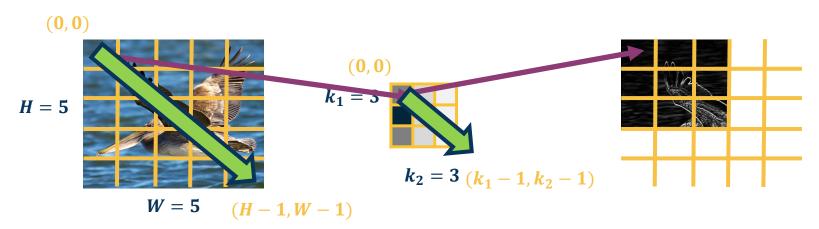
$$k_2 = 3 \quad (k_1 - 1, k_2 - 1)$$

$$W = 5 \quad \left(\frac{H-1}{2}, \frac{W-1}{2}\right)$$

$$y(0,0) = x(-2,-2)k(2,2) + x(-2,-1)k(2,1) + x(-2,0)k(2,0) + x(-2,1)k(2,-1) + x(-2,2)k(2,-2) + \dots$$



$$y(r,c) = (x * k)(r,c) = \sum_{a=0}^{k_1-1} \sum_{b=0}^{k_2-1} x(r+a,c+b) k(a,b)$$



Since we will be learning these kernels, this change does not matter!



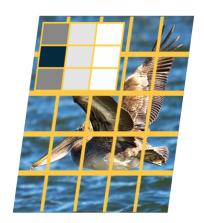
$$X(0:2,0:2) = \begin{bmatrix} 200 & 150 & 150 \\ 100 & 50 & 100 \\ 25 & 25 & 10 \end{bmatrix} \qquad K' = \begin{bmatrix} 1 & 0 & -1 \\ 2 & 0 & -2 \\ 1 & 0 & -1 \end{bmatrix}$$

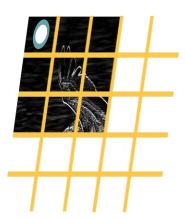
$$\mathsf{K}' = \begin{bmatrix} 1 & 0 & -1 \\ 2 & 0 & -2 \\ 1 & 0 & -1 \end{bmatrix}$$



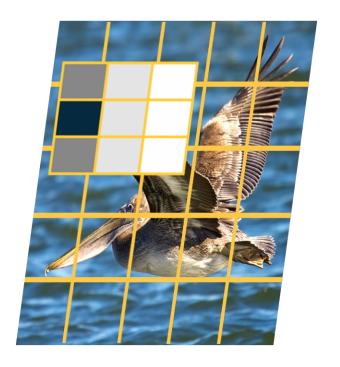
 $X(0:2,0:2) \cdot K' = 65 + bias$ 

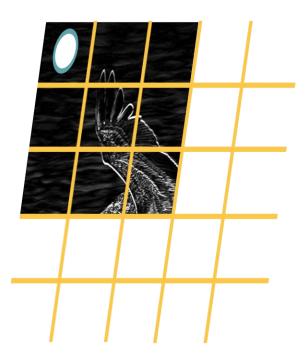
Dot product (element-wise multiply and sum)



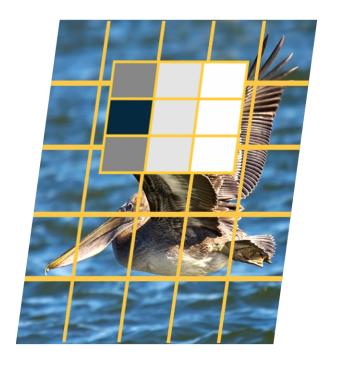


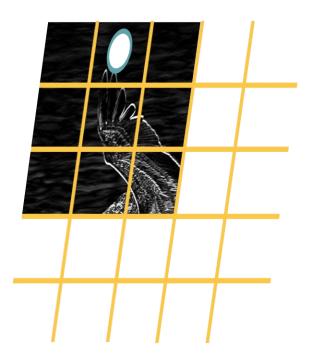




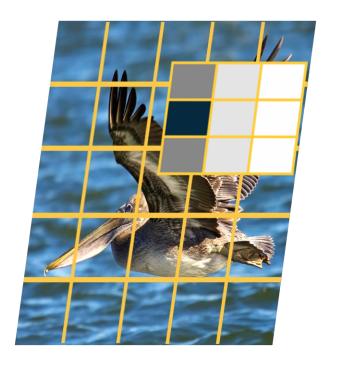


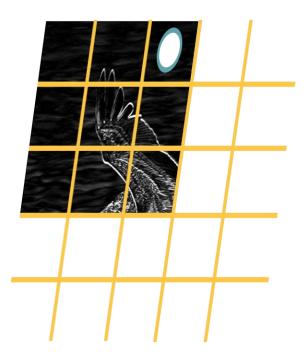




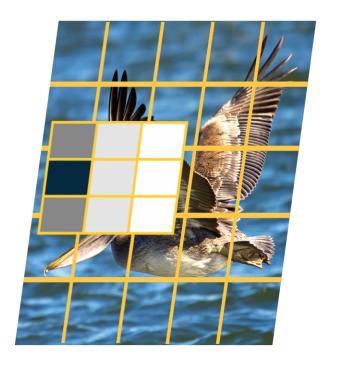


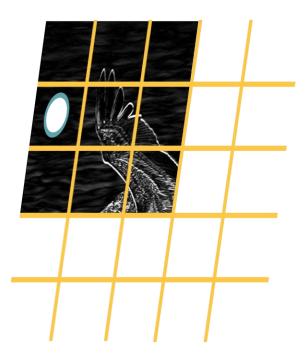




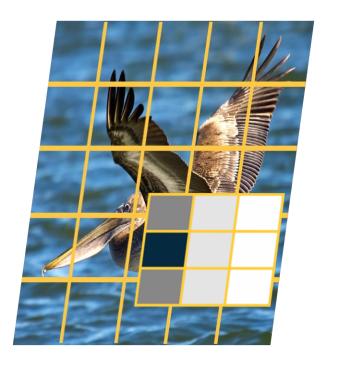


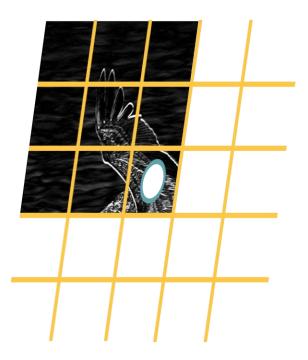














# Why Bother with Convolutions?

Convolutions are just **simple linear operations** 

Why bother with this and not just say it's a linear layer with small receptive field?

