Topics:

- Recurrent Neural Networks
- Long Short-Term Memory

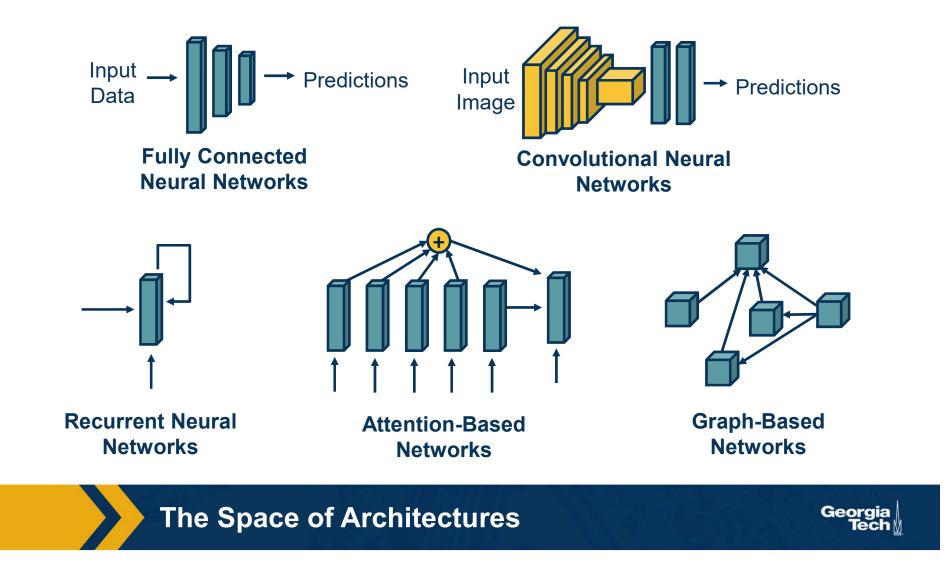
# CS 4644-DL / 7643-A ZSOLT KIRA

### • Assignment 3

- Due March 14th 11:59pm EST.
- See <a href="https://piazza.com/class/ky0k0ha5vgy1mk?cid=176">https://piazza.com/class/ky0k0ha5vgy1mk?cid=176</a>
  - (note: ignore logistics on that slide deck)
- Do not use grace period as extension! Submit \*something\* on time and continue to refine it.

### • Projects

• Project proposal due **March 13<sup>th</sup>** (into grace period)



# (Vanilla) Recurrent Neural Network

The state consists of a single *"hidden"* vector **h**:

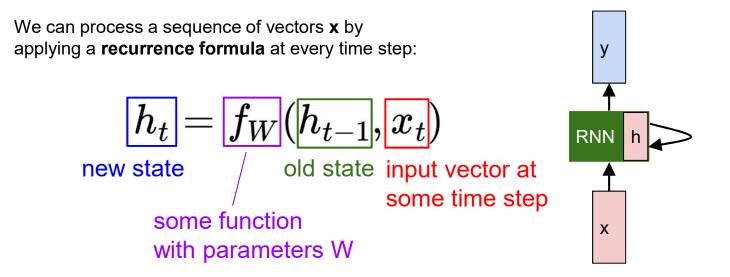
$$y \qquad y_{t} = W_{hy}h_{t} + b_{y}$$

$$h_{t} = \tanh(W_{hh}h_{t-1} + W_{xh}x_{t})$$

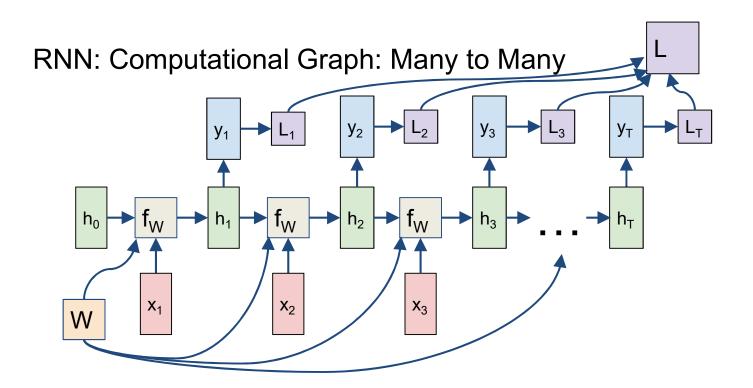
$$= \tanh\left(\left(W_{hh} \quad W_{hx}\right) \begin{pmatrix}h_{t-1}\\x_{t}\end{pmatrix}\right)$$

$$= \tanh\left(W\begin{pmatrix}h_{t-1}\\x_{t}\end{pmatrix}\right)$$

Sometimes called a "Vanilla RNN" or an "Elman RNN" after Prof. Jeffrey Elman Slide Credit: Fei-Fei Li, Justin Johnson, Serena Yeung, CS 231n Georgia Tech **Recurrent Neural Network** 

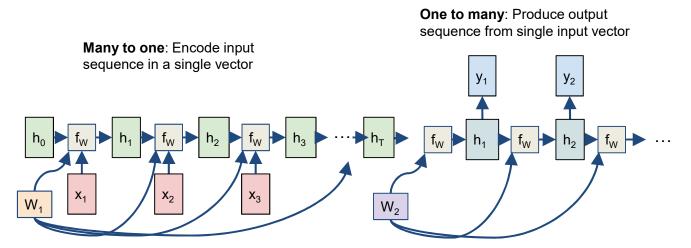








## Sequence to Sequence: Many-to-one + one-to-many





## Example: Character-level Language Model

Vocabulary: [h,e,l,o]

Example training sequence: **"hello"** 

input layer	1 0 0	0 1 0 0	 0 0 1 0	0 0 1 0	
input chars:	"h"	"e"	"["	"I"	

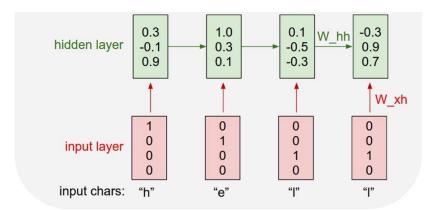


## Example: Character-level Language Model

$$h_t = \tanh(W_{hh}h_{t-1} + W_{xh}x_t + b_h)$$

Vocabulary: [h,e,l,o]

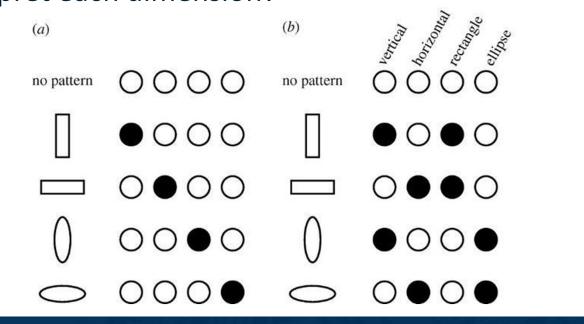
Example training sequence: "hello"





# **Distributed Representations Toy Example**

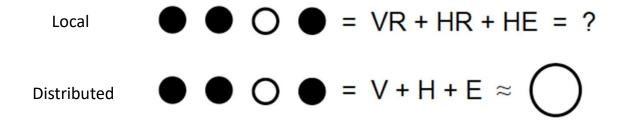
• Can we interpret each dimension?



