

Optimizing Constraint Solving to Better Support Symbolic Execution

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Background: Dynamic Symbolic Execution

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01. foo(int a, int b, int c, int d) {  
02.   if (c > a)  
03.     int e=d+10  
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Inputs: a=4, b= 5, c=6, d=1

Executed branches:

Symbolic state:

Path condition (PC):

DSE:

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$(c_0 > b_0) \wedge (b_0 > 5)$

$(c_0 \leq b_0)$

Symbolic Execution

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  (if (= 1 n)
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    (let [divisors (filter #(= 0 (rem n %)) (range 2 n))]
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Program

Symbolic Execution



Program

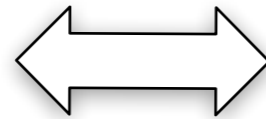


Symbolic
executor

Symbolic Execution

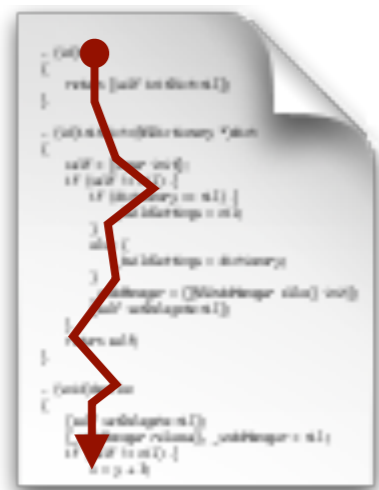


Program



Symbolic
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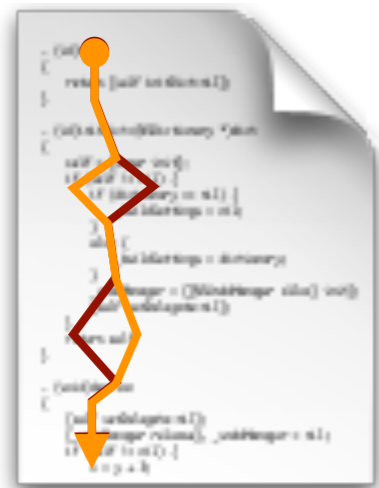


Program

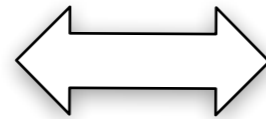


Symbolic
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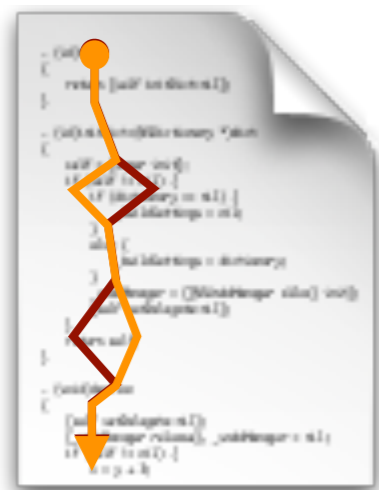


Program

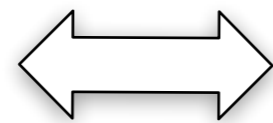


Symbolic
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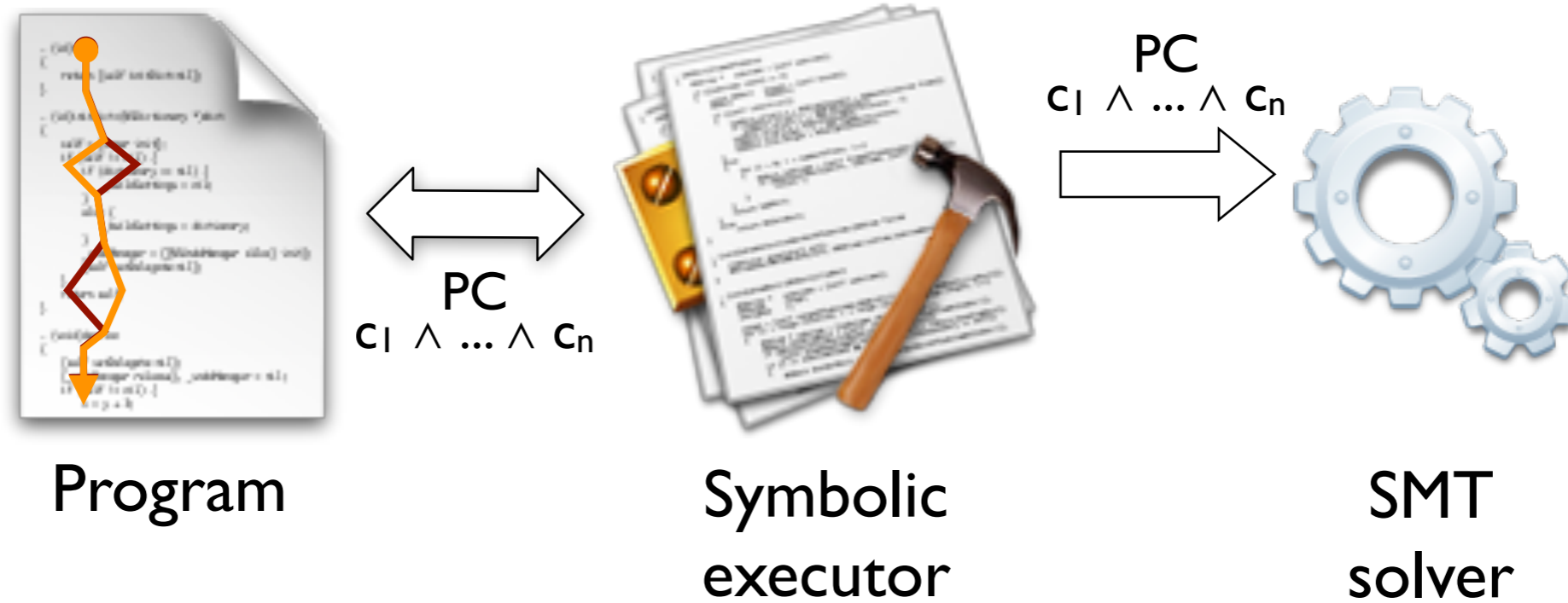


PC
 $C_1 \wedge \dots \wedge C_n$

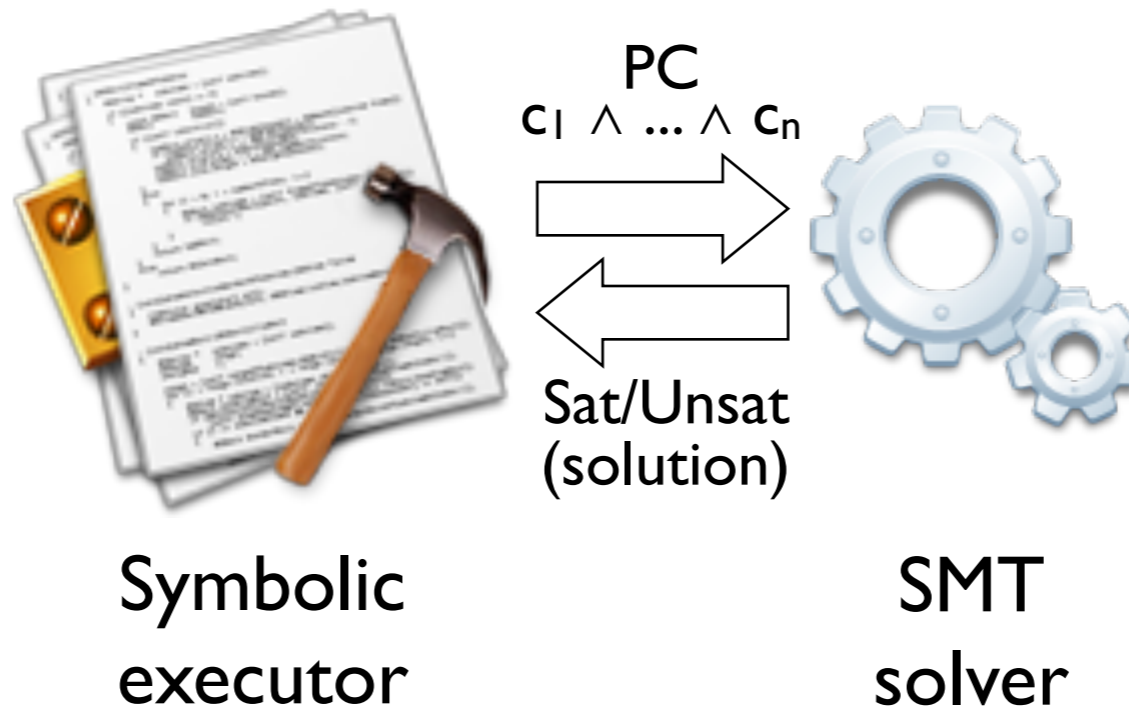


Symbolic
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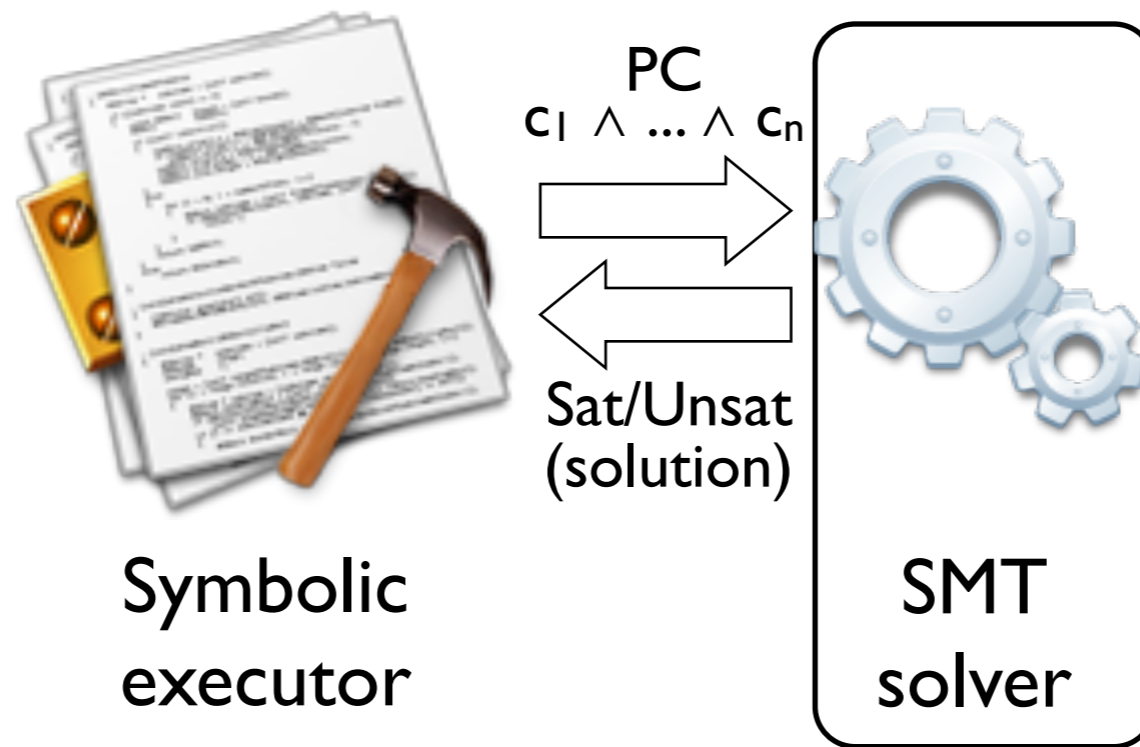
Symbolic Execution



Symbolic Execution and SMT Solving



Symbolic Execution and SMT Solving



Symbolic Execution and SMT Solving



Symbolic
executor

PC
 $C_1 \wedge \dots \wedge C_n$
→
←
Sat/Unsat
(solution)



What Are We Missing?

- **Context information** (e.g., existence of previous solutions for similar PCs)
- **Domain knowledge** (e.g., programs' specific properties)

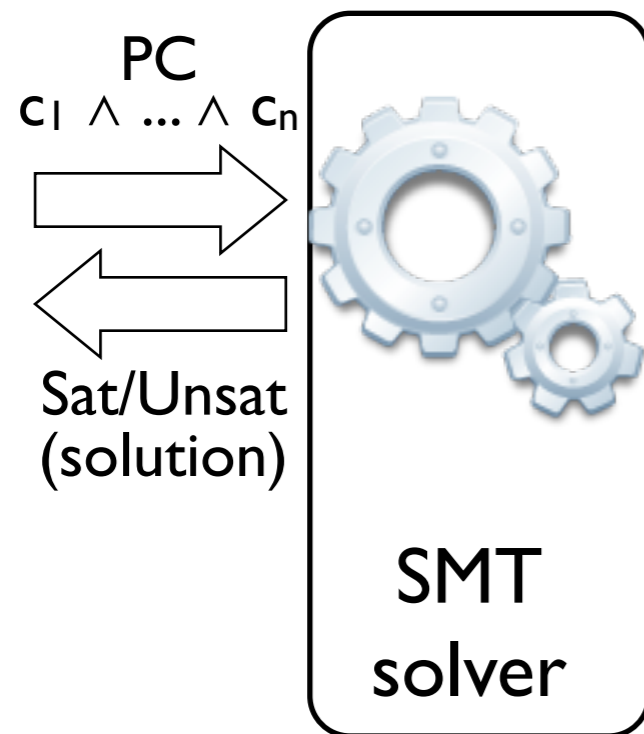
State of the Art

- Some techniques present initial solutions (domain-based constraint optimizations)
- But:
 - What is the effectiveness of these techniques?
 - What other techniques could be used?
 - Would symbolic execution actually benefit from these techniques?