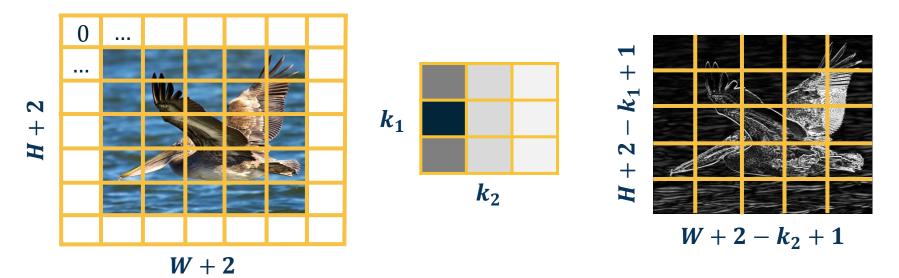
- There is a duality between them during backpropagation
- Convolutions have various mathematical properties people care about
- This is historically how it was inspired





We can **pad the images** to make the output the same size:

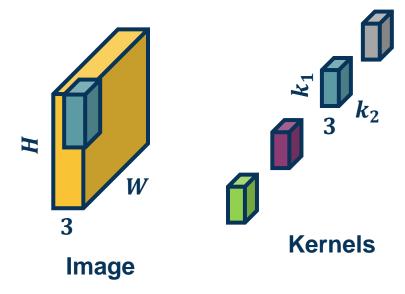
- Zeros, mirrored image, etc.
- ullet Note padding often refers to pixels added to **one size** (P = 1 here)

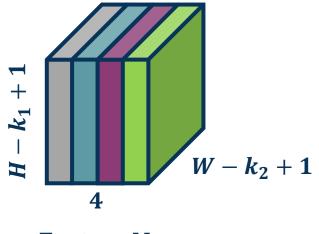


# We can have multiple kernels per layer

We stack the feature maps together at the output

Number of channels in output is equal to *number* of kernels





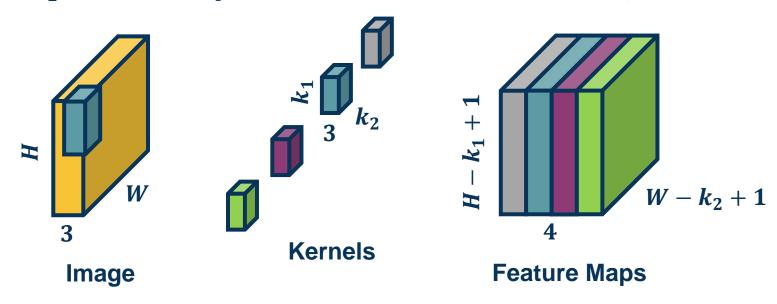
**Feature Maps** 



# Number of parameters with N filters is: $N * (k_1 * k_2 * 3 + 1)$

# Example:

$$k_1 = 3, k_2 = 3, N = 4 input channels = 3, then  $(3 * 3 * 3 + 1) * 4 = 112$$$





Need to incorporate all upstream gradients:

$$\left\{\frac{\partial L}{\partial y(0,0)}, \frac{\partial L}{\partial y(0,1)}, \dots, \frac{\partial L}{\partial y(H,W)}\right\}$$

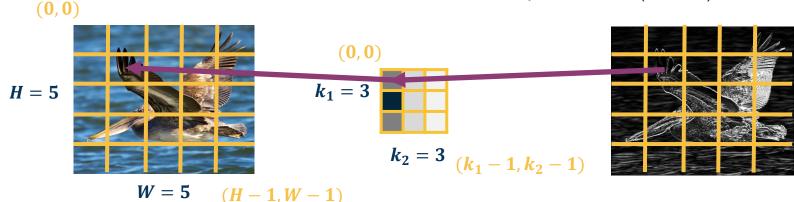
Chain Rule:

$$\frac{\partial L}{\partial k(a,b)} = \sum_{r=0}^{H-1} \sum_{c=0}^{W-1} \frac{\partial L}{\partial y(r,c)} \frac{\partial y(r,c)}{\partial k(a,b)}$$

Sum over all output pixels

Upstream gradient (known)

We will compute



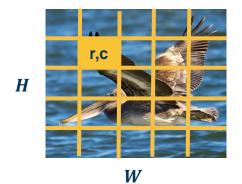
$$\frac{\partial y(r,c)}{\partial k(a,b)} = ?$$

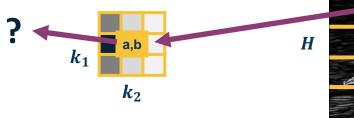
#### Reasoning:

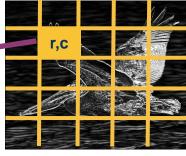
- Cross-correlation is just "dot product" of kernel and input patch (weighted sum)
- When at pixel y(r,c), kernel is on input x such that k(0,0) is multiplied by x(r,c)
- But we want derivative w.r.t. k(a, b)
  - k(0,0) \* x(r,c), k(1,1) \* x(r+1,c+1), k(2,2) \* x(r+2,c+2) => ingeneral k(a,b) \* x(r+a,c+b)
  - Just like before in fully connected layer, partial derivative w.r.t. k(a, b) only has this term (other x terms go away because not multiplied by k(a, b)).











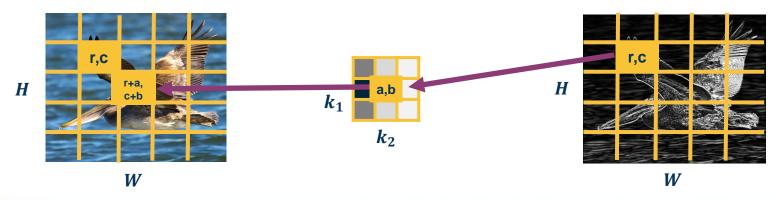
W

$$\frac{\partial y(r,c)}{\partial k(a,b)} = x(r+a,c+b)$$

$$\frac{\partial L}{\partial k(a,b)} = \sum_{r=0}^{H-1} \sum_{c=0}^{W-1} \frac{\partial L}{\partial y(r,c)} x(r+a,c+b)$$

# Does this look familiar?

Cross-correlation between upstream gradient and input! (until  $k_1 \times k_2$  output)



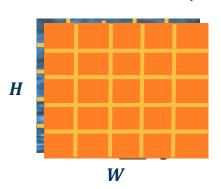


#### **Forward Pass**

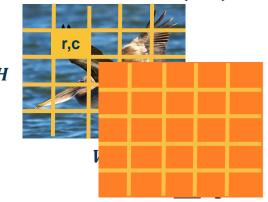


r,c a,b

Backward Pass k(0,0)



Backward Pass k(2,2)



### Does this look familiar?

Cross-correlation between upstream gradient and input! (until  $k_1 \times k_2$  output)





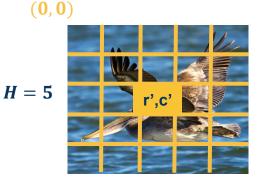
$$\frac{\partial L}{\partial x} = \frac{\partial L}{\partial y} \quad \frac{\partial y}{\partial x}$$

Gradient for input (to pass to prior layer)

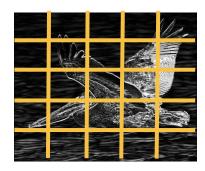
Calculate one pixel at a time  $\frac{\partial L}{\partial x(r',c')}$ 

What does this input pixel affect at the output?