(C) Dhruv Batra Slide Credit: Moontae Lee



# Power of distributed representations!

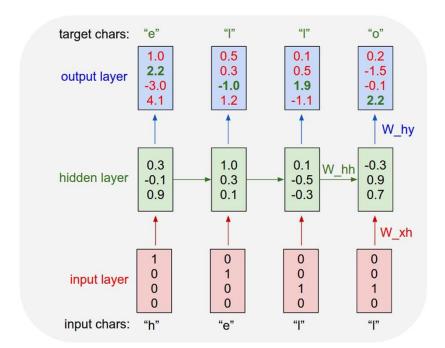


(C) Dhruv Batra Slide Credit: Moontae Lee Georgia Tech

## Example: Character-level Language Model

Vocabulary: [h,e,l,o]

Example training sequence: **"hello"** 



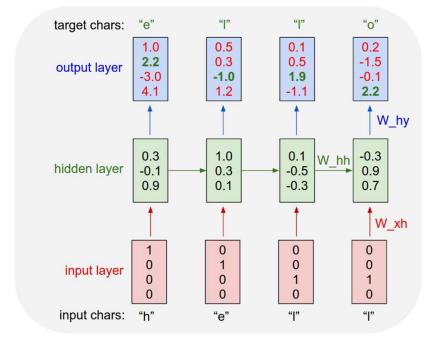
Georgia Tech

# Training Time: MLE / "Teacher Forcing"

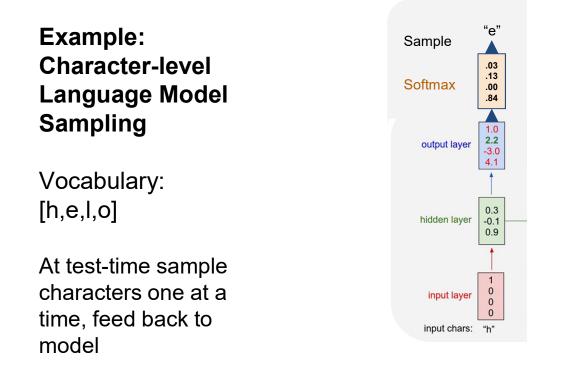
## Example: Character-level Language Model

Vocabulary: [h,e,l,o]

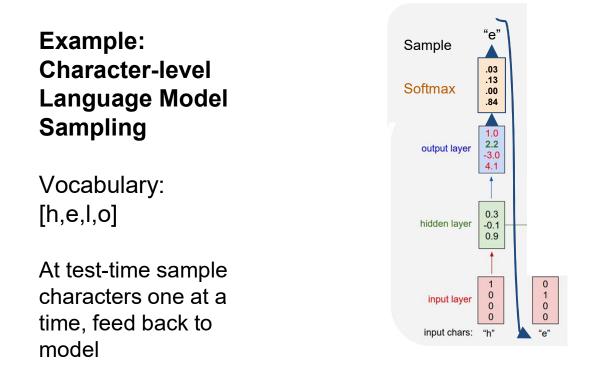
Example training sequence: "hello"



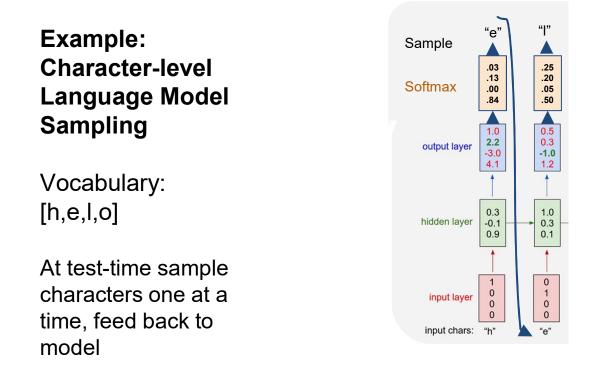




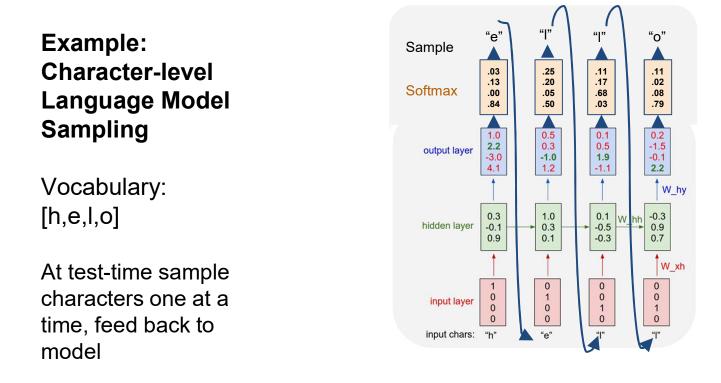




Georgia Tech



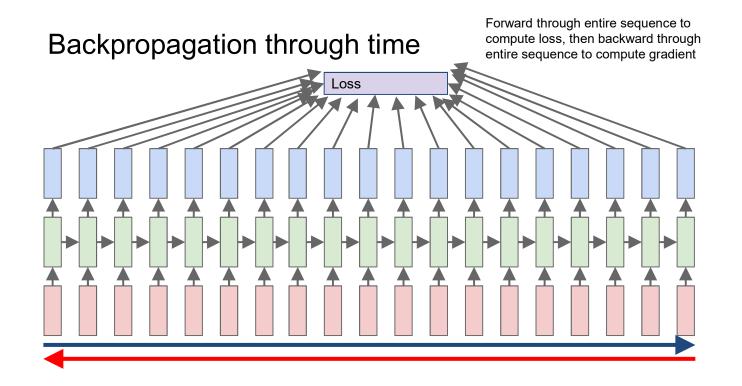




Georgia Tech

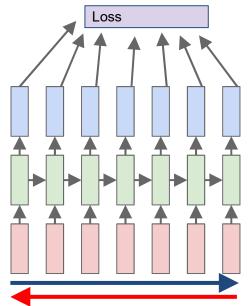








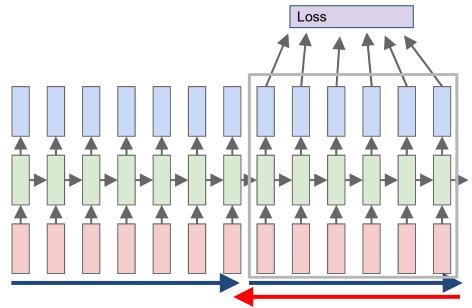
## Truncated Backpropagation through time