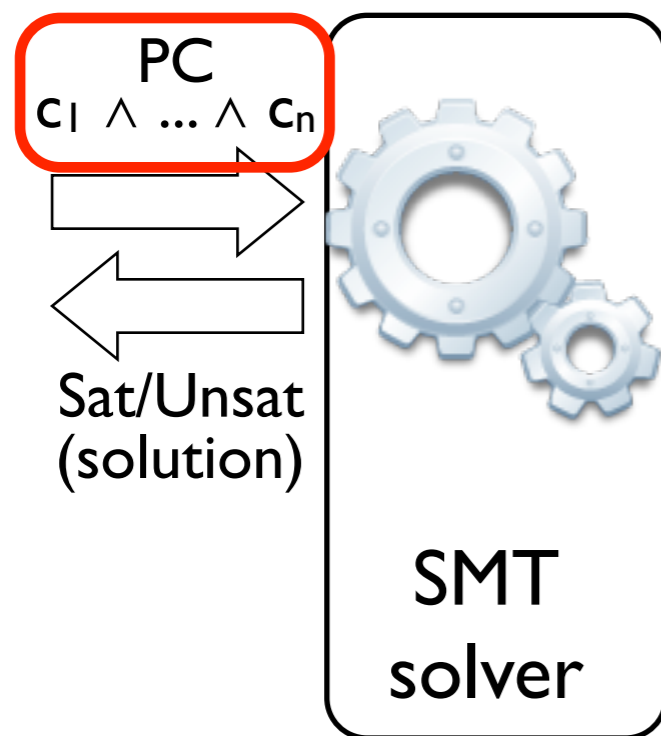
Our Goal

- Initial investigation of these questions by
 - proposing a **novel constraint optimization technique** for dynamic symbolic execution: **DomainReduce**
 - performing an **empirical evaluation** to assess new and existing optimizations empirically

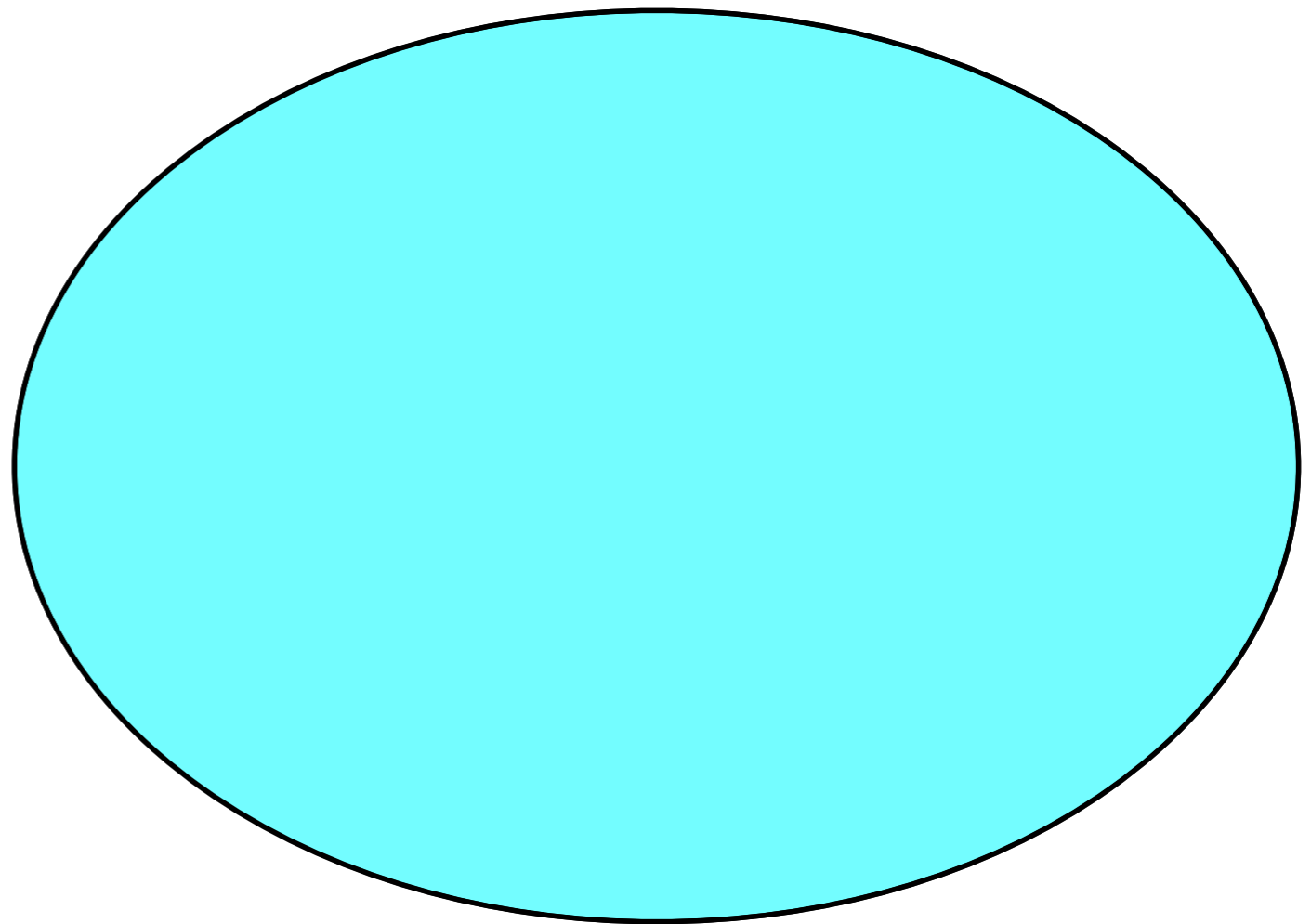
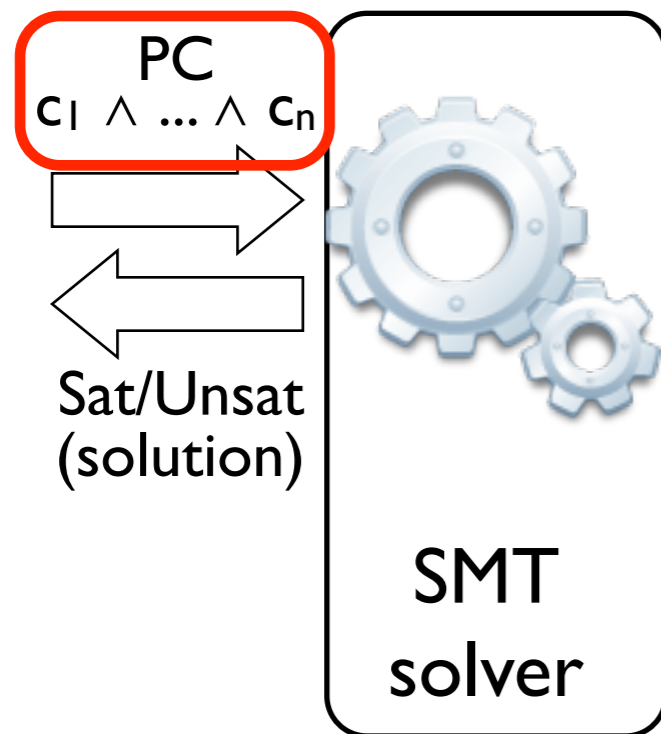
DomainReduce: Intuitive View



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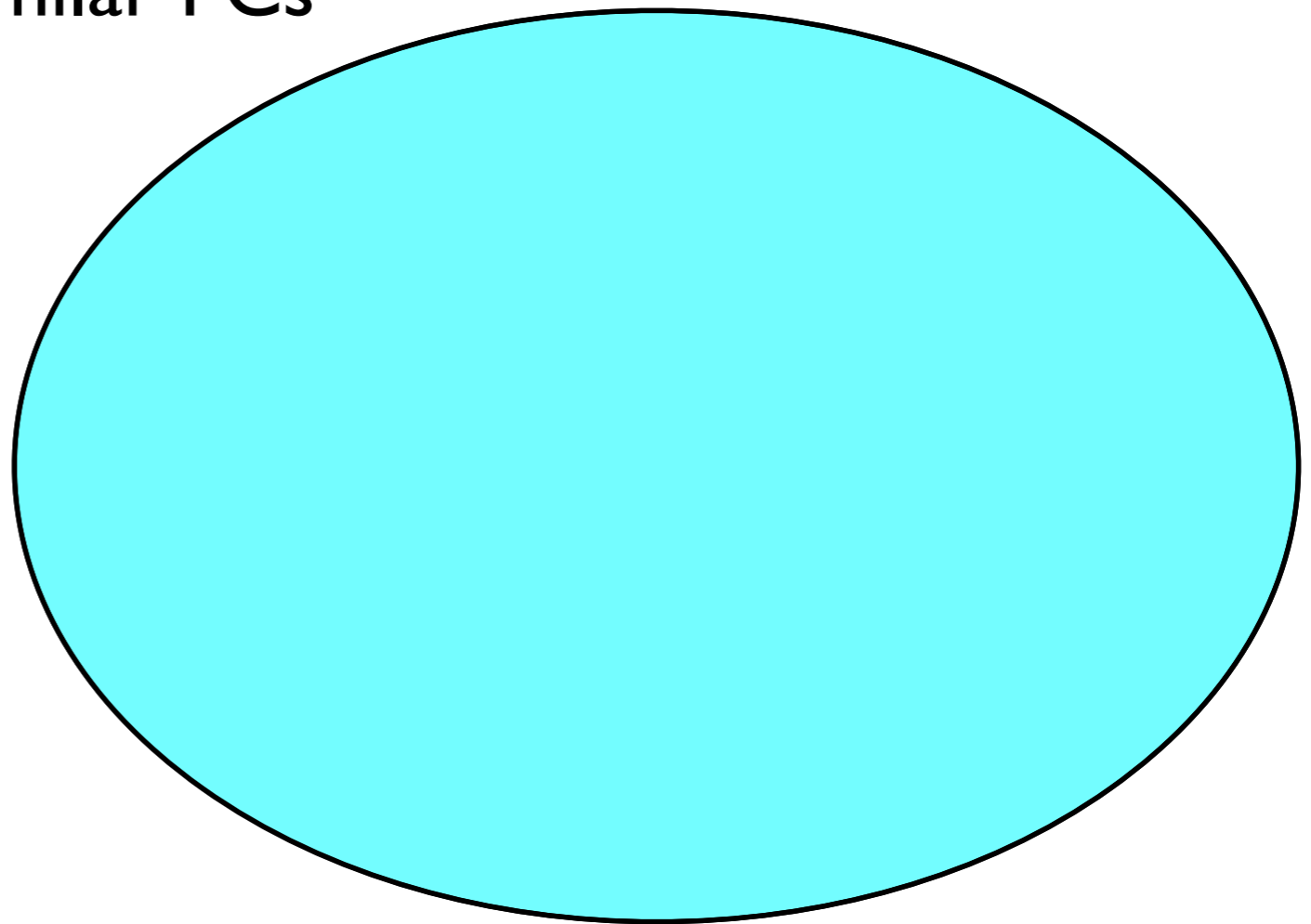
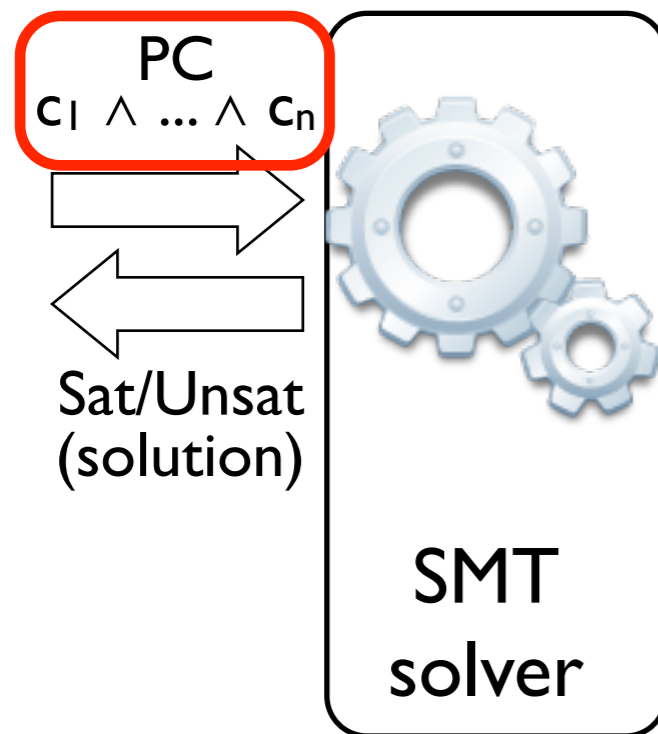