Neighborhood around it (where part of the kernel touches it)

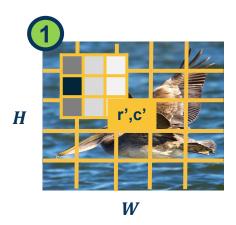


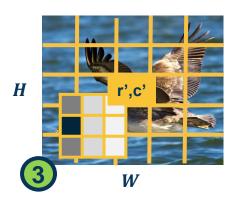
$$k_1 = 3$$
 $k_2 = 3$ 
 $(k_1 - 1, k_2 - 1)$ 

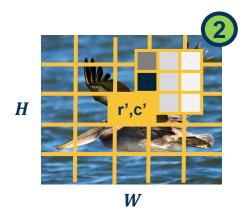


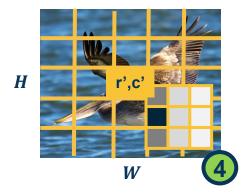
$$W=5 \qquad (H-1,W-1)$$



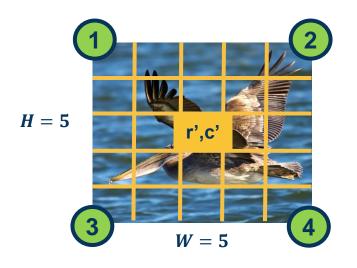








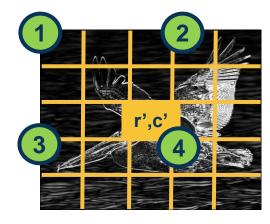




$$k_1 = 3$$

$$k_2 = 3$$

$$(r'-k_1+1, c'-k_2+1)$$



This is where the corresponding locations are for the **output** 

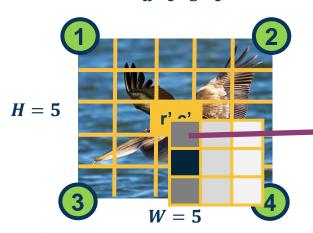


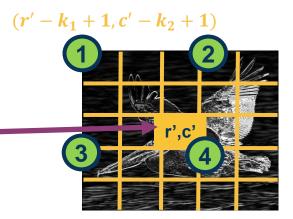
Chain rule for affected pixels (sum gradients):

$$\frac{\partial L}{\partial x(r',c')} = \sum_{Pixels \, p} \frac{\partial L}{\partial y(p)} \frac{\partial y(p)}{\partial x(r',c')}$$

$$x(r',c') * k(0,0) \Rightarrow y(r',c')$$
$$x(r',c') * k(1,1) \Rightarrow ?$$

$$\frac{\partial L}{\partial x(r',c')} = \sum_{a=0}^{k_1-1} \sum_{b=0}^{k_2-1} \frac{\partial L}{\partial y(?,?)} \frac{\partial y(?,?)}{\partial x(r',c')}$$





Chain rule for affected pixels (sum gradients):

$$\frac{\partial L}{\partial x(r',c')} = \sum_{Pixels \, p} \frac{\partial L}{\partial y(p)} \frac{\partial y(p)}{\partial x(r',c')}$$

$$\frac{\partial L}{\partial x(r',c')} = \sum_{q=0}^{k_1-1} \sum_{b=0}^{k_2-1} \frac{\partial L}{\partial y(?,?)} \frac{\partial y(?,?)}{\partial x(r',c')}$$

$$H = 5$$

$$W = 5$$

$$x(r',c') * k(0,0) \Rightarrow y(r',c')$$

$$x(r',c') * k(1,1) \Rightarrow y(r'-1,c'-1)$$
...
$$x(r',c') * k(a,b) \Rightarrow y(r'-a,c'-b)$$

$$(r'-k_1+1,c'-k_2+1)$$

1

2

 $r',c'$ 

3

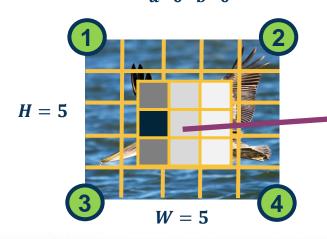


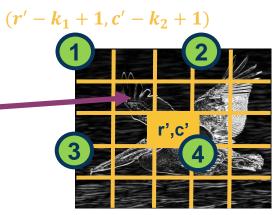
Chain rule for affected pixels (sum gradients):

$$\frac{\partial L}{\partial x(r',c')} = \sum_{Pixels\ p} \frac{\partial L}{\partial y(p)} \frac{\partial y(p)}{\partial x(r',c')}$$

Let's derive it analytically this time (as opposed to visually)

$$\frac{\partial L}{\partial x(r',c')} = \sum_{a=0}^{k_1-1} \sum_{b=0}^{k_2-1} \frac{\partial L}{\partial y(r'-a,c'-b)} \frac{\partial y(r'-a,c'-b)}{\partial x(r',c')}$$







Definition of cross-correlation (use a', b' to distinguish from prior variables):

$$y(r',c') = (x*k)(r',c') = \sum_{a'=0}^{k_1-1} \sum_{b'=0}^{k_2-1} x(r'+a',c'+b') k(a',b')$$

Plug in what we actually wanted:

$$y(r'-a,c'-b)=(x*k)(r',c')=\sum_{a'=0}^{k_1-1}\sum_{b'=0}^{k_2-1}x(r'-a+a',c'-b+b')\ k(a',b')$$

What is 
$$\frac{\partial y(r'-a,c'-b)}{\partial x(r',c')} = \mathbf{k}(a,b)$$
 (we want term with  $x(r',c')$  in it; this happens when  $\mathbf{a} = \mathbf{a}'$  and  $\mathbf{b} = \mathbf{b}'$ )

Plugging in to earlier equation:

$$\frac{\partial L}{\partial x(r',c')} = \sum_{a=0}^{k_1-1} \sum_{b=0}^{k_2-1} \frac{\partial L}{\partial y(r'-a,c'-b)} \frac{\partial y(r'-a,c'-b)}{\partial x(r',c')}$$

$$=\sum_{a=0}^{k_1-1}\sum_{b=0}^{k_2-1}\frac{\partial L}{\partial y(r'-a,c'-b)}k(a,b)$$

Again, all operations can be implemented via matrix multiplications (same as FC layer)!

Does this look familiar?

Convolution between upstream gradient and kernel!