Run forward and backward through chunks of the sequence instead of whole sequence

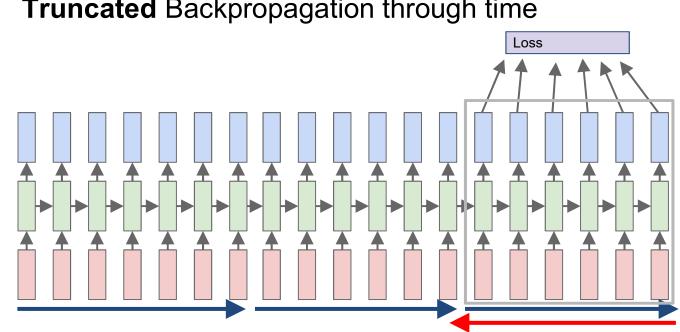


## Truncated Backpropagation through time



Carry hidden states forward in time forever, but only backpropagate for some smaller number of steps





Truncated Backpropagation through time



### min-char-rnn.py gist: 112 lines of Python

rectangle and a set of the s

<code-block></code>

(https://gist.github.com/karpathy/d4dee 566867f8291f086)



Slide Credit: Fei-Fei Li, Justin Johnson, Serena Yeung, CS 231n

Georgia Tech

### THE SONNETS

### by William Shakespeare

From fairest creatures we desire increase, That thereby beauty's rose might never die, But as the riper should by time decease, His tender heir might bear his memory: But thou, contracted to thine own bright eyes, Feed'st thy light's flame with self-substantial fuel, Making a famine where abundance lies, Thyself thy foe, to thy sweet self too cruel: Thou that art now the world's fresh ornament, And only herald to the gaudy spring, Within thise own bud buriest thy content, And tender chur mak'st waste in niggarding: Pity the world, or else this glutton be, To cat the world's due, by the grave and thee.

When forty winters shall besiege thy brow, And dig deep trenches in thy beauty's field, Thy youth's proud livery so gazed on now, Will be a tatter'd weed of small worth held: Then being asked, where all thy beauty lies, Where all the treasure of thy lusty days; To say, within thine own deep sunken eyes. Were an all-eating shame, and thriftless praise. How much more praise deserved thy beauty's use, If thou couldst answer This fair child of mine Shall sum my count, and make my old excuse'. Proving his beauty by succession thine! This were to be new made when thou art old, And see thy blood warm when thou feel's it cold. y RNN



Georgia Tech

at first:	tyntd-iafhatawiaoihrdemot lytdws e ,tfti, astai f ogoh eoase rrranbyne 'nhthnee e plia tklrgd t o idoe ns,smtt h ne etie h,hregtrs nigtike,aoaenns lng				
	train more				
	"Tmont thithey" fomesscerliund Keushey. Thom here sheulke, anmerenith ol sivh I lalterthend Bleipile shuwy fil on aseterlome coaniogennc Phe lism thond hon at. MeiDimorotion in ther thize."				
	↓ train more				
	Aftair fall unsuch that the hall for Prince Velzonski's that me of her hearly, and behs to so arwage fiving were to it beloge, pavu say falling misfort how, and Gogition is so overelical and ofter.				
	↓ train more				
	"Why do what that day," replied Natasha, and wishing to himself the fact the princess, Princess Mary was easier, fed in had oftened him. Pierre aking his soul came to the packs and drove up his father-in-law women.				



Georgia Tech

### PANDARUS:

Alas, I think he shall be come approached and the day When little srain would be attain'd into being never fed, And who is but a chain and subjects of his death, I should not sleep.

### Second Senator:

They are away this miseries, produced upon my soul, Breaking and strongly should be buried, when I perish The earth and thoughts of many states.

### DUKE VINCENTIO:

Well, your wit is in the care of side and that.

### Second Lord:

They would be ruled after this chamber, and my fair nues begun out of the fact, to be conveyed, Whose noble souls I'll have the heart of the wars.

Clown:

Come, sir, I will make did behold your worship.

### VIOLA: I'll drink it.

#### VIOLA:

Why, Salisbury must find his flesh and thought That which I am not aps, not a man and in fire, To show the reining of the raven and the wars To grace my hand reproach within, and not a fair are hand, That Caesar and my goodly father's world; When I was heaven of presence and our fleets, We spare with hours, but cut thy council I am great, Murdered and by thy master's ready there My power to give thee but so much as hell: Some service in the noble bondman here, Would show him to her wine.

### KING LEAR:

O, if you were a feeble sight, the courtesy of your law, Your sight and several breath, will wear the gods With his heads, and my hands are wonder'd at the deeds, So drop upon your lordship's head, and your opinion Shall be against your honour.



### The Stacks Project: open source algebraic geometry textbook

wse chapter	rs			Parts
	-			1. Preliminaries
art	Chapter	online TeX sour	ce view pdf	2. Schemes
reliminaries				3. Topics in Scheme Theory
	1. Introduction	online tex 🖓	pdf >	4. Algebraic Spaces
	2. Conventions	online tex 🖓	pdf >>	<ol> <li><u>Topics in Geometry</u></li> <li><u>Deformation Theory</u></li> </ol>
	3. Set Theory	online tex	pdf >	7. Algebraic Stacks
	4. Categories	online tex	pdf >>	8. Miscellany
	5. Topology	online tex	pdf >>	Statistics
	6. Sheaves on Spaces	online tex	pdf >	Staustics
	7. Sites and Sheaves	online tex 🖓	pdf >>	The Stacks project now consists of
	8. Stacks	online tex 🖓	pdf >>	<ul> <li>455910 lines of code</li> </ul>
	9. Fields	online tex	pdf >>	<ul> <li>14221 tags (56 inactive tags)</li> </ul>
	10. Commutative Algebra	online tex 🖓	pdf >>	<ul> <li>2366 sections</li> </ul>