DomainReduce: Intuitive View



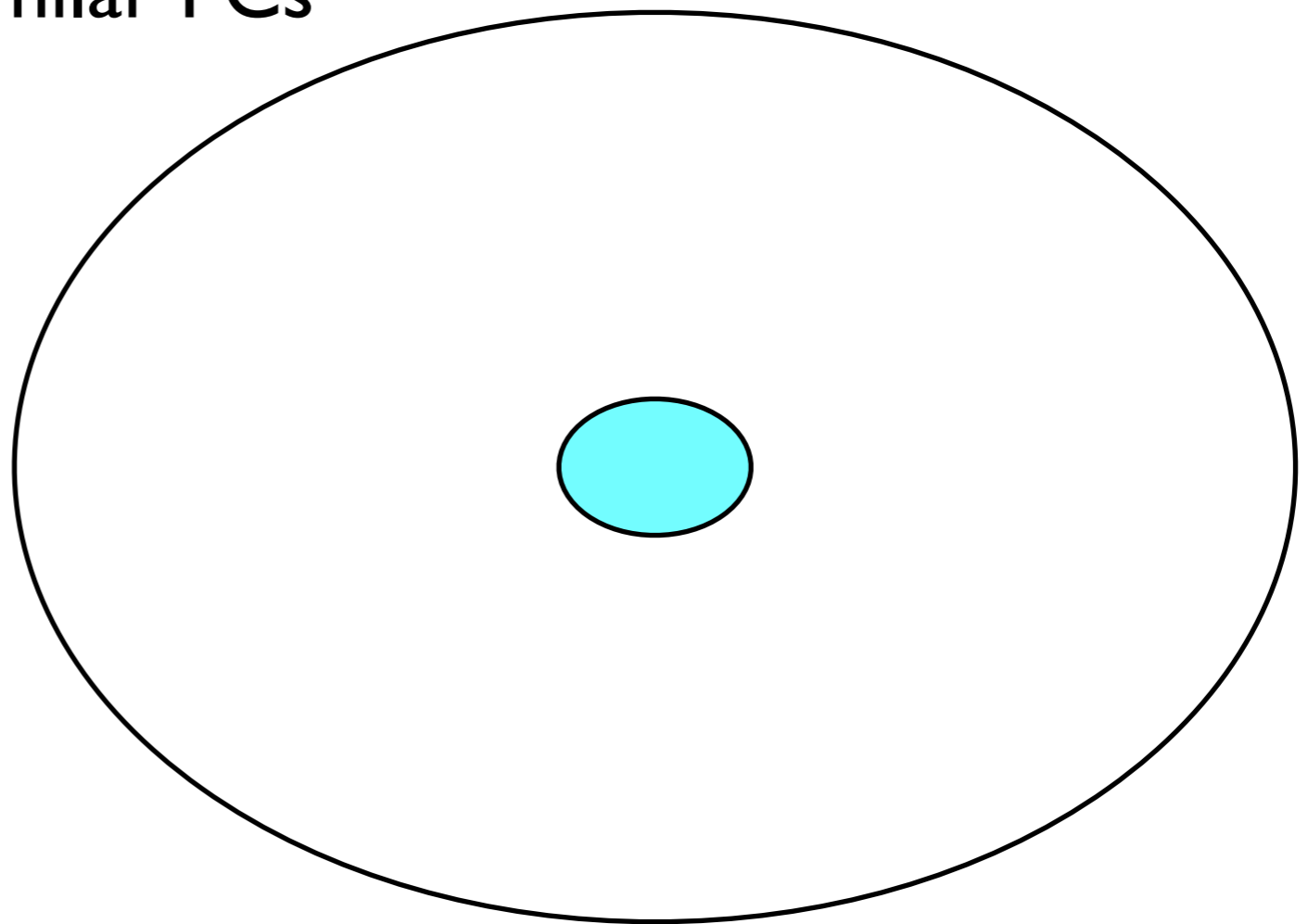
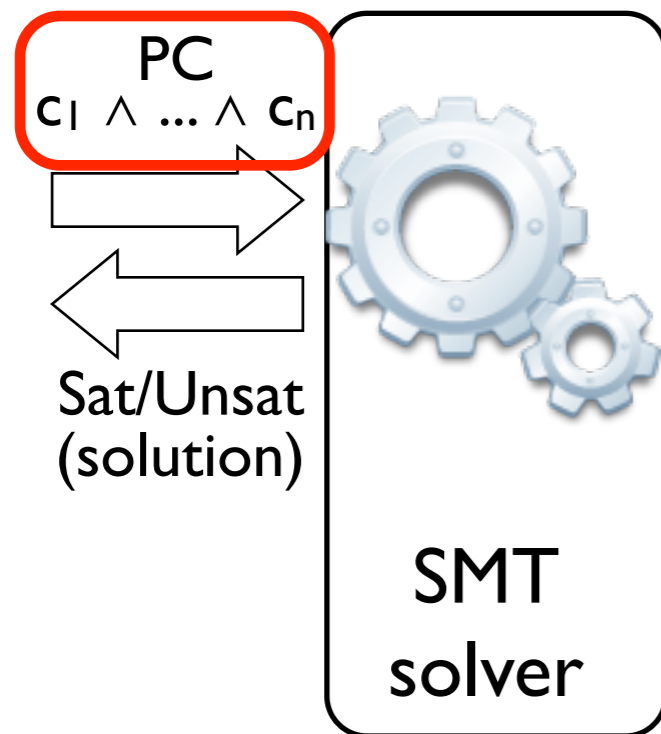
DomainReduce: Intuitive View

Restrict domain of constraints to be solved
by leveraging solutions of similar PCs



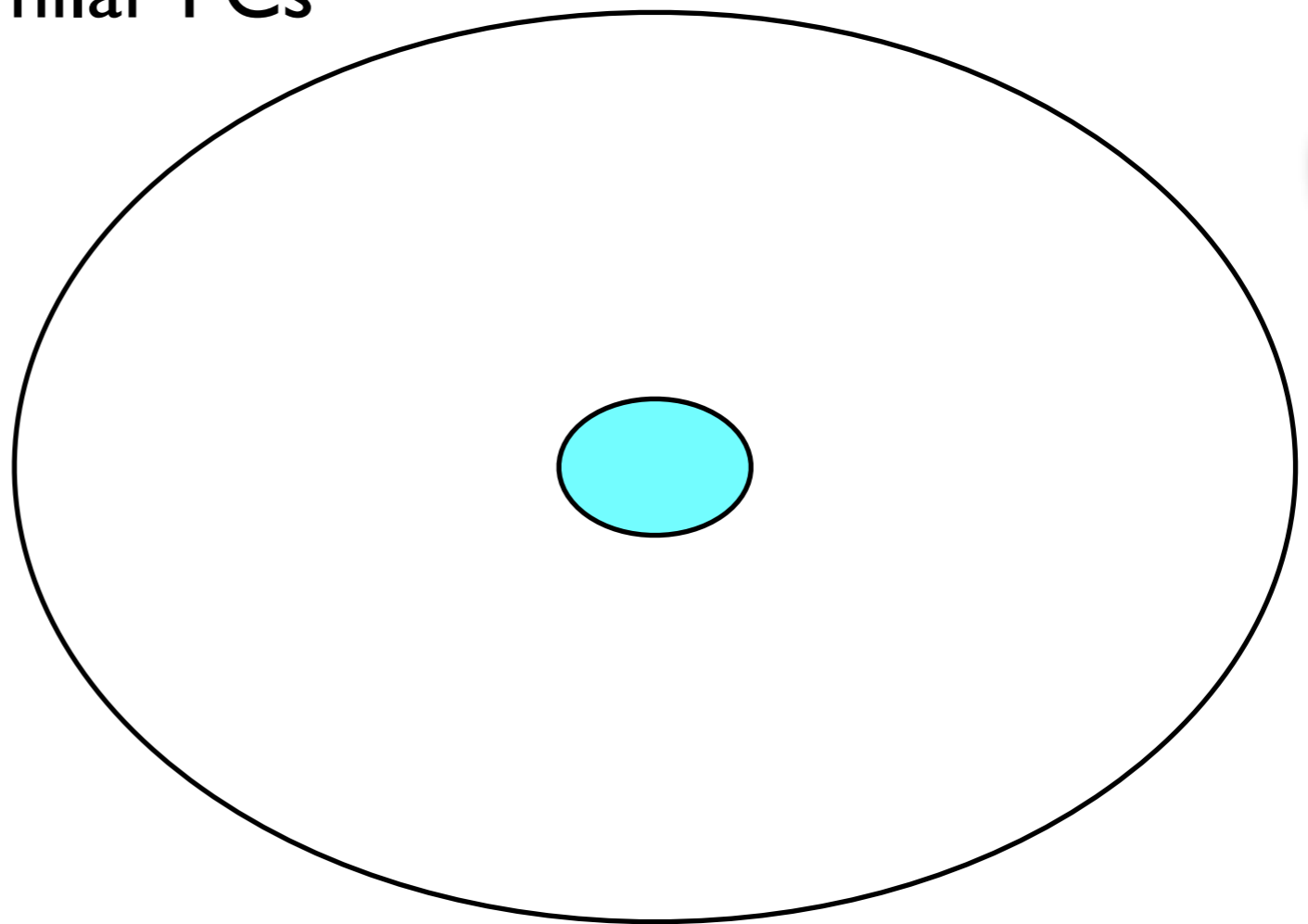
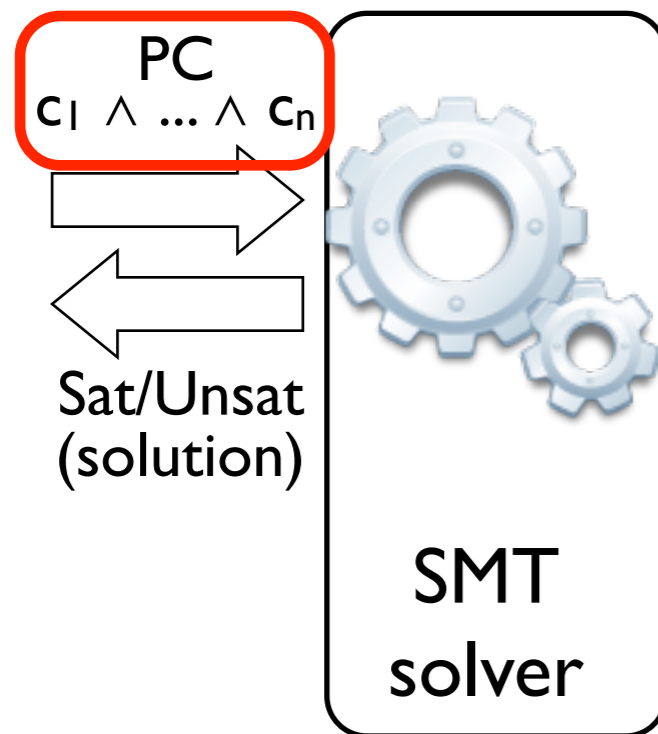
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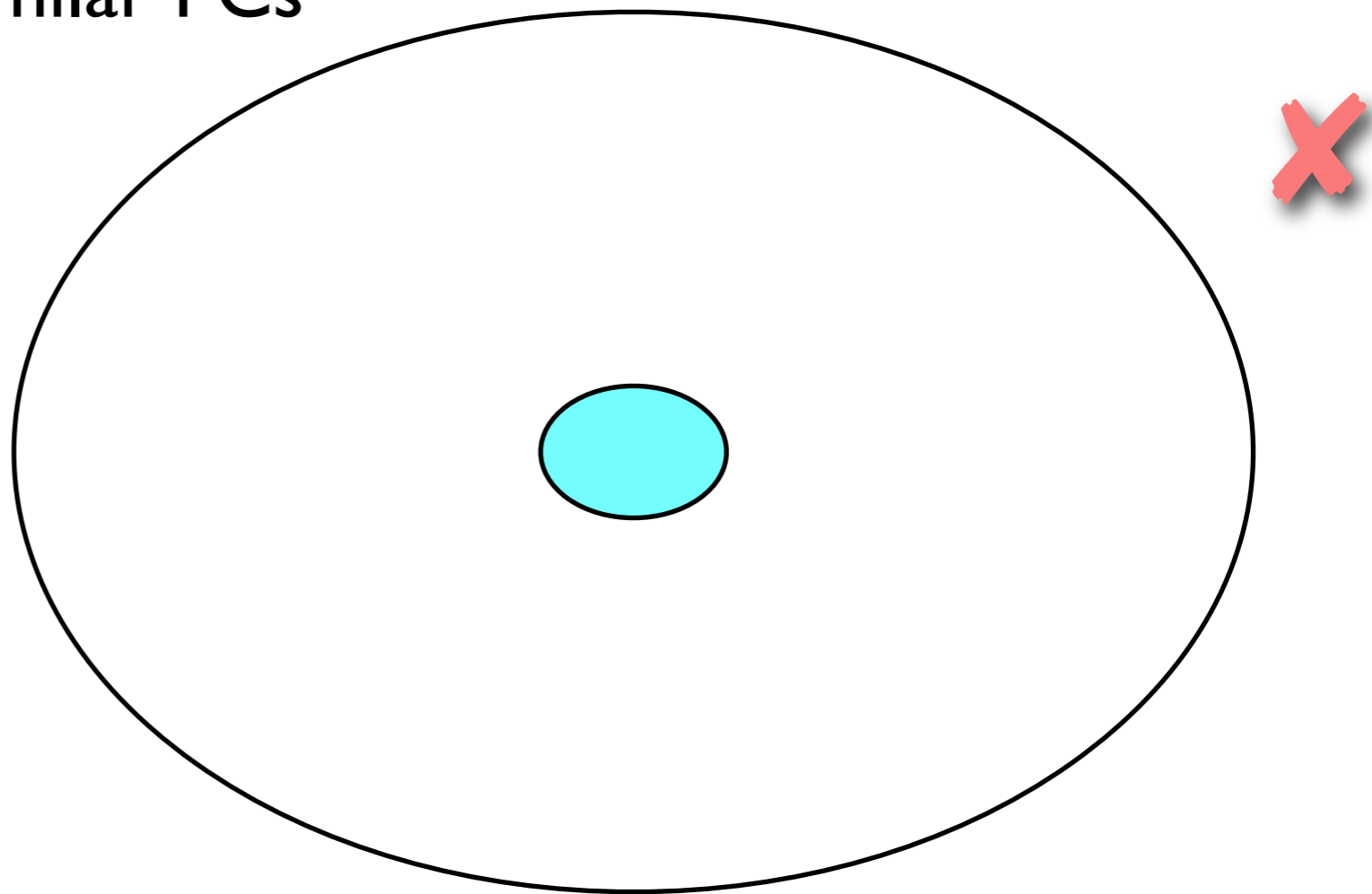
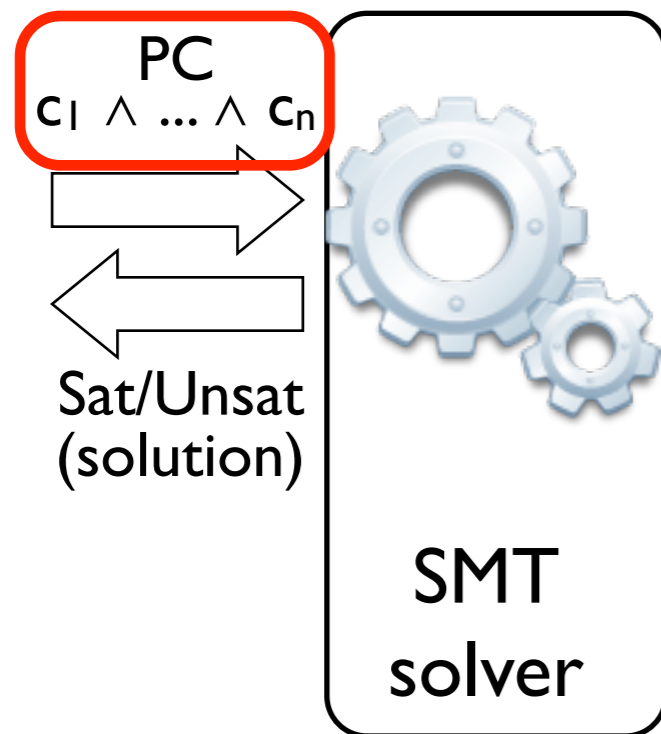
DomainReduce: Intuitive View