(can implement by flipping kernel and cross- correlation)



- Convolutions are mathematical descriptions of striding linear operation
- In practice, we implement cross-correlation neural networks! (still called convolutional neural networks due to history)
  - Can connect to convolutions via duality (flipping kernel)
  - Convolution formulation has mathematical properties explored in ECE
- Duality for forwards and backwards:
  - Forward: Cross-correlation
  - Backwards w.r.t. K: Cross-correlation b/w upstream gradient and input
  - Backwards w.r.t. X: Convolution b/w upstream gradient and kernel
    - In practice implement via cross-correlation and flipped kernel
- All operations still implemented via efficient linear algebra (e.g. matrixmatrix multiplication)

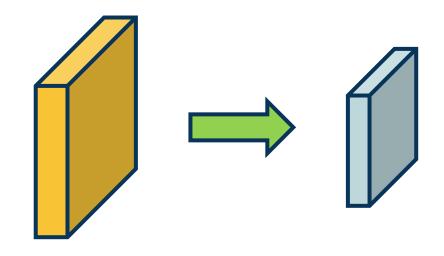


# Pooling Layers



- Dimensionality reduction is an important aspect of machine learning
- Can we make a layer to explicitly down-sample image or feature maps?

Yes! We call one class of these operations pooling operations



#### **Parameters**

- kernel\_size the size of the window to take a max over
- **stride** the stride of the window. Default value is kernel\_size
- padding implicit zero padding to be added on both sides

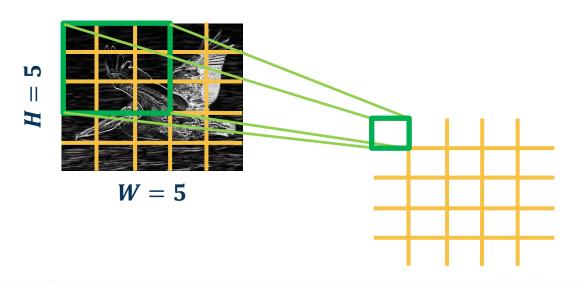
From: https://pytorch.org/docs/stable/generated/torch.nn.MaxPool2d.html#torch.html#to



# **Example:** Max pooling

Stride window across image but perform per-patch max operation

$$X(0:2,0:2) = \begin{bmatrix} 200 & 150 & 150 \\ 100 & 50 & 100 \\ 25 & 25 & 10 \end{bmatrix} \implies \max(0:2,0:2) = 200$$



How many learned parameters does this layer have?

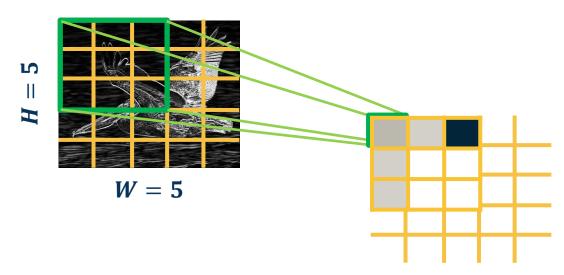
None!



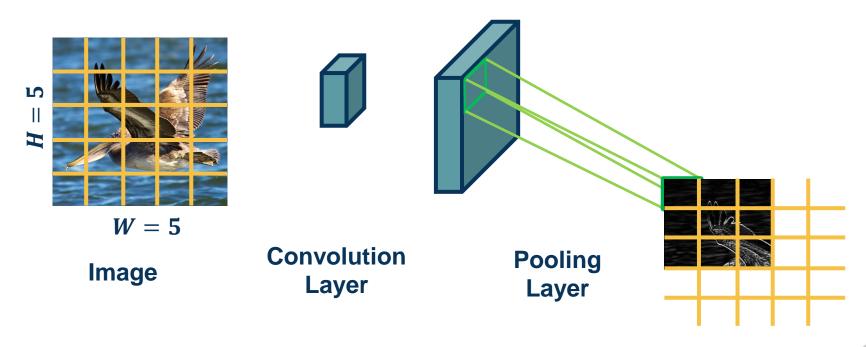
# Not restricted to max; can use any differentiable function

Not very common in practice

$$X(0:2,0:2) = \begin{bmatrix} 200 & 150 & 150 \\ 100 & 50 & 100 \\ 25 & 25 & 10 \end{bmatrix} \implies \text{average}(0:2,0:2) = \frac{1}{N} \sum_{i} \sum_{j} x(i,j) = 90$$



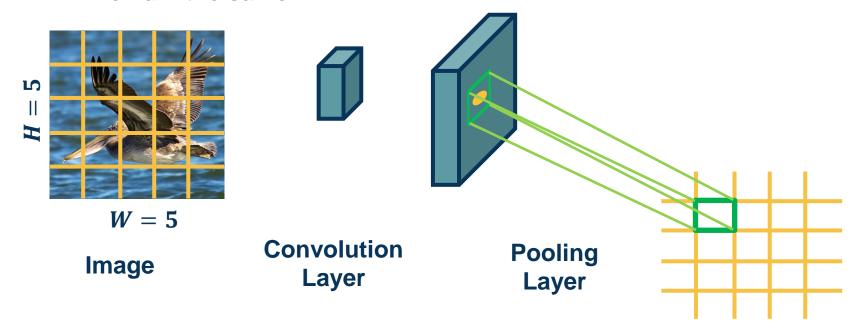
Since the **output** of convolution and pooling layers are **(multi-channel) images**, we can sequence them just as any other layer





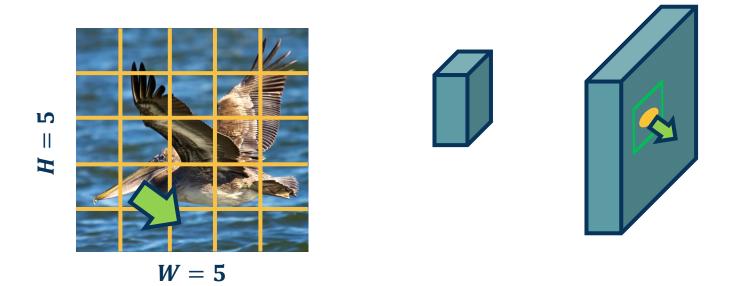
#### This combination adds some **invariance** to translation of the features

If feature (such as beak) translated a little bit, output values still
 remain the same



# Convolution by itself has the property of equivariance

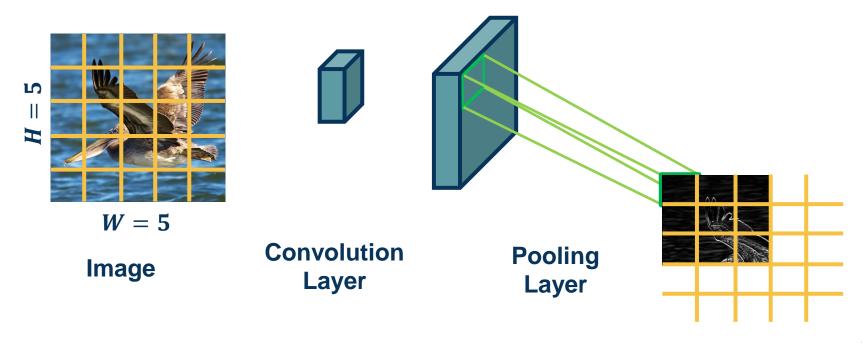
 If feature (such as beak) translated a little bit, output values move by the same translation