Slide Credit: Fei-Fei Li, Justin Johnson, Serena Yeung, CS 231n

Georgia

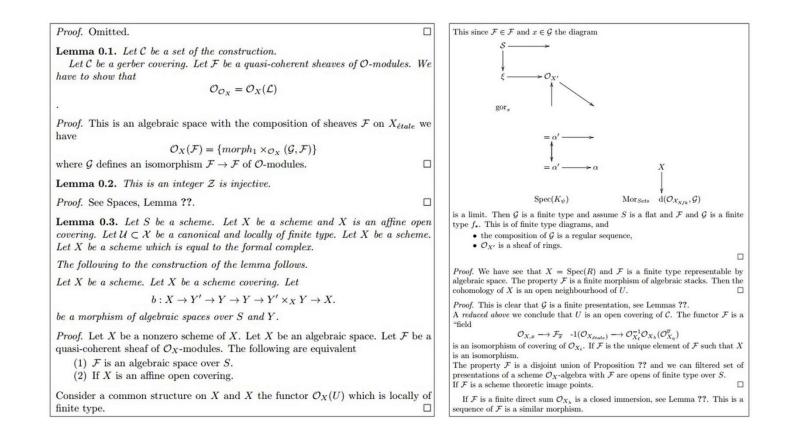
verse, the groupoid scheme S.	For $\bigoplus_{n=1,\dots,m}$ where $\mathcal{L}_{m_{\bullet}} = 0$ , hence we can find a closed subset $\mathcal{H}$ in $\mathcal{H}$ and and by sets $\mathcal{F}$ on $X, U$ is a closed immersion of $S$ , then $U \to T$ is a separated algebraic ace. roof. Proof of (1). It also start we get $S = \operatorname{Spec}(R) = U \times_X U \times_X U$ and the comparicoly in the fibre product covering we have to prove the lemma merated by $\prod Z \times_U U \to V$ . Consider the maps $M$ along the set of points $h_{fppf}$ and $U \to U$ is the fibre category of $S$ in $U$ in Section, ?? and the fact that by $U$ affine, see Morphisms, Lemma ??. Hence we obtain a scheme $S$ and any we subset $W \subset U$ in $Sh(G)$ such that $\operatorname{Spec}(R') \to S$ is smooth or an $U = \bigcup U_i \times_{S_i} U_i$ hich has a nonzero morphism we may assume that $f_i$ is of finite presentation over We claim that $\mathcal{O}_{X,x}$ is a scheme where $x, x', s'' \in S'$ such that $\mathcal{O}_{X,x'} \to \mathcal{O}'_{X',x'}$ is parated. By Algebra, Lemma ?? we can define a map of complexes $\operatorname{GL}_{S'}(x'/S'')$ of we win. $\Box$ to prove study we see that $\mathcal{F} _U$ is a covering of $\mathcal{X}'$ , and $\mathcal{T}_i$ is an object of $\mathcal{F}_{X/S}$ for > 0 and $\mathcal{F}_p$ exists and let $\mathcal{F}_i$ be a presheaf of $\mathcal{O}_{X}$ -modules on $C$ as a $\mathcal{F}$ -module. particular $\mathcal{F} = U/\mathcal{F}$ we have to show that $\widetilde{M^{\bullet}} = \mathbb{T}^{\bullet} \otimes_{\operatorname{Spec}(k)} \mathcal{O}_{S,s} - i_X^{-1}\mathcal{F})$ a unique morphism of algebraic stacks. Note that $\operatorname{Arrows} = (Sch/S)_{fppf}^{opp}, (Sch/S)_{fppf}$ d $V = \Gamma(S, \mathcal{O}) \longmapsto (U, \operatorname{Spec}(A))$ an open subset of $X$ . Thus $U$ is affine. This is a continuous map of $X$ is the	$ \begin{array}{llllllllllllllllllllllllllllllllllll$
Proof. See discussion of sheaves of sets.	verse, the groupoid scheme S.	



regular over S.

Slide Credit: Fei-Fei Li, Justin Johnson, Serena Yeung, CS 231n

Georgia Tech





This repository Sear	ch Explore Gist B	llog Help	餐 ka	arpathy +- 🚅 🌣	E
torvalds / linux		Watch - 3,711	★ Star	23,054 ¥ Fork	9,141
inux kernel source tree					
3 520,037 commits	1 branch 📎 420 releases	5,039 contribut	ors	<> Code	
្រ ទ្រ branch: master -	linux / +		:=	ាំ Pull requests	74
Merge branch 'drm-fixes' of	git://people.freedesktop.org/~airlied/linux				
torvalds authored 9 hou	urs ago	latest commit 4b170	6927d 🗟	4~ Pulse	
Documentation	Merge git://git.kernel.org/pub/scm/linux/kernel/git/nab/targ	et-pending 6	days ago	r uise	
arch	Merge branch 'x86-urgent-for-linus' of git://git.kernel.org/pub/scm/l a day ag			Graphs	
block	block: discard bdi_unregister() in favour of bdi_destroy() 9 days ago				
crypto	Merge git://git.kernel.org/pub/scm/linux/kernel/git/herbert/crypto-2.6 10 days ago				
drivers	Merge branch 'drm-fixes' of git://people.freedesktop.org/~d	airlied/linux 9 h	ours ago	https://github.c	G
firmware	firmware/ihex2fw.c: restore missing default in switch statement 2 months ago			You can clone with HTTPS SSH, or Subversion.	
ts	vfs: read file_handle only once in handle_to_path 4 days ago				
include	Merge branch 'perf-urgent-for-linus' of git://git.kernel.org/p	ub/scm/	a day ago 🖉 Clone in Desk		
init init	init: fix regression by supporting devices with major:minor:	offset fo a m	a month ago		
in ins	Mana branch Yar linus' of ait-liait kannal analauk/somiliau	ukomol o n	costh cos	•	



```
static void do_command(struct seq_file *m, void *v)
{
                                                                  Generated
 int column = 32 << (cmd[2] & 0x80);</pre>
 if (state)
   cmd = (int)(int_state ^ (in_8(&ch->ch_flags) & Cmd) ? 2 : 1); C COCE
  else
   seq = 1;
 for (i = 0; i < 16; i++) {
   if (k & (1 << 1))
     pipe = (in_use & UMXTHREAD_UNCCA) +
        ((count & 0x0000000fffffff8) & 0x000000f) << 8;
   if (count == 0)
     sub(pid, ppc_md.kexec_handle, 0x2000000);
   pipe_set_bytes(i, 0);
 }
 /* Free our user pages pointer to place camera if all dash */
  subsystem_info = &of_changes[PAGE_SIZE];
 rek_controls(offset, idx, &soffset);
  /* Now we want to deliberately put it to device */
 control_check_polarity(&context, val, 0);
 for (i = 0; i < COUNTER; i++)</pre>
   seq_puts(s, "policy ");
```



```
1*
   Copyright (c) 2006-2010, Intel Mobile Communications. All rights reserved.
 * This program is free software; you can redistribute it and/or modify it
 * under the terms of the GNU General Public License version 2 as published by
 * the Free Software Foundation.
         This program is distributed in the hope that it will be useful,
 * but WITHOUT ANY WARRANTY; without even the implied warranty of
    MERCHANTABILITY or FITNESS FOR A PARTICULAR PURPOSE. See the
 *
 *
   GNU General Public License for more details.
 * You should have received a copy of the GNU General Public License
     along with this program; if not, write to the Free Software Foundation,
 *
 * Inc., 675 Mass Ave, Cambridge, MA 02139, USA.
*/
#include <linux/kexec.h>
#include <linux/errno.h>
```