Restrict domain of constraints to be solved
by leveraging solutions of similar PCs



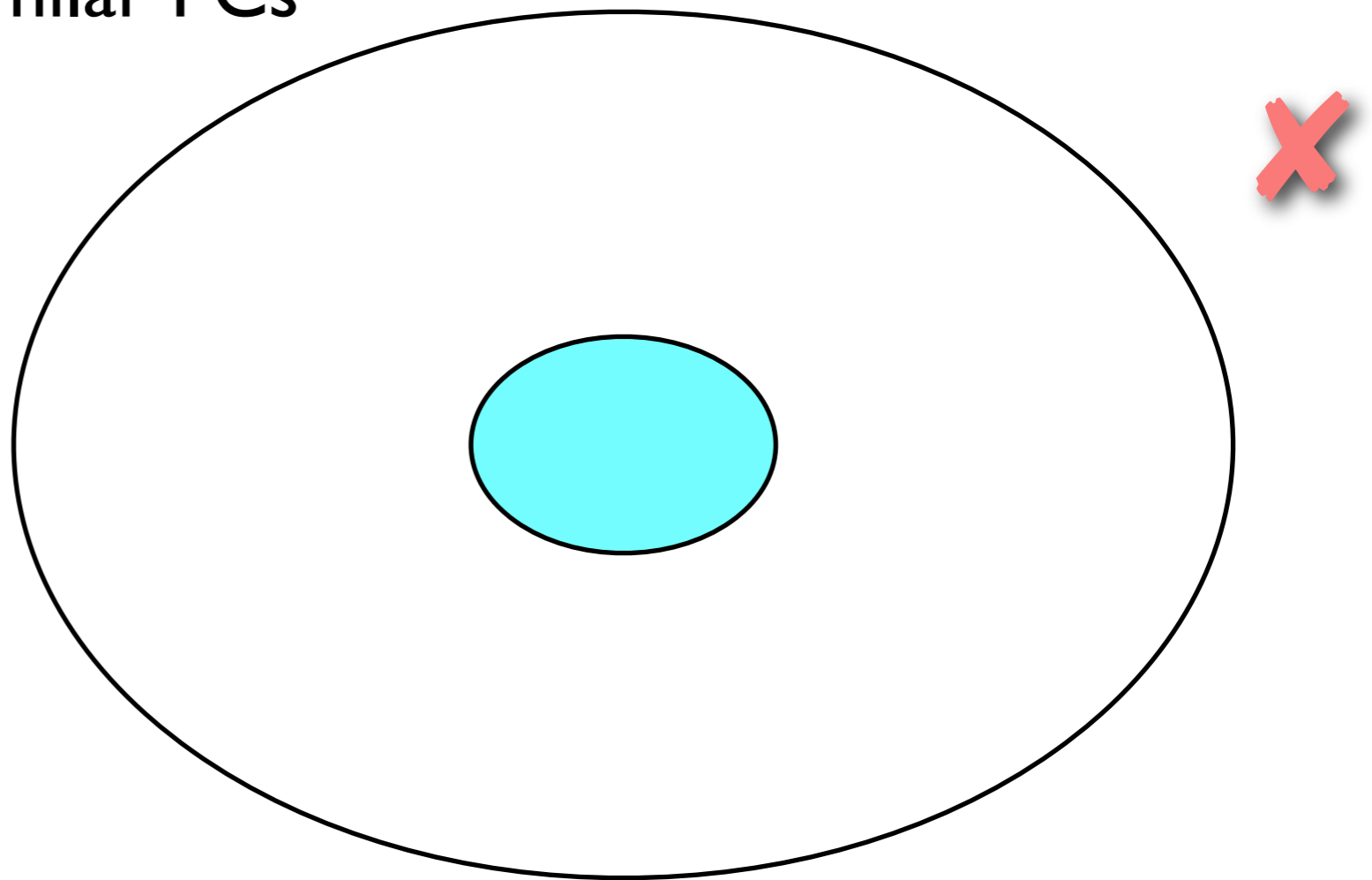
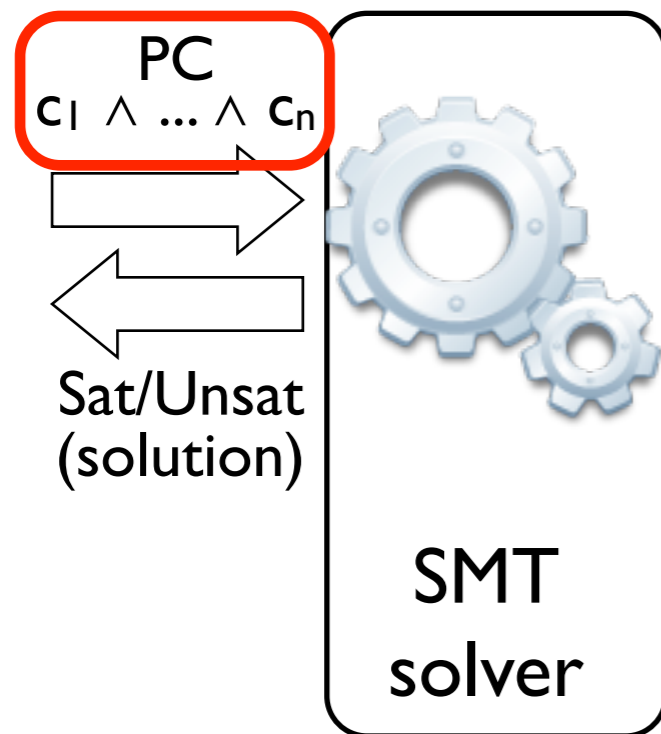
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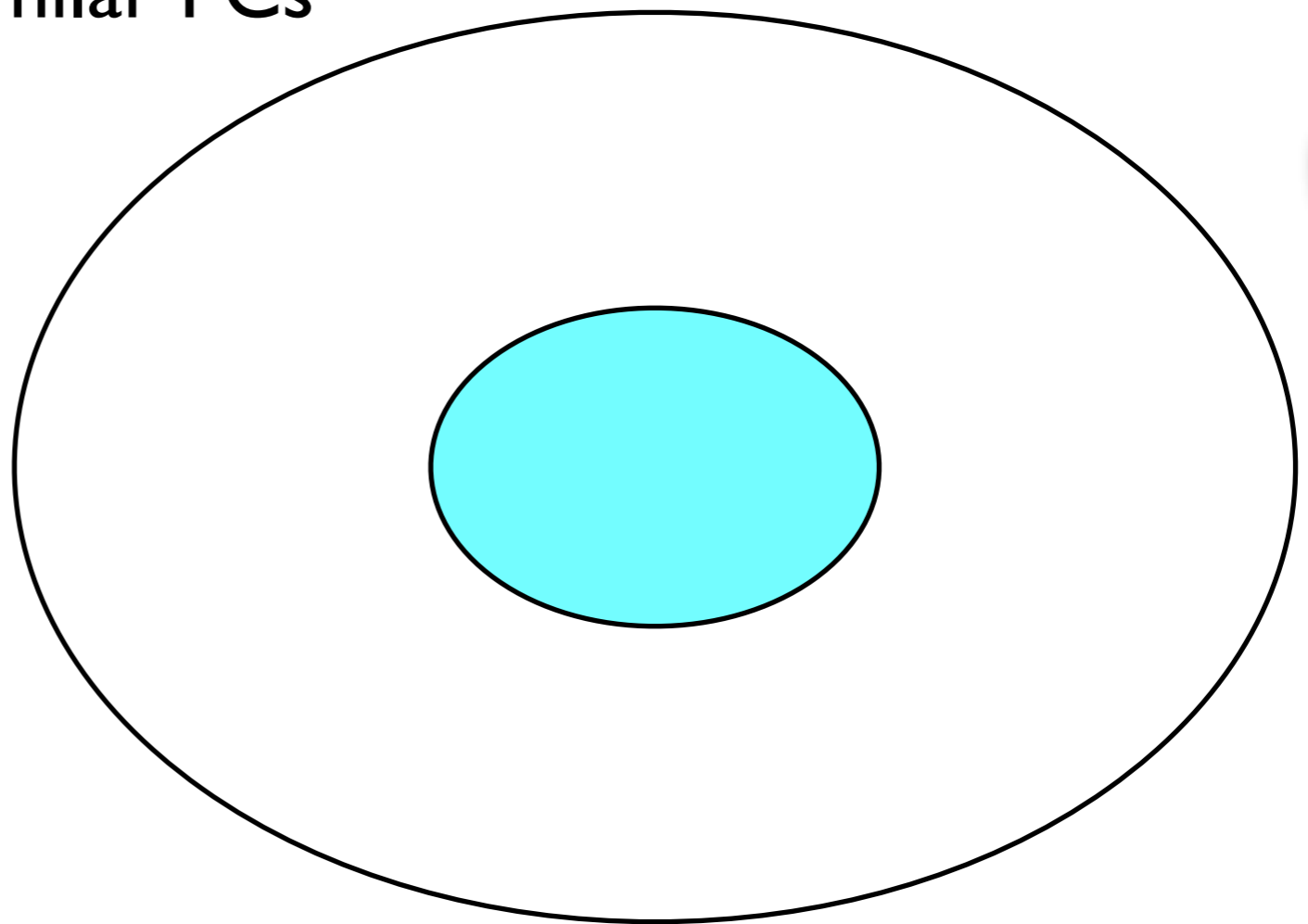
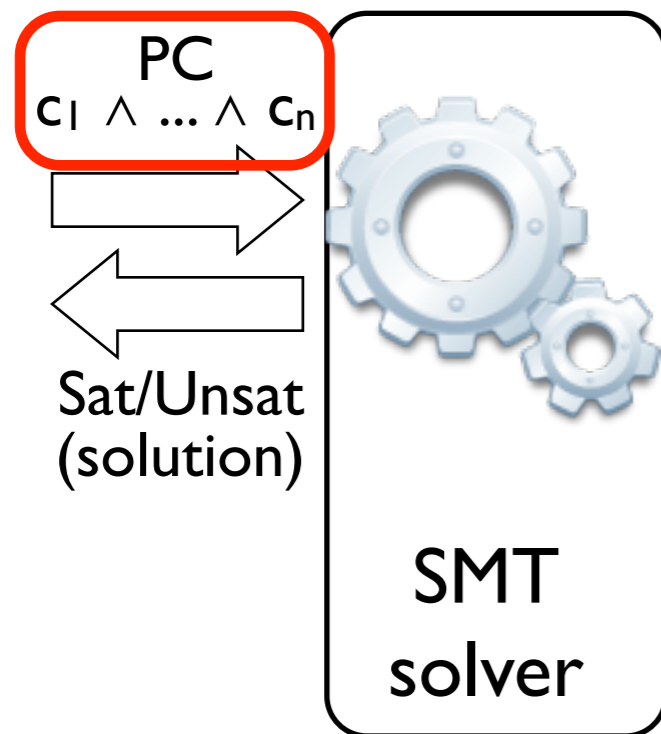
DomainReduce: Intuitive View