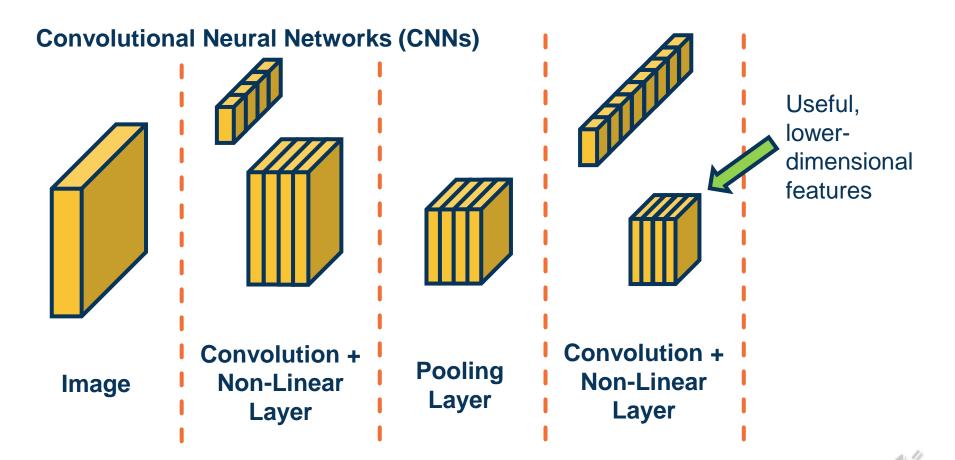
Simple
Convolutional
Neural
Networks



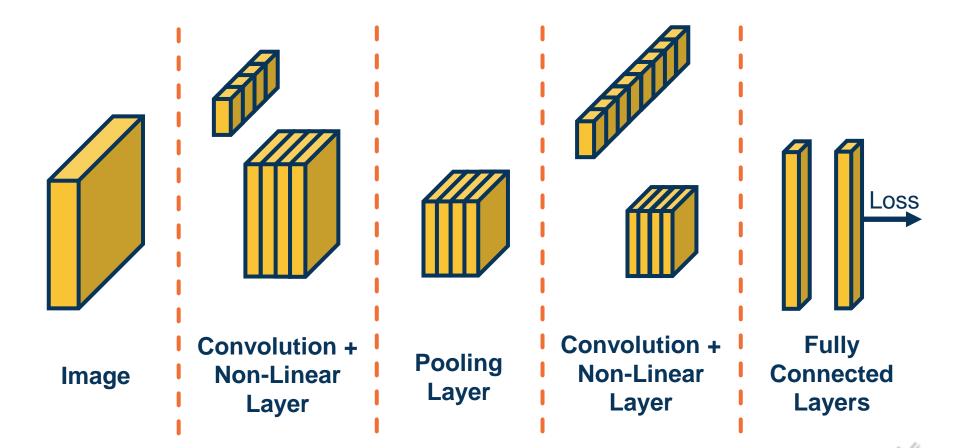
Since the **output** of convolution and pooling layers are **(multi-channel) images**, we can sequence them just as any other layer