#include <linux/errno.h>
#include <linux/io.h>
#include <linux/platform\_device.h>
#include <linux/multi.h>
#include <linux/ckevent.h>

```
#include <asm/io.h>
#include <asm/prom.h>
#include <asm/e820.h>
#include <asm/system_info.h>
#include <asm/setew.h>
#include <asm/pgproto.h>
```

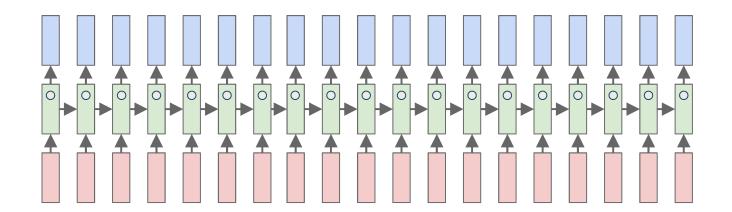
Georgia Tech

```
#include <asm/io.h>
#include <asm/prom.h>
#include <asm/e820.h>
#include <asm/system_info.h>
#include <asm/setew.h>
#include <asm/pgproto.h>
#define REG_PG vesa_slot_addr_pack
#define PFM_NOCOMP AFSR(0, load)
#define STACK DDR(type)
                           (func)
#define SWAP_ALLOCATE(nr)
                             (e)
#define emulate_sigs() arch_get_unaligned_child()
#define access_rw(TST) asm volatile("movd %%esp, %0, %3" :: "r" (0)); \
 if (__type & DO_READ)
static void stat_PC_SEC __read_mostly offsetof(struct seq_argsqueue, \
         pC>[1]);
static void
os_prefix(unsigned long sys)
{
#ifdef CONFIG_PREEMPT
 PUT_PARAM_RAID(2, sel) = get_state_state();
 set_pid_sum((unsigned long)state, current_state_str(),
          (unsigned long)-1->lr_full; low;
```



}

# Searching for interpretable cells

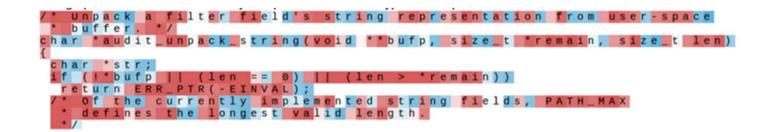


Karpathy, Johnson, and Fei-Fei: Visualizing and Understanding Recurrent Networks, ICLR Workshop 2016





# Searching for interpretable cells



Karpathy, Johnson, and Fei-Fei: Visualizing and Understanding Recurrent Networks, ICLR Workshop 2016 Figures copyright Karpathy, Johnson, and Fei-Fei, 2015; reproduced with



Georgi

# Searching for interpretable cells



Kutuzov, shrugging his shoulders, replied with his subtle penetrating smile: "I meant merely to say what I said."

### quote detection cell

Karpathy, Johnson, and Fei-Fei: Visualizing and Understanding Recurrent Networks, ICLR Workshop 2016 Figures copyright Karpathy, Johnson, and Fei-Fei, 2015; reproduced with permission



Slide Credit: Fei-Fei Li, Justin Johnson, Serena Yeung, CS 231n

Georg

#### Cell sensitive to position in line:

The sole importance of the crossing of the Berezina lies in the fact that it plainly and indubitably proved the fallacy of all the plans for cutting off the enemy's retreat and the soundness of the only possible line of action--the one Kutuzov and the general mass of the army demanded--namely, simply to follow the enemy up. The French crowd fled at a continually increasing speed and all its energy was directed to reaching its goal. It fled like a wounded animal and it was impossible to block its path. This was shown not so much by the arrangements it made for crossing as by what took place at the bridges. When the bridges broke down, unarmed soldiers, people from Moscow and women with children who were with the French transport, all--carried on by vis inertiae-pressed forward into boats and into the ice-covered water and did not,

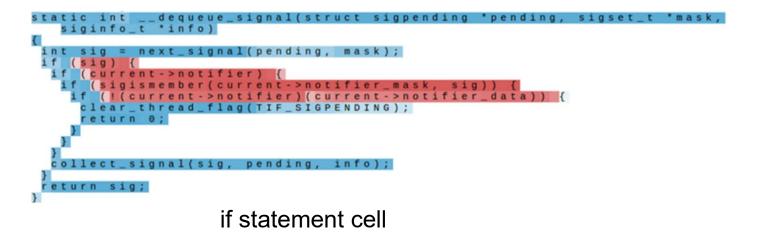
#### line length tracking cell

Karpathy, Johnson, and Fei-Fei: Visualizing and Understanding Recurrent Networks, ICLR Workshop 2016 Figures copyright Karpathy, Johnson, and Fei-Fei, 2015; reproduced with permission



Slide Credit: Fei-Fei Li, Justin Johnson, Serena Yeung, CS 231n

Georgi



Karpathy, Johnson, and Fei-Fei: Visualizing and Understanding Recurrent Networks, ICLR Workshop 2016 Figures copyright Karpathy, Johnson, and Fei-Fei, 2015; reproduced with permission