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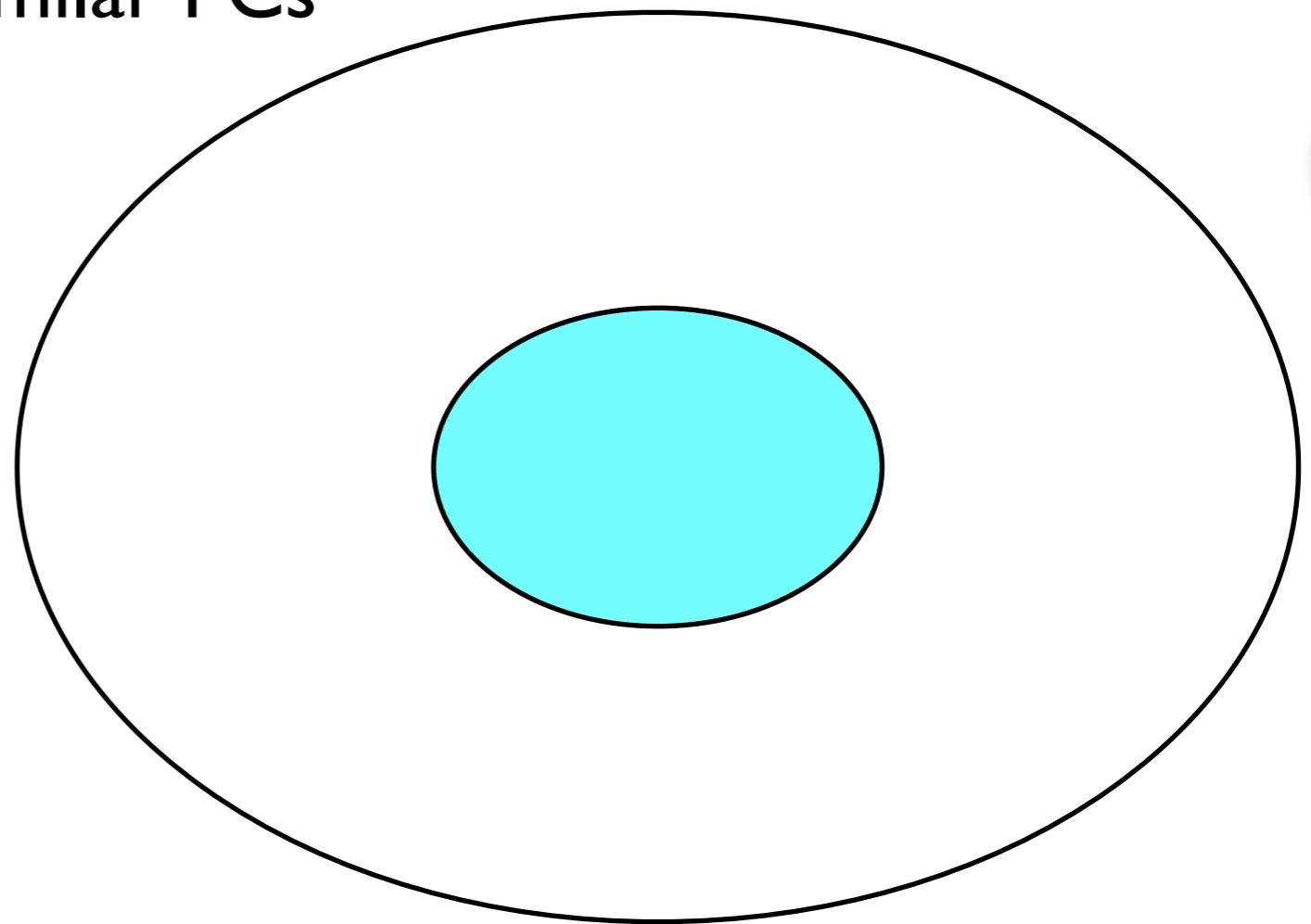
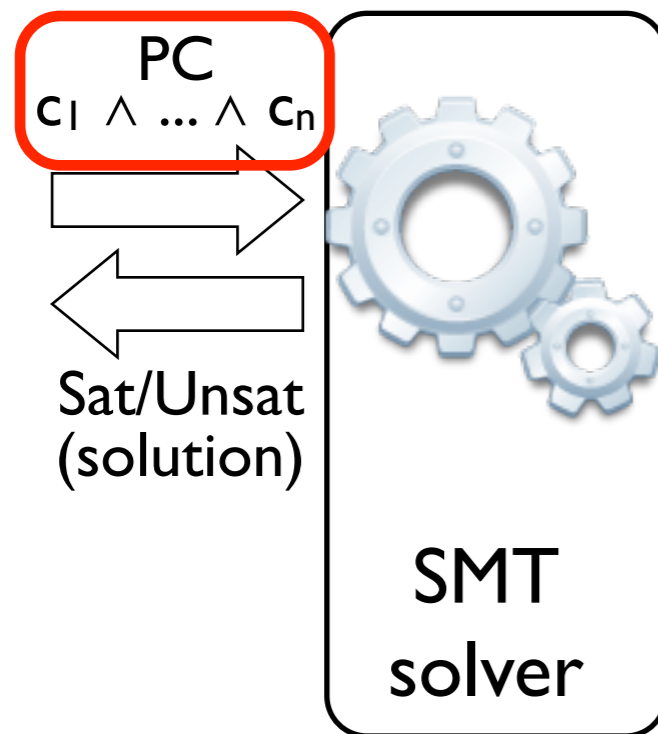
DomainReduce: Intuitive View

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DomainReduce: Intuitive View

Restrict domain of constraints to be solved
by leveraging solutions of similar PCs



Trade-off speed/likelihood of finding solutions

DomainReduce Example

(with dependencies)

DomainReduce Example

(with dependencies)

$$(c_0 > a_0) \wedge (b_0 \leq 5) \wedge (a_0 < d_0 + 10) \wedge (b_0 < c_0)$$

DomainReduce Example

(with dependencies)

$$\begin{cases} (c_0 > a_0) \wedge (b_0 \leq 5) \wedge (a_0 < d_0 + 10) \wedge (b_0 < c_0) \\ a_0 = 4, b_0 = 5, c_0 = 6, d_0 = 1 \end{cases}$$

DomainReduce Example

(with dependencies)

$$\begin{cases} (c_0 > a_0) \wedge (b_0 \leq 5) \wedge (a_0 < d_0 + 10) \wedge (b_0 < c_0) \\ a_0 = 4, b_0 = 5, c_0 = 6, d_0 = 1 \end{cases}$$

$$(c_0 > a_0) \wedge (b_0 \leq 5) \wedge (a_0 < d_0 + 10) \wedge (b_0 \geq c_0)$$

DomainReduce Example

(with dependencies)

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DomainReduce Example

(with dependencies)

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(with dependencies)

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With dependencies:

$$(c_0 > a_0) \wedge (b_0 \leq 5) \wedge (a_0 < d_0 + 10) \wedge (b_0 \geq c_0)$$
$$(5 > 4) \wedge (b_0 \leq 5) \wedge (4 < 1 + 10) \wedge (b_0 \geq 6)$$

DomainReduce Example

(with dependencies)

$$\begin{cases} (c_0 > a_0) \wedge (b_0 \leq 5) \wedge (a_0 < d_0 + 10) \wedge (b_0 < c_0) \\ a_0 = 4, b_0 = 5, c_0 = 6, d_0 = 1 \end{cases}$$

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(with dependencies)

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DomainReduce Example

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$$\begin{cases} (c_0 > a_0) \wedge (b_0 \leq 5) \wedge (a_0 < d_0 + 10) \wedge (b_0 < c_0) \\ a_0 = 4, b_0 = 5, c_0 = 6, d_0 = 1 \end{cases}$$

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DomainReduce Example

(without dependencies)

$$\begin{cases} (c_0 > a_0) \wedge (b_0 \leq 5) \wedge (a_0 < d_0 + 10) \wedge (b_0 < c_0) \\ a_0 = 4, b_0 = 5, c_0 = 6, d_0 = 1 \end{cases}$$

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DomainReduce Example

(without dependencies)

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$$(c_0 > 4) \wedge (5 \leq 5) \wedge (4 < 1 + 10) \wedge (5 \geq c_0)$$



Terminology

- **Target constraint:**
negated constraint
- **Target variables:**
variables in negated constraint
- **Direct dependency (c_a, c_b):**
 $\text{vars}(c_a) \cap \text{vars}(c_b) \neq 0$
- **Indirect dependency (c_a, c_b):**
 $\text{vars}(c_a) \cap \text{vars}(c_1) \neq 0, \text{vars}(c_1) \cap \text{vars}(c_2) \neq 0, \dots,$
 $\text{vars}(c_n) \cap \text{vars}(c_b) \neq 0$

DomainReduce Algorithm

- $s = 1$
- until sat or $s = \text{max}$ or time limit reached
 - select next subset TV of target variables of size s
 - if no more subsets, increase s and reiterate
 - identify variables dependents on TV and add them to TV
 - keep variables in TV symbolic
 - concretize all other variables
 - invoke solver

Other Techniques Considered

- **Incremental solving** (Sen et al, 2005)
 - Eliminates irrelevant constraints
 - Analogous to worst case for DomainReduce with dependencies
- **Subsumption** (Godefroid et al, 2008)
 - Eliminates implied constraints in input-bound loops
 - In hindsight, not really a constraint-optimization approach

Empirical Evaluation

Goal: Quantitative initial investigation of the usefulness of constraint optimization

RQ1: Are constraint optimization techniques effective?

RQ2: How do the different techniques compare to each other?

Experimental Infrastructure

- **Tool**
Customized JFuzz/JPF framework
- **Software subjects**
HTMLParser
XMLParser
K-Nearest Neighbor
- **Solvers**
CVC3, Z3
- **Data**
- Ten input sets per subject
- Over 5,000 real path conditions; \forall technique and constraint:
 - Number of path conditions solved by the technique
 - Time necessary to solve the condition (10 minutes timeout)

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Infrastructure and data freely available online:
<http://www.cc.gatech.edu/~ikpeme/software/>

Study Results I

(# constraints processed)

Subjects	PCs considered	PCs successfully processed									
		unsat+sat									
		No optimization		Subsumption		Incremental solving		DomainReduce with dependencies		DomainReduce without dependencies	
cvc3	z3	cvc3	z3	cvc3	z3	cvc3	z3	cvc3	z3	cvc3	z3
HTMLParser	1879	1879 43+1836	1879 43+1836	1879 43+1836	1879 43+1836	1879 43+1836	1879 43+1836	1879 43+1836	1879 43+1836	1879 43+1836	1879 43+1836
XMLParser	1881	473 49+424	1881 49+1832	473 49+424	1881 49+1832	1881 49+1832	1881 49+1832	1881 49+1832	1881 49+1832	1881 49+1832	1881 49+1832
K-NN	1930	261 261+0	936 936+0	261 261+0	936 936+0	271 271+0	937 937+0	262 262+0	878 878+0	111 0+111	0 0+0

Study Results I

(# constraints processed)

Subjects	PCs considered	PCs successfully processed									
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		No optimization		Subsumption		Incremental solving		DomainReduce with dependencies		DomainReduce without dependencies	
cvc3	z3	cvc3	z3	cvc3	z3	cvc3	z3	cvc3	z3	cvc3	z3
HTMLParser	1879	1879 43+1836	1879 43+1836	1879 43+1836	1879 43+1836	1879 43+1836	1879 43+1836	1879 43+1836	1879 43+1836	1879 43+1836	1879 43+1836
XMLParser	1881	473 49+424	1881 49+1832	473 49+424	1881 49+1832	1881 49+1832	1881 49+1832	1881 49+1832	1881 49+1832	1881 49+1832	1881 49+1832
K-NN	1930	261 261+0	936 936+0	261 261+0	936 936+0	271 271+0	937 937+0	262 262+0	878 878+0	111 0+111	0 0+0