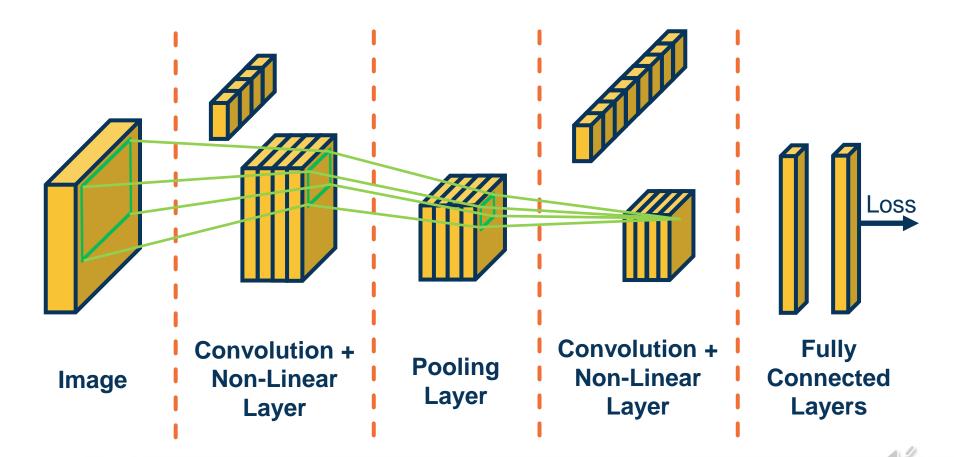


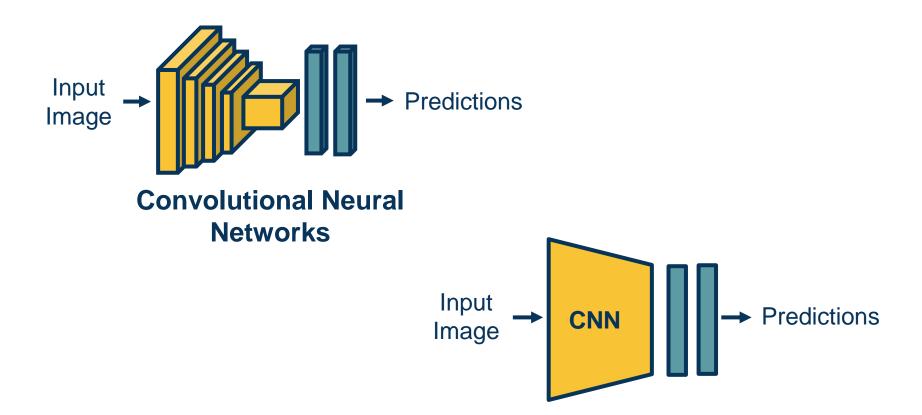














#### These architectures have existed **since 1980s**

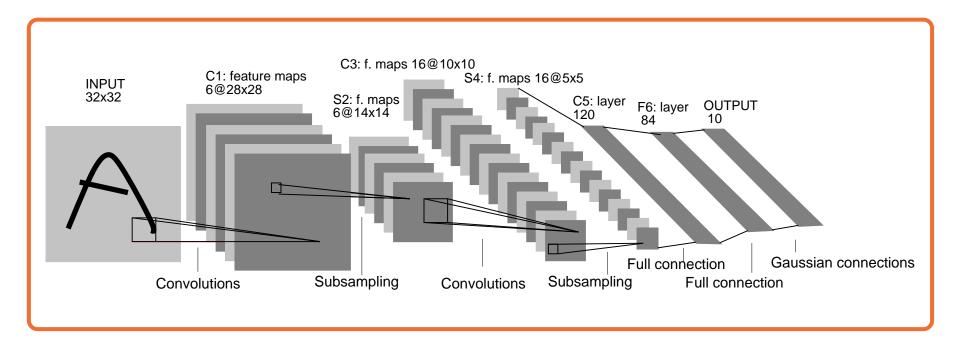


Image Credit: Yann LeCun, Kevin Murchy



### **Handwriting Recognition**

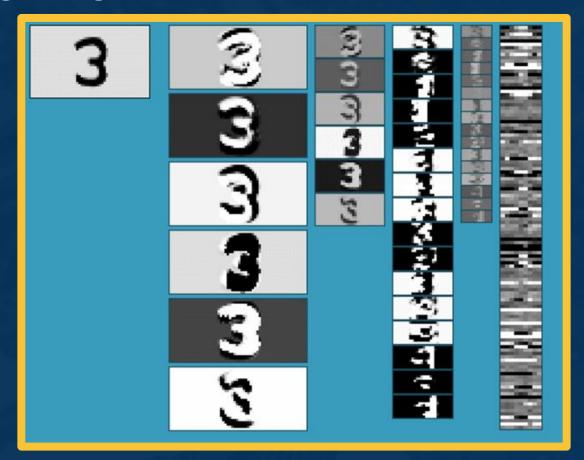


Image Credit:
Yann LeCun
Georgan

### **Translation Equivariance (Conv Layers) & Invariance (Output)**



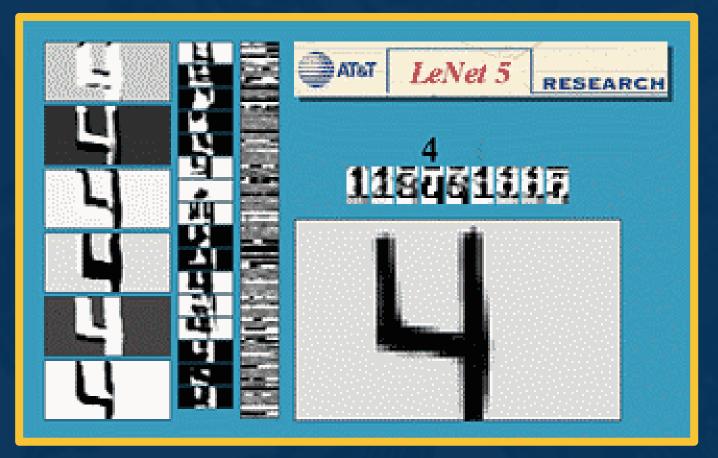


### (Some) Rotation Invariance





### (Some) Scale Invariance





# Advanced Convolutional Networks



### **The Importance of Benchmarks**

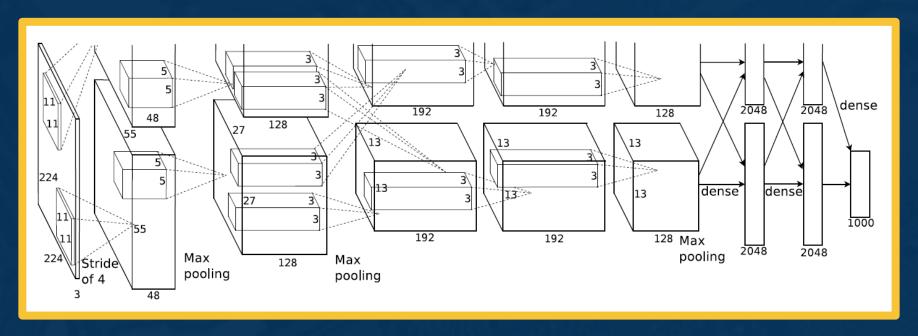








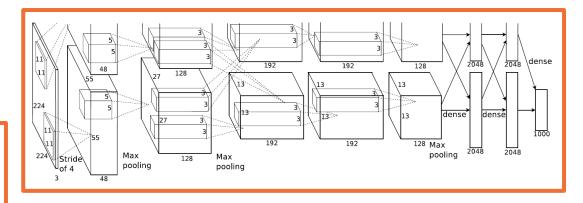
### **AlexNet - Architecture**



From: Krizhevsky et al., ImageNet Classification with Deep ConvolutionalNeural Networks, 2012.



Full (simplified) AlexNet architecture:
[227x227x3] INPUT
[55x55x96] CONV1: 96 11x11 filters at stride 4, pad 0
[27x27x96] MAX POOL1: 3x3 filters at stride 2
[27x27x96] NORM1: Normalization layer
[27x27x256] CONV2: 256 5x5 filters at stride 1, pad 2
[13x13x256] MAX POOL2: 3x3 filters at stride 2
[13x13x256] NORM2: Normalization layer
[13x13x384] CONV3: 384 3x3 filters at stride 1, pad 1
[13x13x384] CONV4: 384 3x3 filters at stride 1, pad 1
[13x13x256] CONV5: 256 3x3 filters at stride 1, pad 1
[6x6x256] MAX POOL3: 3x3 filters at stride 2
[4096] FC6: 4096 neurons
[4096] FC7: 4096 neurons
[1000] FC8: 1000 neurons (class scores)



#### **Key aspects:**

- ReLU instead of sigmoid or tanh
- Specialized normalization layers
- PCA-based data augmentation
- Dropout
- Ensembling

From: Slides by Fei-Fei Li, Justin Johnson, Serena Yeung, CS 231r



```
(not counting biases)
INPUT: [224x224x3]
                     memory: 224*224*3=150K params: 0
CONV3-64: [224x224x64] memory: 224*224*64=3.2M params: (3*3*3)*64 = 1,728
CONV3-64: [224x224x64] memory: 224*224*64=3.2M params: (3*3*64)*64 = 36.864
POOL2: [112x112x64] memory: 112*112*64=800K params: 0
CONV3-128: [112x112x128] memory: 112*112*128=1.6M params: (3*3*64)*128 = 73,728
CONV3-128: [112x112x128] memory: 112*112*128=1.6M params: (3*3*128)*128 = 147,456
POOL2: [56x56x128] memory: 56*56*128=400K params: 0
CONV3-256: [56x56x256] memory: 56*56*256=800K params: (3*3*128)*256 = 294,912
CONV3-256: [56x56x256] memory: 56*56*256=800K params: (3*3*256)*256 = 589.824
CONV3-256: [56x56x256] memory: 56*56*256=800K params: (3*3*256)*256 = 589,824
POOL2: [28x28x256] memory: 28*28*256=200K params: 0
CONV3-512: [28x28x512] memory: 28*28*512=400K params: (3*3*256)*512 = 1.179.648
CONV3-512: [28x28x512] memory: 28*28*512=400K params: (3*3*512)*512 = 2,359,296
CONV3-512: [28x28x512] memory: 28*28*512=400K params: (3*3*512)*512 = 2,359,296
POOL2: [14x14x512] memory: 14*14*512=100K params: 0
CONV3-512: [14x14x512] memory: 14*14*512=100K params: (3*3*512)*512 = 2,359,296
CONV3-512: [14x14x512] memory: 14*14*512=100K params: (3*3*512)*512 = 2,359,296
CONV3-512: [14x14x512] memory: 14*14*512=100K params: (3*3*512)*512 = 2,359,296
POOL2: [7x7x512] memory: 7*7*512=25K params: 0
FC: [1x1x4096] memory: 4096 params: 7*7*512*4096 = 102,760,448
FC: [1x1x4096] memory: 4096 params: 4096*4096 = 16,777,216
FC: [1x1x1000] memory: 1000 params: 4096*1000 = 4.096.000
```