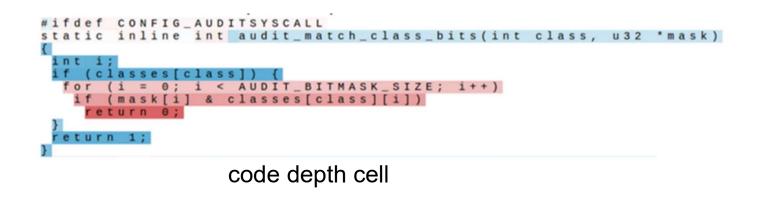


Cell that turns on inside comments and quotes:

Con that tamb on molde commente and quotes.	
/* Duplicate LSM field information. The lsm_rule is opaque,	50
* re-initialized. */	
static inline int audit_dupe_lsm_field(struct audit_field *df,	
struct audit_field *sf)	
schuce about_hera si)	
int ret = $0;$	
char *lsm_str;	
/ • our own copy of lsm_str •/	
lsm_str = kstrdup(sf->lsm_str, GFP_KERNEL);	
if (unlikely(!lsm_str))	
return - ENOMEM:	
df->lsm_str = lsm_str;	
/* our own (refreshed) copy of lsm_rule */	
ret = security_audit_rule_init(df->type, df->op, df->lsm_str	
(void **)&df->lsm_rule);	
/* Keep currently invalid fields around in case they	
* become valid after a policy reload. */	
if (ret == -EINVAL) {	
pr_warn("audit rule for LSM \'%s\' is invalid\n",	
df->1sm_str);	
ret = 0;	
quote/comment cell	

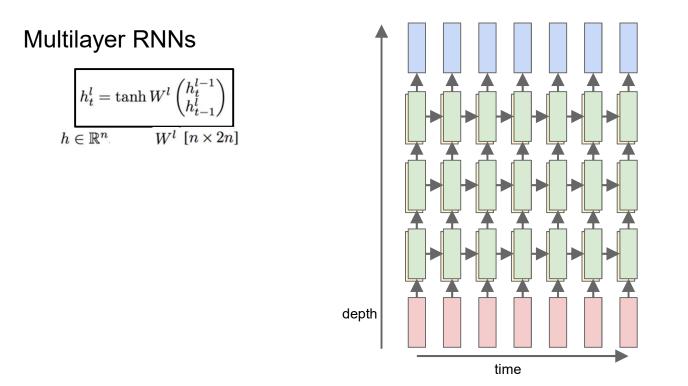
Karpathy, Johnson, and Fei-Fei: Visualizing and Understanding Recurrent Networks, ICLR Workshop 2016 Figures copyright Karpathy, Johnson, and Fei-Fei, 2015; reproduced with permission





Karpathy, Johnson, and Fei-Fei: Visualizing and Understanding Recurrent Networks, ICLR Workshop 2016 Figures copyright Karpathy, Johnson, and Fei-Fei, 2015; reproduced with permission

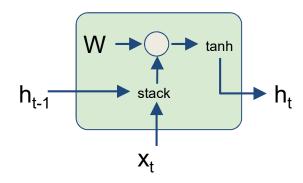






Slide Credit: Fei-Fei Li, Justin Johnson, Serena Yeung, CS 231n

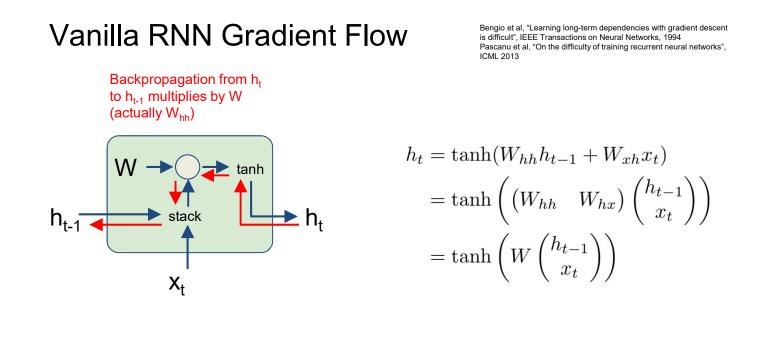
Bengio et al, "Learning long-term dependencies with gradient descent is difficult", IEEE Transactions on Neural Networks, 1994 Pascanu et al, "On the difficulty of training recurrent neural networks", ICML 2013



$$h_{t} = \tanh(W_{hh}h_{t-1} + W_{xh}x_{t})$$
$$= \tanh\left(\left(W_{hh} \quad W_{hx}\right) \begin{pmatrix} h_{t-1} \\ x_{t} \end{pmatrix}\right)$$
$$= \tanh\left(W\begin{pmatrix} h_{t-1} \\ x_{t} \end{pmatrix}\right)$$

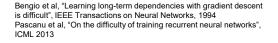
Georgia Tech

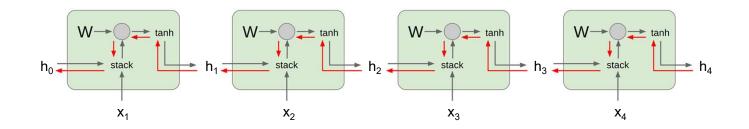
Slide Credit: Fei-Fei Li, Justin Johnson, Serena Yeung, CS 231n





Slide Credit: Fei-Fei Li, Justin Johnson, Serena Yeung, CS 231n

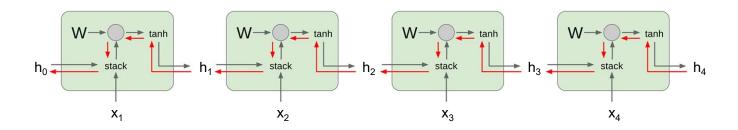




Computing gradient of  $h_0$  involves many factors of W (and repeated tanh)



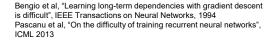
Bengio et al, "Learning long-term dependencies with gradient descent is difficult", IEEE Transactions on Neural Networks, 1994 Pascanu et al, "On the difficulty of training recurrent neural networks", ICML 2013

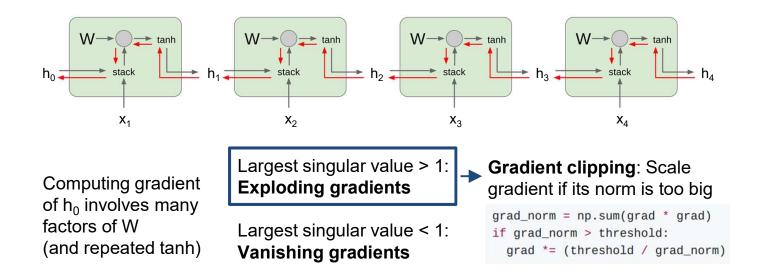


Computing gradient of  $h_0$  involves many factors of W (and repeated tanh) Largest singular value > 1: Exploding gradients