- Results for HTMLParser not compelling

Study Results I

(# constraints processed)

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		No optimization		Subsumption		Incremental solving		DomainReduce with dependencies		DomainReduce without dependencies	
cvc3	z3	cvc3	z3	cvc3	z3	cvc3	z3	cvc3	z3	cvc3	z3
HTMLParser	1879	1879 43+1836	1879 43+1836	1879 43+1836	1879 43+1836	1879 43+1836	1879 43+1836	1879 43+1836	1879 43+1836	1879 43+1836	1879 43+1836
XMLParser	1881	473 49+424	1881 49+1832	473 49+424	1881 49+1832	1881 49+1832	1881 49+1832	1881 49+1832	1881 49+1832	1881 49+1832	1881 49+1832
K-NN	1930	261 261+0	936 936+0	261 261+0	936 936+0	271 271+0	937 937+0	262 262+0	878 878+0	111 0+111	0 0+0

- Results for HTMLParser not compelling
- Optimizations ineffective for Z3

Study Results I

(# constraints processed)

Subjects	PCs considered	PCs successfully processed unsat+sat									
		No optimization		Subsumption		Incremental solving		DomainReduce with dependencies		DomainReduce without dependencies	
		cvc3	z3	cvc3	z3	cvc3	z3	cvc3	z3	cvc3	z3
HTMLParser	1879	1879 43+1836	1879 43+1836	1879 43+1836	1879 43+1836	1879 43+1836	1879 43+1836	1879 43+1836	1879 43+1836	1879 43+1836	1879 43+1836
XMLParser	1881	473 49+424	1881 49+1832	473 49+424	1881 49+1832	1881 49+1832	1881 49+1832	1881 49+1832	1881 49+1832	1881 49+1832	1881 49+1832
K-NN	1930	261 261+0	936 936+0	261 261+0	936 936+0	271 271+0	937 937+0	262 262+0	878 878+0	111 0+111	0 0+0

- Results for HTMLParser not compelling
- Optimizations ineffective for Z3
 - Useless or ineffective, with one exception

Study Results I

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		unsat+sat									
		No optimization		Subsumption		Incremental solving		DomainReduce with dependencies		DomainReduce without dependencies	
cvc3	z3	cvc3	z3	cvc3	z3	cvc3	z3	cvc3	z3		
HTMLParser	1879	1879 43+1836	1879 43+1836	1879 43+1836	1879 43+1836	1879 43+1836	1879 43+1836	1879 43+1836	1879 43+1836	1879 43+1836	1879 43+1836
XMLParser	1881	473 49+424	1881 49+1832	473 49+424	1881 49+1832	1881 49+1832	1881 49+1832	1881 49+1832	1881 49+1832	1881 49+1832	1881 49+1832
K-NN	1930	261 261+0	936 936+0	261 261+0	936 936+0	271 271+0	937 937+0	262 262+0	878 878+0	111 0+111	0 0+0

- Results for HTMLParser not compelling
- Optimizations ineffective for Z3
 - Useless or ineffective, with one exception
- DomainReduce produces negative results for K-NN (worst case)

Study Results I

(# constraints processed)

Subjects	PCs considered	PCs successfully processed									
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		No optimization		Subsumption		Incremental solving		DomainReduce with dependencies		DomainReduce without dependencies	
cvc3	z3	cvc3	z3	cvc3	z3	cvc3	z3	cvc3	z3		
HTMLParser	1879	1879 43+1836	1879 43+1836	1879 43+1836	1879 43+1836	1879 43+1836	1879 43+1836	1879 43+1836	1879 43+1836	1879 43+1836	1879 43+1836
XMLParser	1881	473 49+424	1881 49+1832	473 49+424	1881 49+1832	1881 49+1832	1881 49+1832	1881 49+1832	1881 49+1832	1881 49+1832	1881 49+1832
K-NN	1930	261 261+0	936 936+0	261 261+0	936 936+0	271 271+0	937 937+0	262 262+0	878 878+0	111 0+111	0 0+0

- Results for HTMLParser not compelling
- Optimizations ineffective for Z3
 - Useless or ineffective, with one exception
 - DomainReduce produces negative results for K-NN (worst case)
- Optimizations effective for CVC3

Study Results I

(# constraints processed)

Subjects	PCs considered	PCs successfully processed										
		unsat+sat										
		No optimization		Subsumption		Incremental solving		DomainReduce with dependencies		DomainReduce without dependencies		
cvc3	z3	cvc3	z3	cvc3	z3	cvc3	z3	cvc3	z3	cvc3	z3	
HTMLParser	1879	1879 43+1836	1879 43+1836	1879 43+1836	1879 43+1836	1879 43+1836	1879 43+1836	1879 43+1836	1879 43+1836	1879 43+1836	1879 43+1836	1879 43+1836
XMLParser	1881	473 49+424	1881 49+1832	473 49+424	1881 49+1832	1881 49+1832	1881 49+1832	1881 49+1832	1881 49+1832	1881 49+1832	1881 49+1832	1881 49+1832
K-NN	1930	261 261+0	936 936+0	261 261+0	936 936+0	271 271+0	937 937+0	262 262+0	878 878+0	111 0+111	0 0+0	0 0+0

- Results for HTMLParser not compelling
- Optimizations ineffective for Z3
 - Useless or ineffective, with one exception
 - DomainReduce produces negative results for K-NN (worst case)
- Optimizations effective for CVC3
 - Small improvement for K-NN

Study Results I

(# constraints processed)

Subjects	PCs considered	PCs successfully processed									
		unsat+sat									
		No optimization		Subsumption		Incremental solving		DomainReduce with dependencies		DomainReduce without dependencies	
cvc3	z3	cvc3	z3	cvc3	z3	cvc3	z3	cvc3	z3		
HTMLParser	1879	1879 43+1836	1879 43+1836	1879 43+1836	1879 43+1836	1879 43+1836	1879 43+1836	1879 43+1836	1879 43+1836	1879 43+1836	1879 43+1836
XMLParser	1881	473 49+424	1881 49+1832	473 49+424	1881 49+1832	1881 49+1832	1881 49+1832	1881 49+1832	1881 49+1832	1881 49+1832	1881 49+1832
K-NN	1930	261 261+0	936 936+0	261 261+0	936 936+0	271 271+0	937 937+0	262 262+0	878 878+0	111 0+111	0 0+0

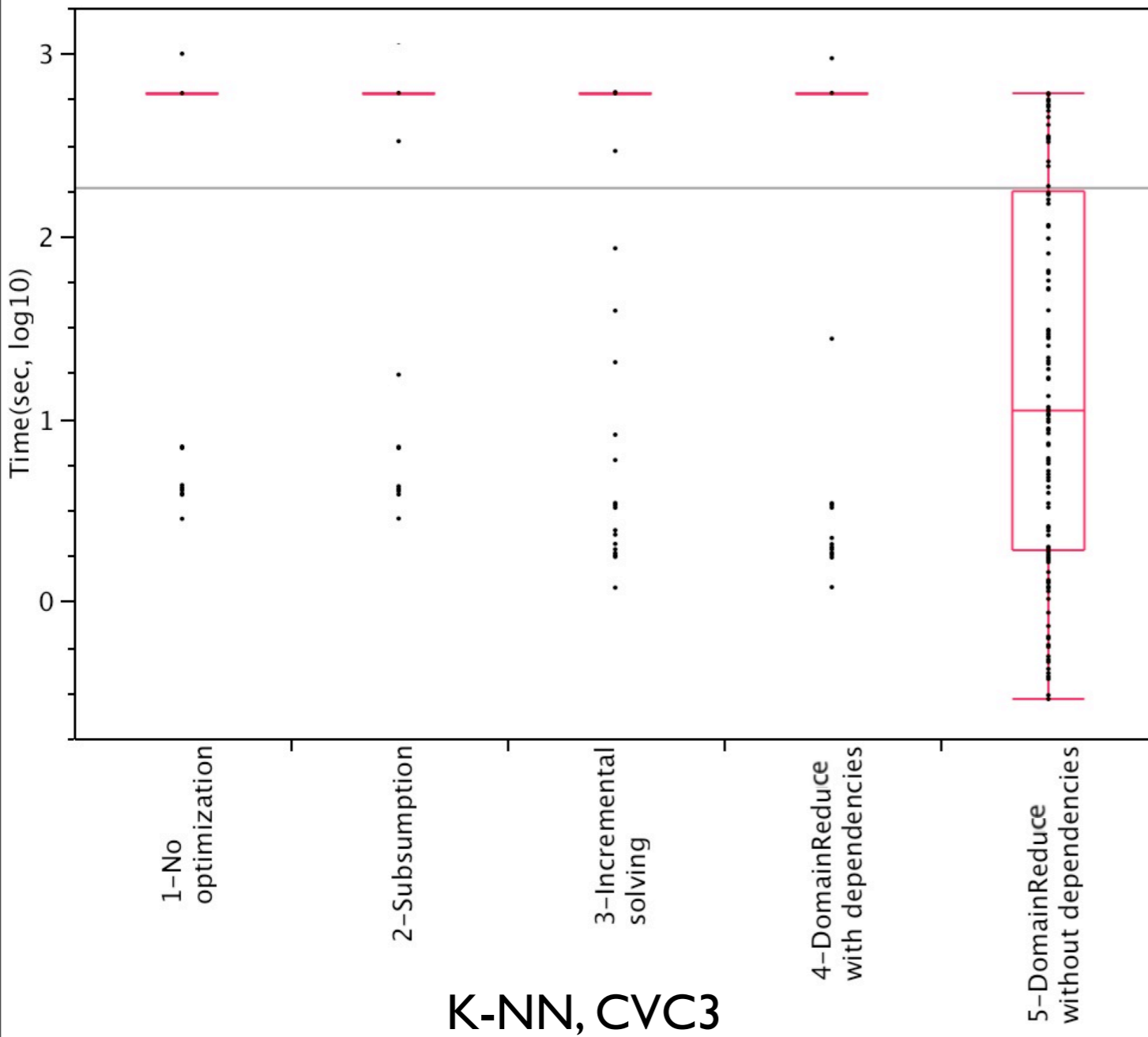
- Results for HTMLParser not compelling
- Optimizations ineffective for Z3
 - Useless or ineffective, with one exception
 - DomainReduce produces negative results for K-NN (worst case)
- Optimizations effective for CVC3
 - Small improvement for K-NN
 - Dramatic improvement for XMLParser (25% ➔ 100%)

Study Results 2

(time to process constraints)