ConvNet Configuration							
A	A-LRN	В	C	D	Е		
11 weight	11 weight	13 weight	16 weight	16 weight	19 weight		
layers	layers	layers	layers	layers	layers		
input (224 × 224 RGB image)							
conv3-64	conv3-64	conv3-64	conv3-64	conv3-64	conv3-64		
	LRN	conv3-64	conv3-64	conv3-64	conv3-64		
			pool				
conv3-128	conv3-128	conv3-128	conv3-128	conv3-128	conv3-128		
		conv3-128	conv3-128	conv3-128	conv3-128		
			pool				
conv3-256	conv3-256	conv3-256	conv3-256	conv3-256	conv3-256		
conv3-256	conv3-256	conv3-256	conv3-256	conv3-256	conv3-256		
			conv1-256	conv3-256	conv3-256		
					conv3-256		
			pool				
conv3-512	conv3-512	conv3-512	conv3-512	conv3-512	conv3-512		
conv3-512	conv3-512	conv3-512	conv3-512	conv3-512	conv3-512		
			conv1-512	conv3-512	conv3-512		
					conv3-512		
			pool				
conv3-512	conv3-512	conv3-512	conv3-512	conv3-512	conv3-512		
conv3-512	conv3-512	conv3-512	conv3-512	conv3-512	conv3-512		
			conv1-512	conv3-512	conv3-512		
					conv3-512		
maxpool							
FC-4096							
FC-4096							
FC-1000							
soft-max							
1							

Table 2: Number of parameters (in millions).

Network	A,A-LRN	В	C	D	Е
Number of parameters	133	133	134	138	144

From: Simonyan & Zimmerman, Very Deep Convolutional Networks for Large-Scale Image Recognition From: Slides by Fei-Fei Li, Justin Johnson, Serena Yeung, CS 231r/



```
(not counting biases)
INPUT: [224x224x3]
                     memory: 224*224*3=150K params: 0
CONV3-64: [224x224x64] memory: 224*224*64=3.2M params: (3*3*3)*64 = 1,728
CONV3-64: [224x224x64] memory: 224*224*64=3.2M params: (3*3*64)*64 = 36,864
POOL2: [112x112x64] memory: 112*112*64=800K params: 0
CONV3-128: [112x112x128] memory: 112*112*128=1.6M params: (3*3*64)*128 = 73,728
CONV3-128: [112x112x128] memory: 112*112*128=1.6M params: (3*3*128)*128 = 147,456
POOL2: [56x56x128] memory: 56*56*128=400K params: 0
CONV3-256: [56x56x256] memory: 56*56*256=800K params: (3*3*128)*256 = 294,912
CONV3-256: [56x56x256] memory: 56*56*256=800K params: (3*3*256)*256 = 589.824
CONV3-256: [56x56x256] memory: 56*56*256=800K params: (3*3*256)*256 = 589,824
POOL2: [28x28x256] memory: 28*28*256=200K params: 0
CONV3-512: [28x28x512] memory: 28*28*512=400K params: (3*3*256)*512 = 1,179,648
CONV3-512: [28x28x512] memory: 28*28*512=400K params: (3*3*512)*512 = 2,359,296
CONV3-512: [28x28x512] memory: 28*28*512=400K params: (3*3*512)*512 = 2,359,296
POOL2: [14x14x512] memory: 14*14*512=100K params: 0
CONV3-512: [14x14x512] memory: 14*14*512=100K params: (3*3*512)*512 = 2,359,296
CONV3-512: [14x14x512] memory: 14*14*512=100K params: (3*3*512)*512 = 2,359,296
CONV3-512: [14x14x512] memory: 14*14*512=100K params: (3*3*512)*512 = 2,359,296
POOL2: [7x7x512] memory: 7*7*512=25K params: 0
FC: [1x1x4096] memory: 4096 params: 7*7*512*4096 = 102,760,448
FC: [1x1x4096] memory: 4096 params: 4096*4096 = 16,777,216
FC: [1x1x1000] memory: 1000 params: 4096*1000 = 4.096.000
```

### Most memory usage in convolution layers

## Most parameters in FC layers

From: Simonyan & Zimmerman, Very Deep Convolutional Networks for Large-Scale Image Recognition From: Slides by Fei-Fei Li, Justin Johnson, Serena Yeung, CS 231r/



### **Key aspects:**

### Repeated application of:

- 3x3 conv (stride of 1, padding of 1)
- 2x2 max pooling (stride 2)

Very large number of parameters

ConvNet Configuration								
A	A-LRN	В	C	D	E			
11 weight	11 weight	13 weight	16 weight	16 weight	19 weight			
layers	layers	layers	layers	layers	layers			
	i	nput (224 × 2	24 RGB image	e)				
conv3-64	conv3-64	conv3-64	conv3-64	conv3-64	conv3-64			
	LRN	conv3-64	conv3-64	conv3-64	conv3-64			
		max	pool		'			
conv3-128	conv3-128	conv3-128	conv3-128	conv3-128	conv3-128			
		conv3-128	conv3-128	conv3-128	conv3-128			
	maxpool							
conv3-256	conv3-256	conv3-256	conv3-256	conv3-256	conv3-256			
conv3-256	conv3-256	conv3-256	conv3-256	conv3-256	conv3-256			
			conv1-256	conv3-256	conv3-256			
					conv3-256			
		max	pool					
conv3-512	conv3-512	conv3-512	conv3-512	conv3-512	conv3-512			
conv3-512	conv3-512	conv3-512	conv3-512	conv3-512	conv3-512			
			conv1-512	conv3-512	conv3-512			
					conv3-512			
			pool					
conv3-512	conv3-512	conv3-512	conv3-512	conv3-512	conv3-512			
conv3-512	conv3-512	conv3-512	conv3-512	conv3-512	conv3-512			
			conv1-512	conv3-512	conv3-512			
					conv3-512			
maxpool								
	FC-4096							
FC-4096								
FC-1000								
soft-max								