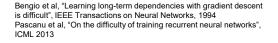
Largest singular value < 1: Vanishing gradients

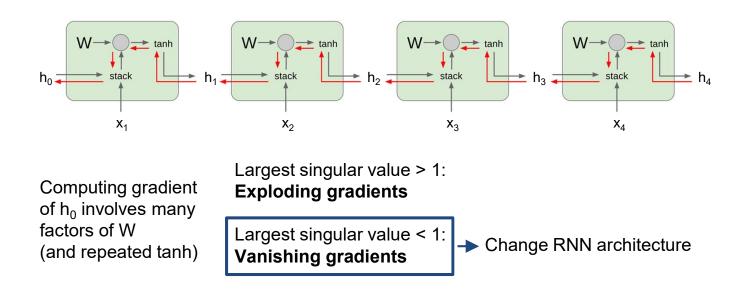














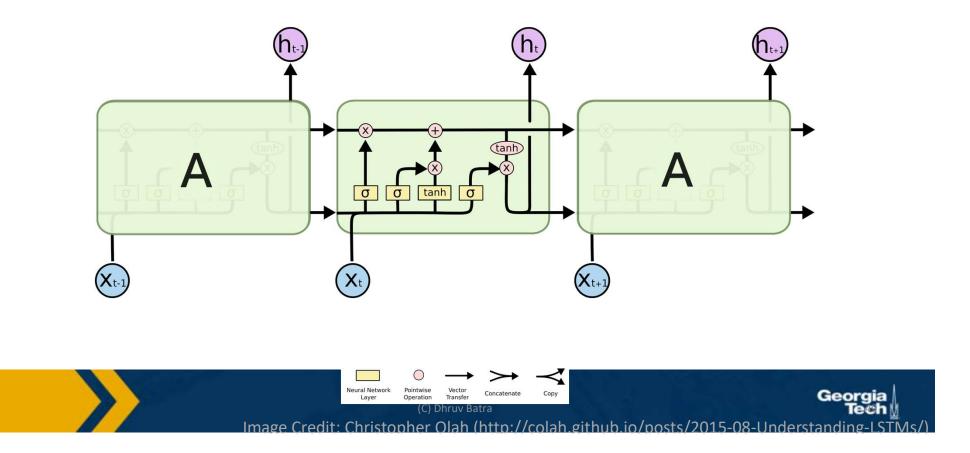
#### Long Short Term Memory (LSTM)

Vanilla RNNLSTM $h_t = \tanh\left(W\begin{pmatrix}h_{t-1}\\x_t\end{pmatrix}\right)$  $\begin{bmatrix}i\\f\\o\\g\end{pmatrix} = \begin{pmatrix}\sigma\\\sigma\\d\\tanh\end{pmatrix}W\begin{pmatrix}h_{t-1}\\x_t\end{pmatrix}$  $c_t = f \odot c_{t-1} + i \odot g$  $h_t = o \odot \tanh(c_t)$ 

Hochreiter and Schmidhuber, "Long Short Term Memory", Neural Computation 1997

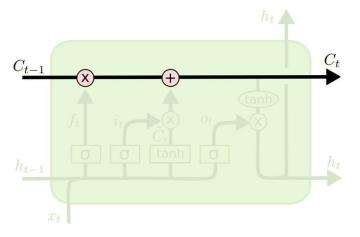


# Meet LSTMs



# LSTMs Intuition: Memory

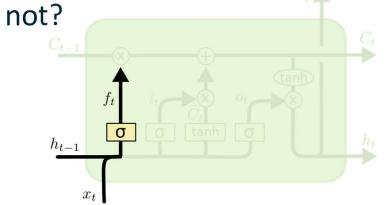
• Cell State / Memory





## LSTMs Intuition: Forget Gate

Should we continue to remember this "bit" of information or

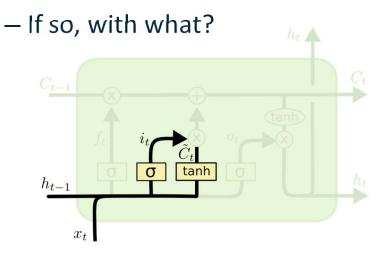


$$f_t = \sigma \left( W_f \cdot [h_{t-1}, x_t] + b_f \right)$$



## LSTMs Intuition: Input Gate

• Should we update this "bit" of information or not?

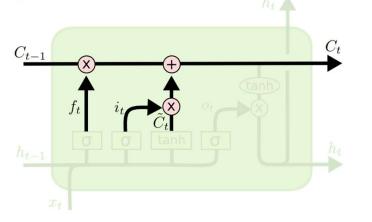


$$i_t = \sigma \left( W_i \cdot [h_{t-1}, x_t] + b_i \right)$$
$$\tilde{C}_t = \tanh(W_C \cdot [h_{t-1}, x_t] + b_C)$$



## LSTMs Intuition: Memory Update

• Forget that + memorize this

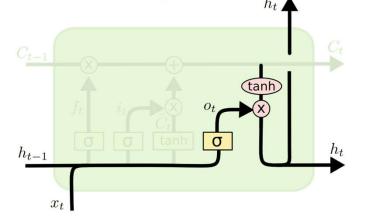


$$C_t = f_t * C_{t-1} + i_t * \tilde{C}_t$$



### LSTMs Intuition: Output Gate

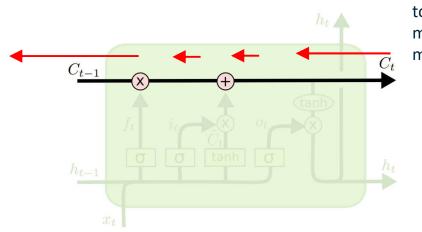
• Should we output this "bit" of information to "deeper" layers?



$$o_t = \sigma \left( W_o \left[ h_{t-1}, x_t \right] + b_o \right)$$
$$h_t = o_t * \tanh \left( C_t \right)$$

(C) Dhruv Batra Image Credit: Christopher Olah (http://colah.github.io/posts/2015-08-Understanding-LSTMs