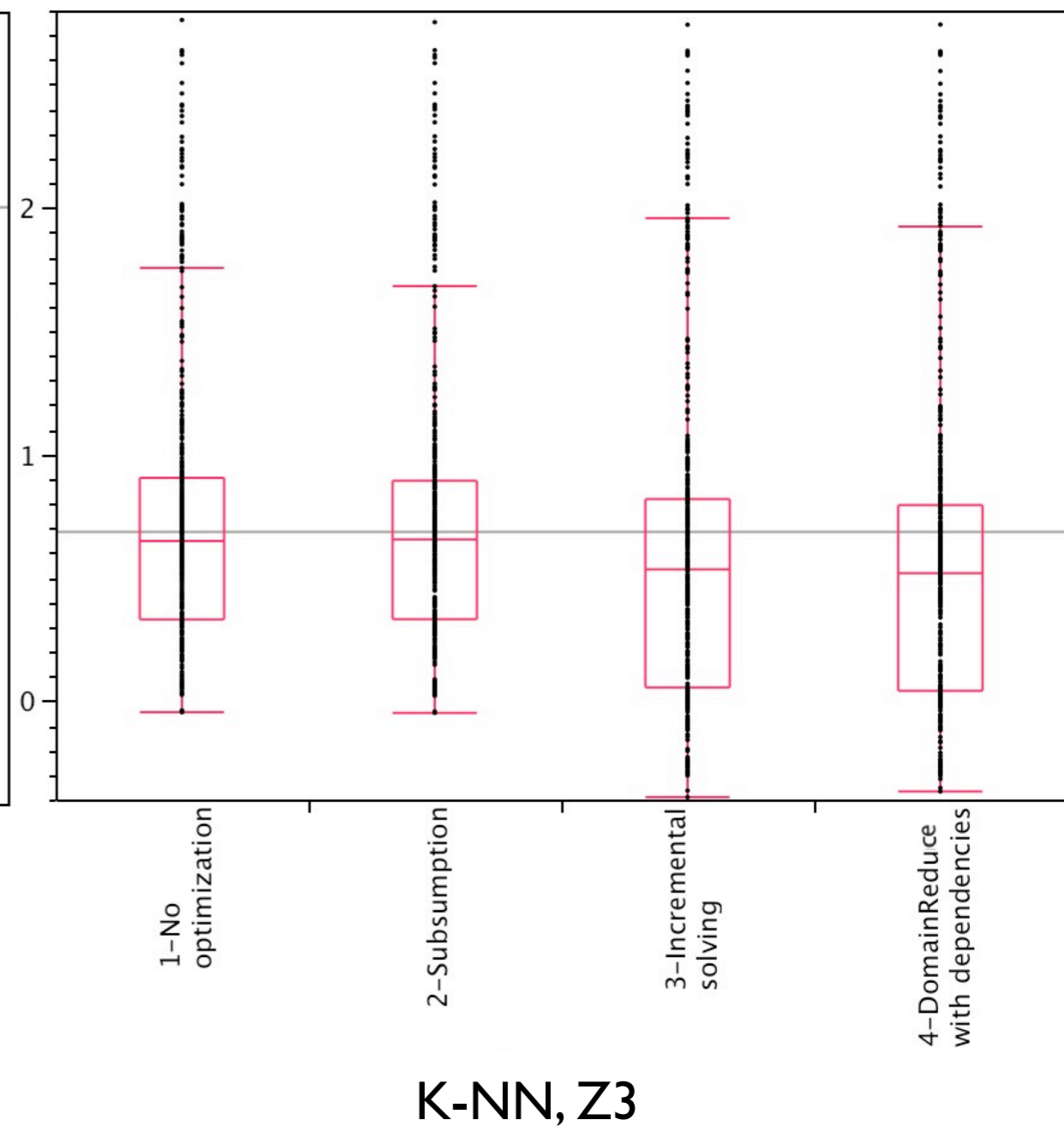
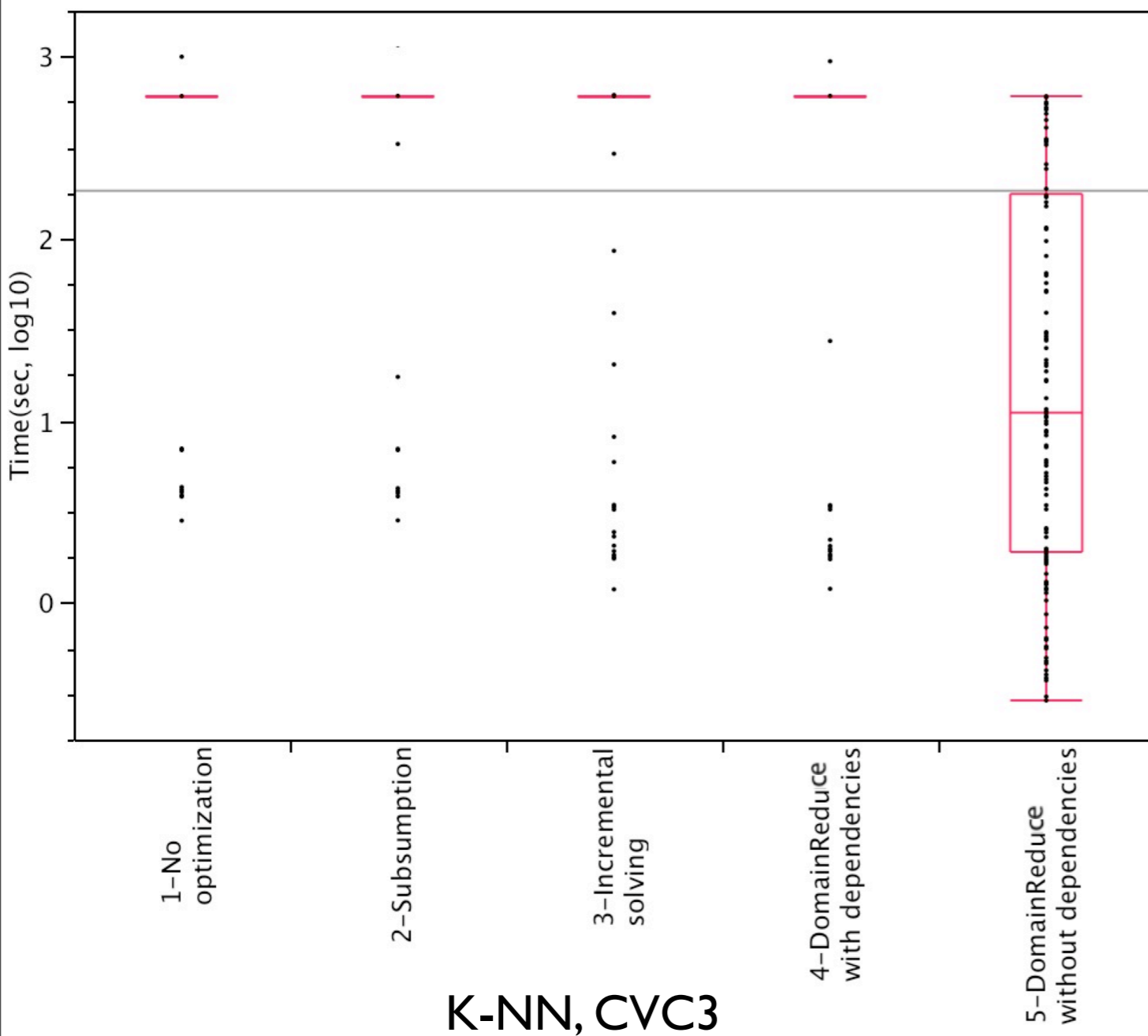
Study Results 2

(time to process constraints)



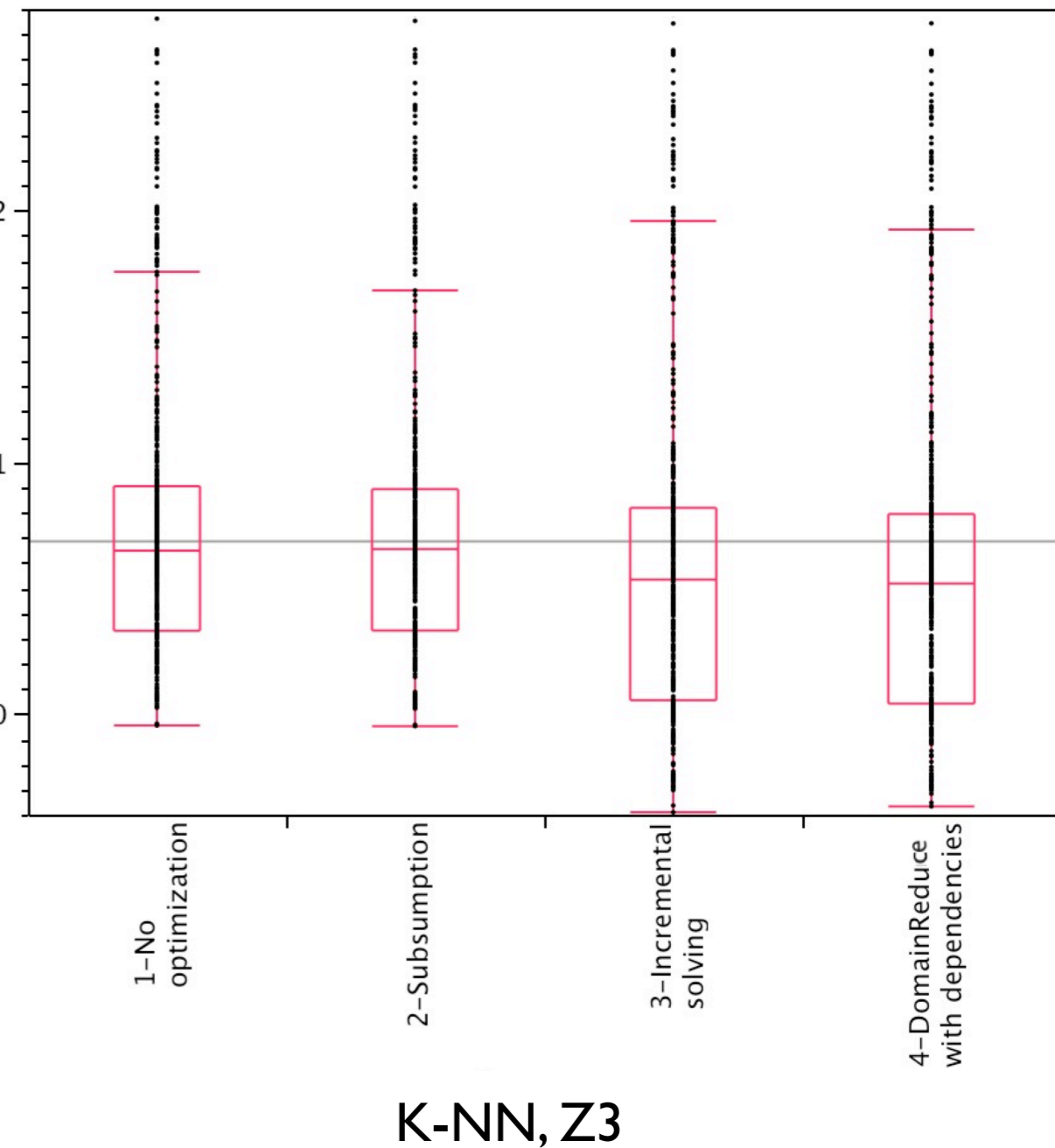
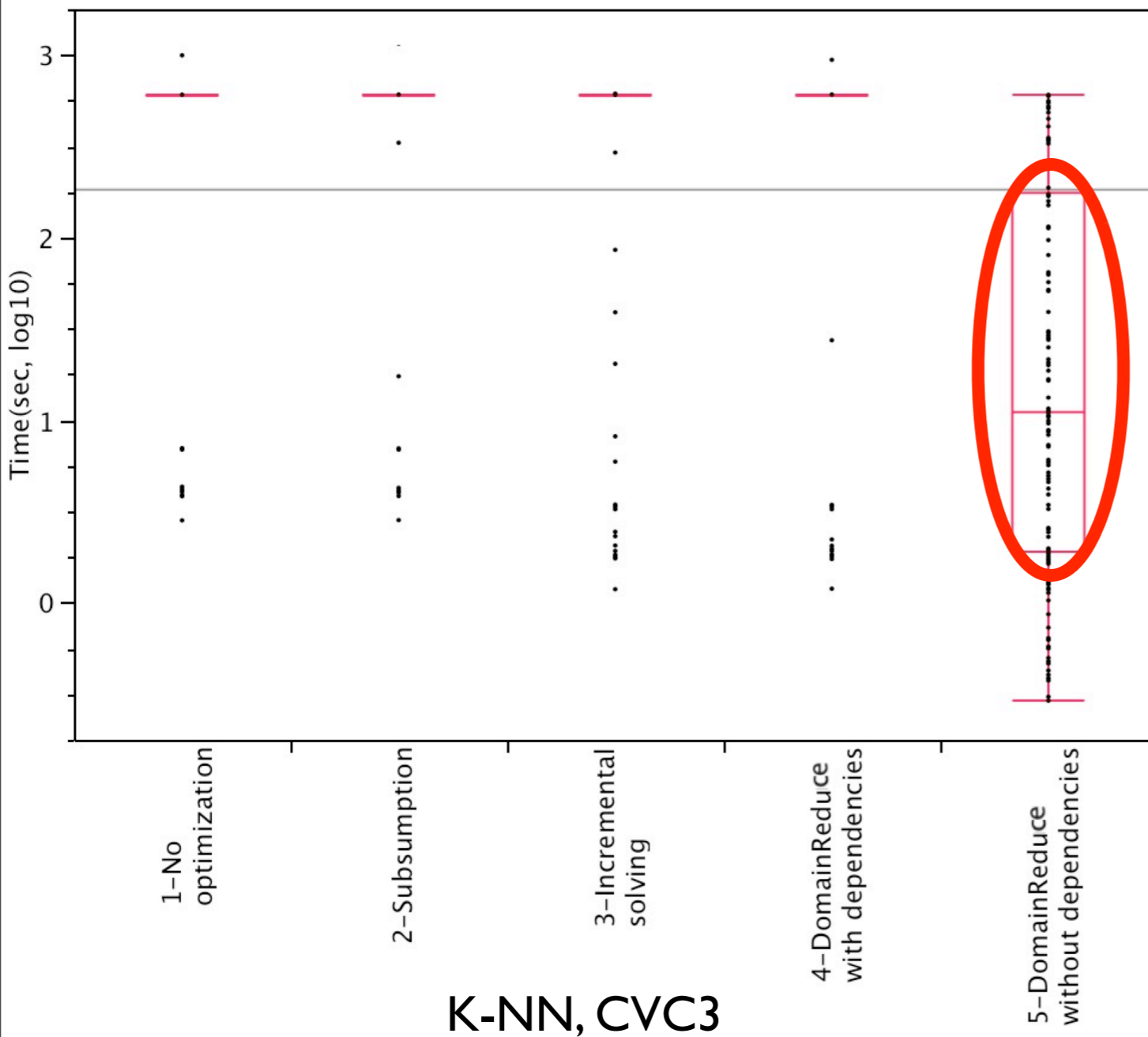
Study Results 2

(time to process constraints)



Study Results 2

(time to process constraints)