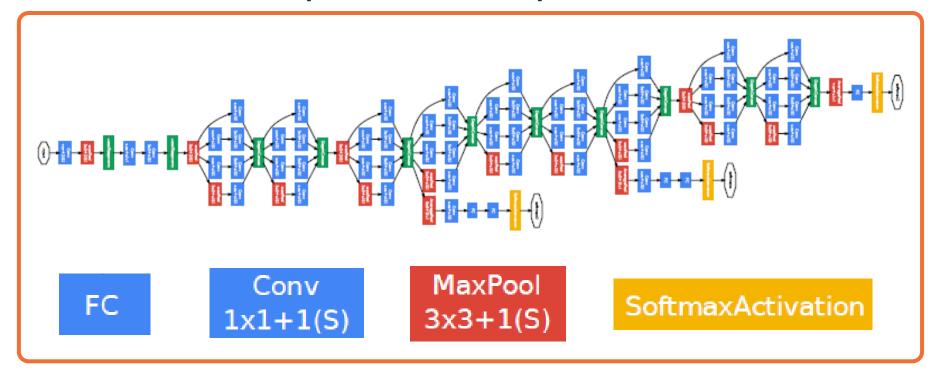
Table 2: Number of parameters (in millions).

Network	A,A-LRN	В	C	D	E
Number of parameters	133	133	134	138	144

From: Simonyan & Zimmerman, Very Deep Convolutional Networks for Large-Scale Image Recognition From: Slides by Fei-Fei Li, Justin Johnson, Serena Yeung, CS 231r/



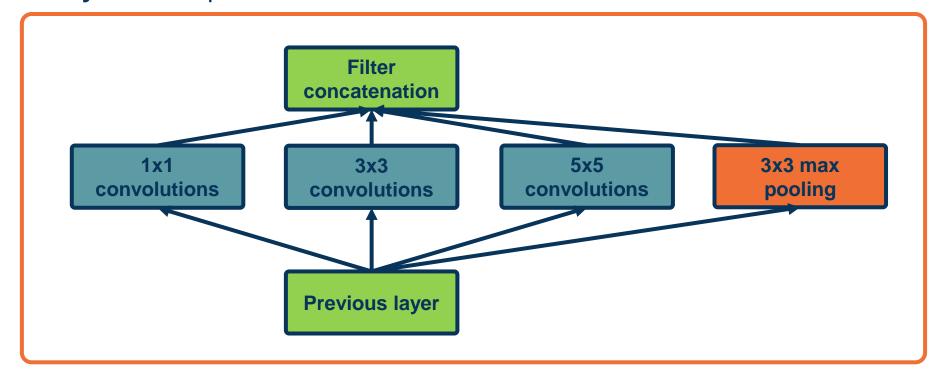
### But have become deeper and more complex



From: Szegedy et al. Going deeper with convolutions



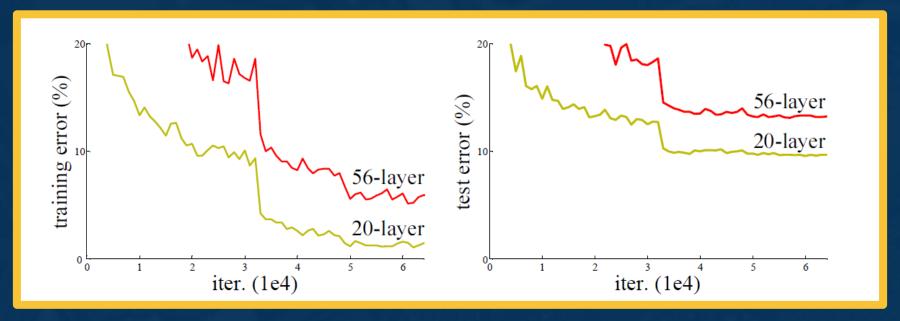
### **Key idea:** Repeated blocks and multi-scale features



From: Szegedy et al. Going deeper with convolutions



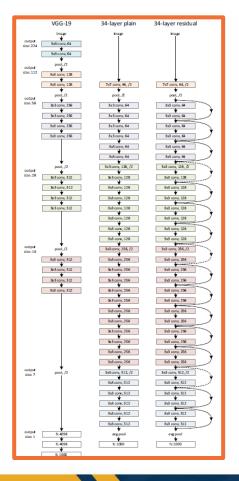
### The Challenge of Depth

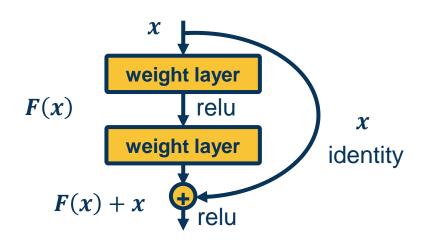


From: He et al., Deep Residual Learning for Image Recognition

Optimizing very deep networks is challenging!







**Key idea**: Allow information from a layer to propagate to any future layer (forward)

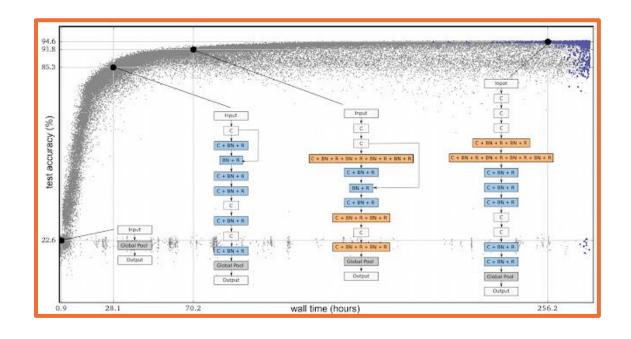
Same is true for gradients!

From: He et al., Deep Residual Learning for Image Recognition



### Several ways to *learn* architectures:

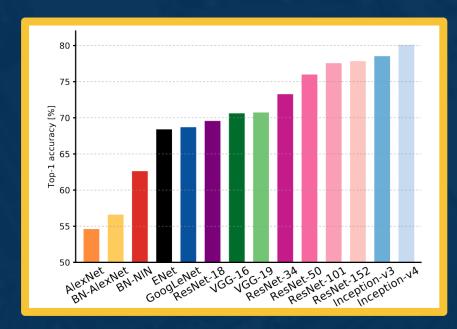
- Evolutionary learning and reinforcement learning
- Prune overparameterized networks
- Learning of repeated blocks typical

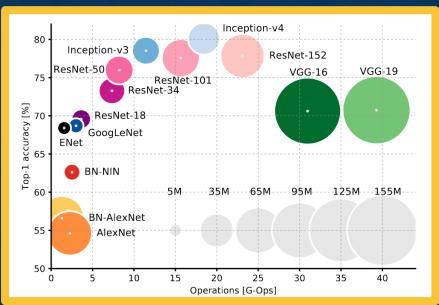


From: https://ai.googleblog.com/2018/03/using-evolutionary-automl-to-discover.html



### **Computational Complexity**





From: An Analysis Of Deep Neural Network Models For Practical Application