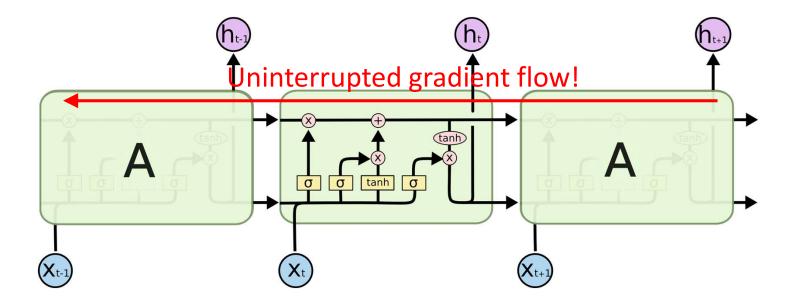
# LSTMs Intuition: Additive Updates



Backpropagation from  $c_t$ to  $c_{t-1}$  only elementwise multiplication by f, no matrix multiply by W

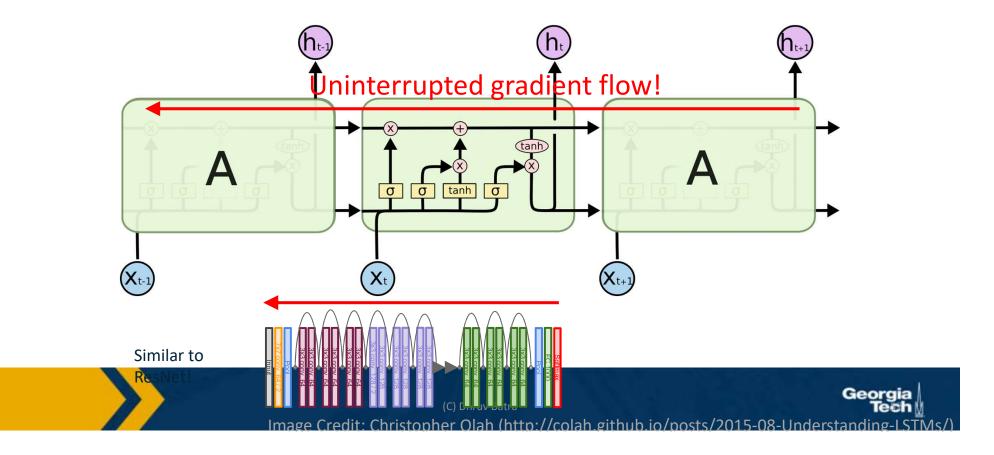
(C) Dhruv Batra (C) Dhruv Batra Image Credit: Christopher Olah (http://colah.github.io/posts/2015-08-Understanding-LSTMs/)

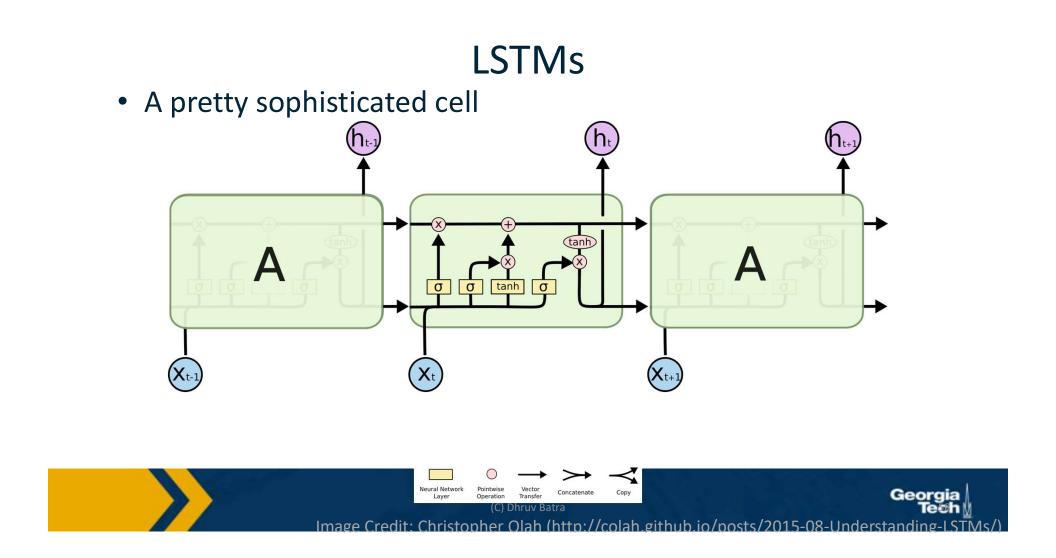
## LSTMs Intuition: Additive Updates





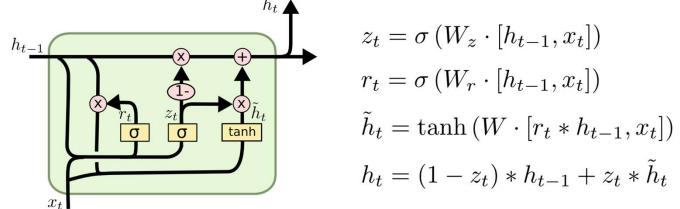
## LSTMs Intuition: Additive Updates





## LSTM Variants: Gated Recurrent Units

- Changes:
  - No explicit memory; memory = hidden output
  - Z = memorize new and forget old





#### **Other RNN Variants**

[An Empirical Exploration of Recurrent Network Architectures, Jozefowicz et al., 2015]

MUT1:

 $z = \operatorname{sigm}(W_{\mathrm{xz}}x_t + b_{\mathrm{z}})$ 

 $r = \operatorname{sigm}(W_{\operatorname{xr}}x_t + W_{\operatorname{hr}}h_t + b_{\operatorname{r}})$ 

 $h_{t+1} = \tanh(W_{\rm hh}(r \odot h_t) + \tanh(x_t) + b_{\rm h}) \odot z$  $+ h_t \odot (1-z)$ 

#### MUT2:

 $z = \operatorname{sigm}(W_{xz}x_t + W_{hz}h_t + b_z)$ 

 $r = \operatorname{sigm}(x_t + W_{\operatorname{hr}}h_t + b_{\operatorname{r}})$ 

- $h_{t+1} = \tanh(W_{\rm hh}(r \odot h_t) + W_{xh}x_t + b_{\rm h}) \odot z$ 
  - +  $h_t \odot (1-z)$

#### MUT3:

- $z = \operatorname{sigm}(W_{xz}x_t + W_{hz} \tanh(h_t) + b_z)$
- $r = \operatorname{sigm}(W_{\operatorname{xr}}x_t + W_{\operatorname{hr}}h_t + b_{\operatorname{r}})$
- $h_{t+1} = \tanh(W_{\rm hh}(r \odot h_t) + W_{xh}x_t + b_{\rm h}) \odot z$ 
  - $+ h_t \odot (1-z)$