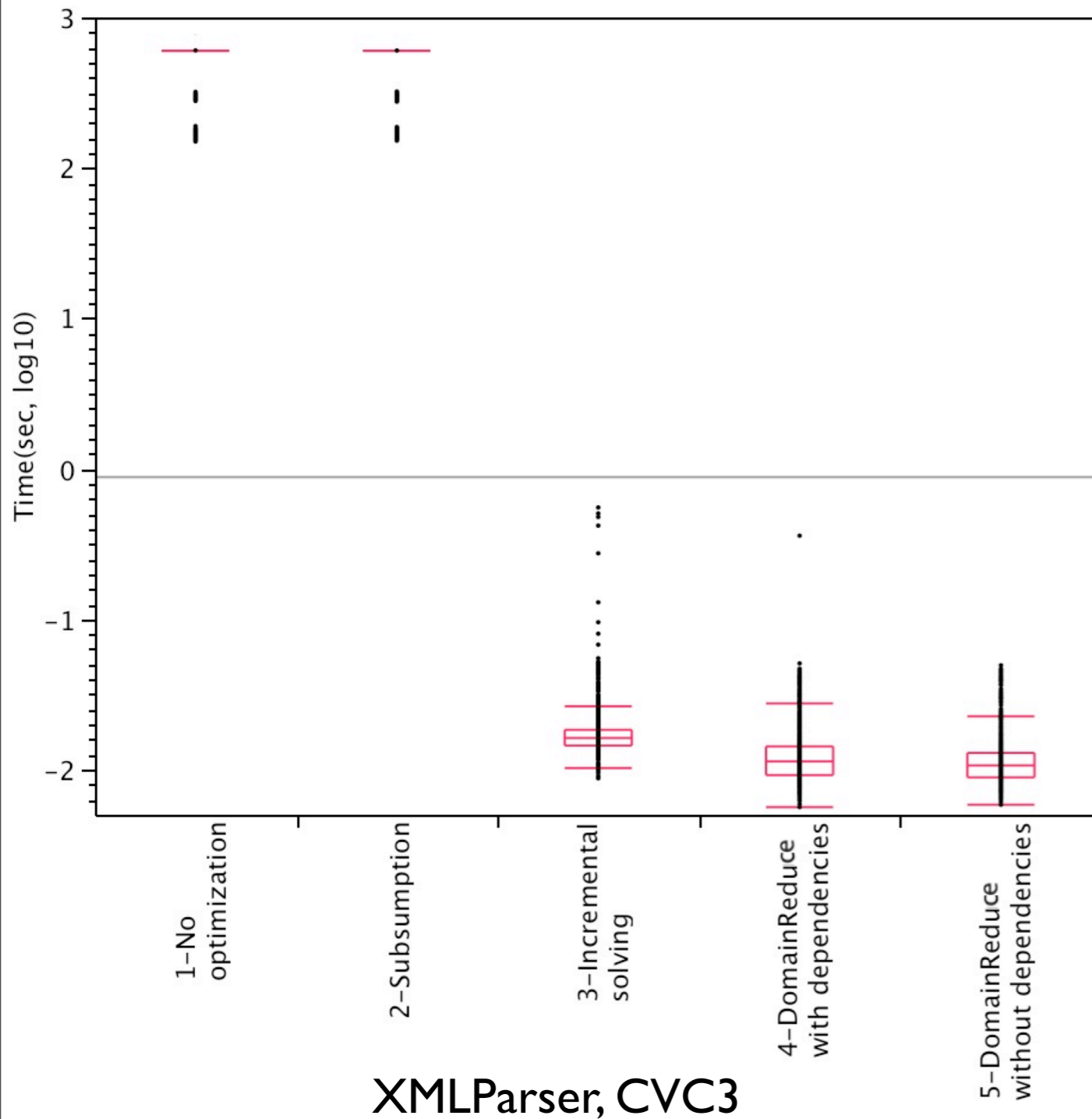


Study Results 2

(time to process constraints)

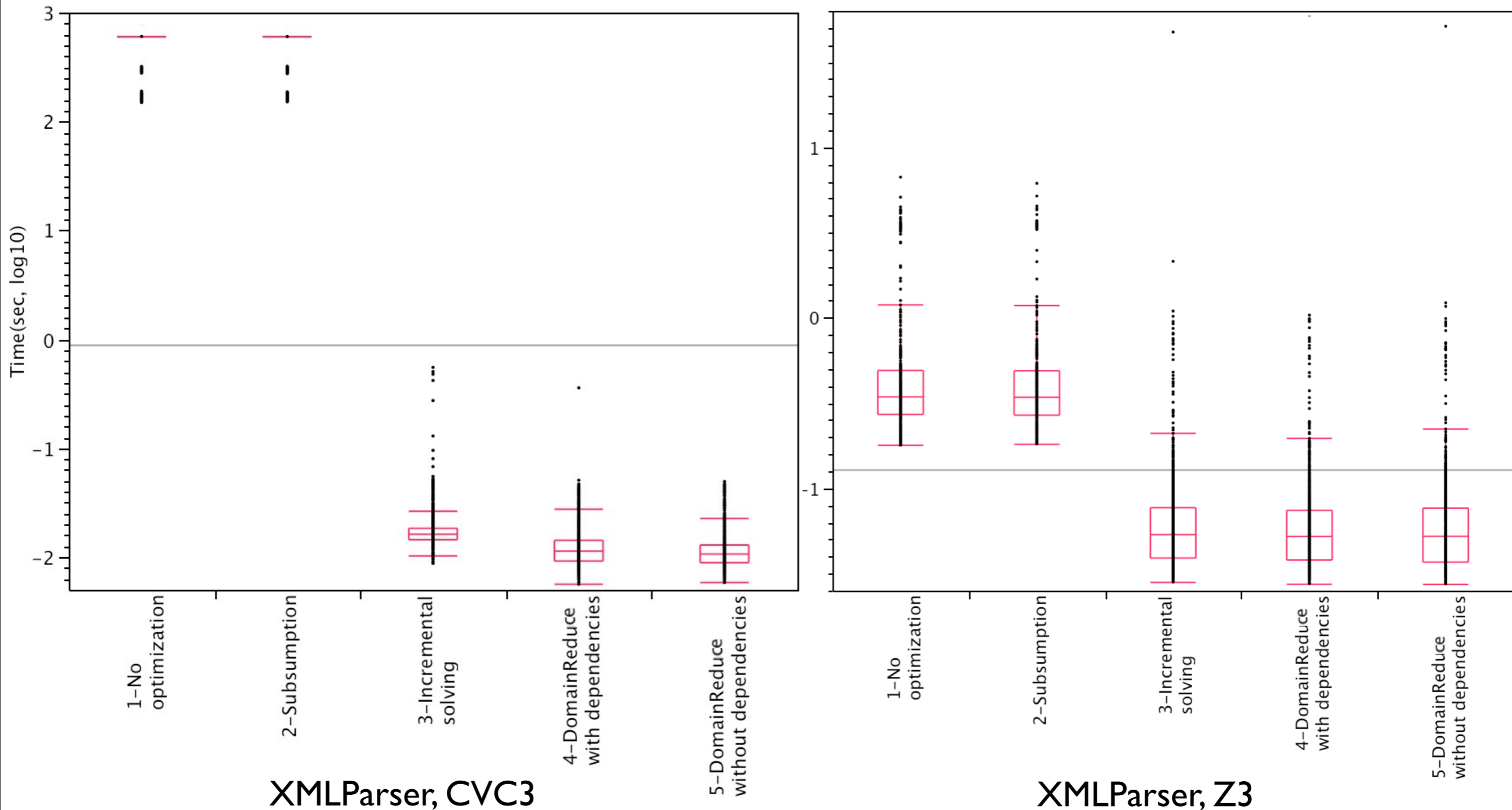
Study Results 2

(time to process constraints)



Study Results 2

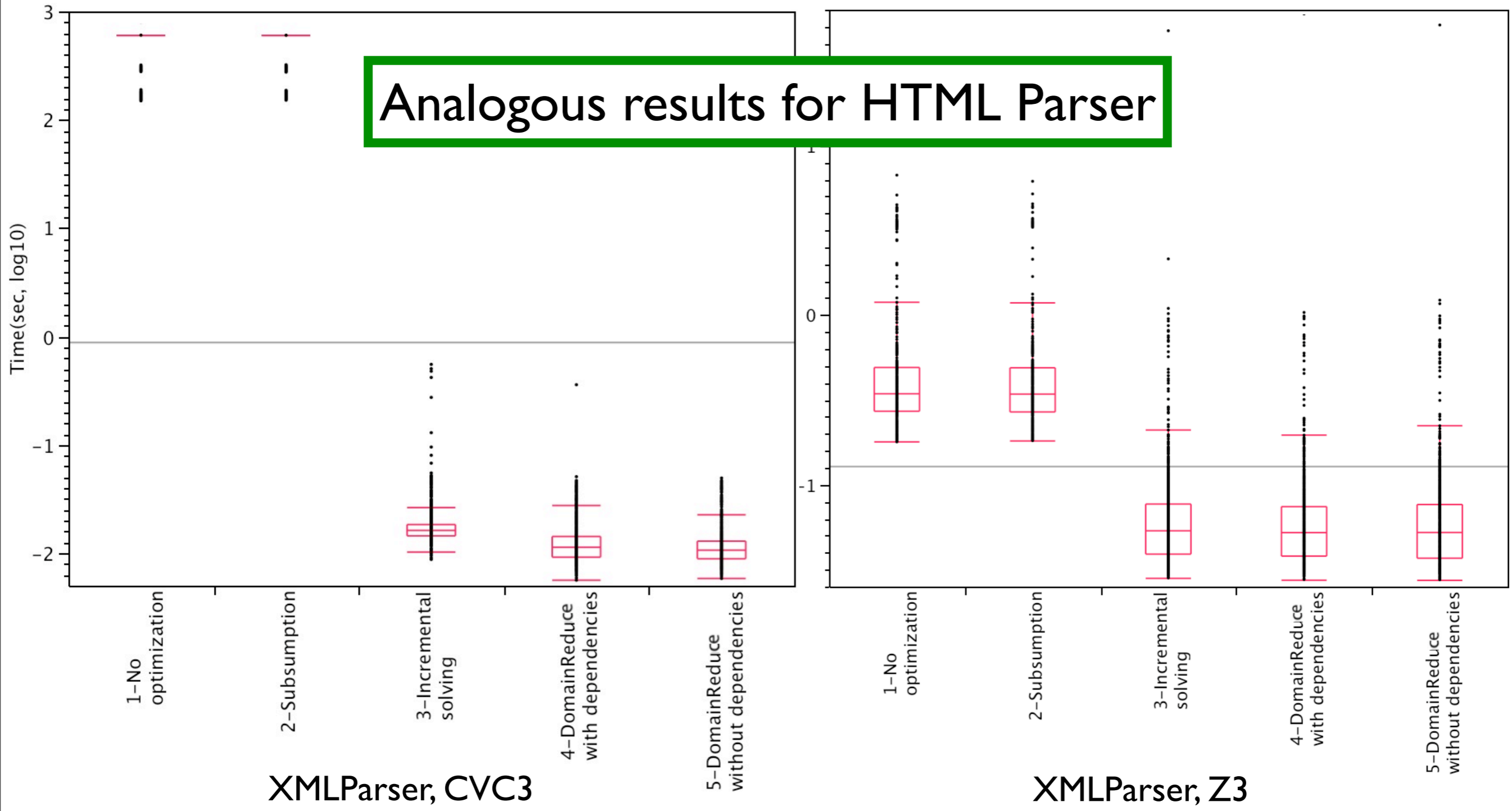
(time to process constraints)



Study Results 2

(time to process constraints)

Analogous results for HTML Parser



Study Results 2

(time to process constraints)

Study Results 2

(time to process constraints)

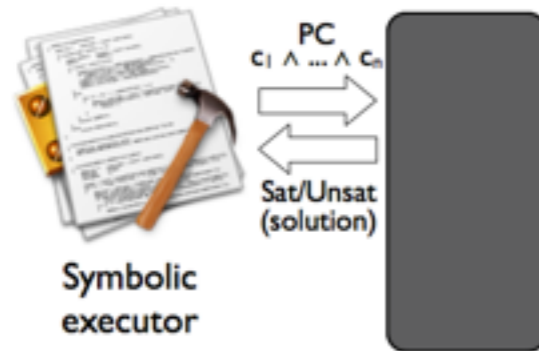
- K-NN
 - All but one optimizations provided no benefits (timeout or unsat after a long time)
 - DomainReduce with no dependencies finds solutions for less constraints, but very quickly, for K-NN and CVC3

Study Results 2

(time to process constraints)

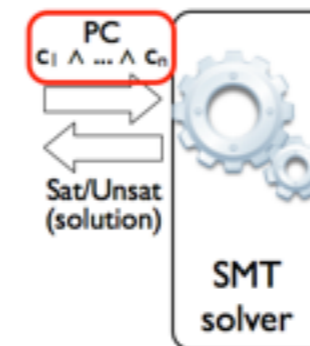
- K-NN
 - All but one optimizations provided no benefits (timeout or unsat after a long time)
 - DomainReduce with no dependencies finds solutions for less constraints, but very quickly, for K-NN and CVC3
- HTMLParser and XMLParser
 - Almost all optimizations improve efficiency of constraint solvers dramatically (several orders of magnitude)

Symbolic Execution and SMT Solving



DomainReduce: Intuitive View

Restrict domain of constraints to be solved by leveraging solution of similar PC



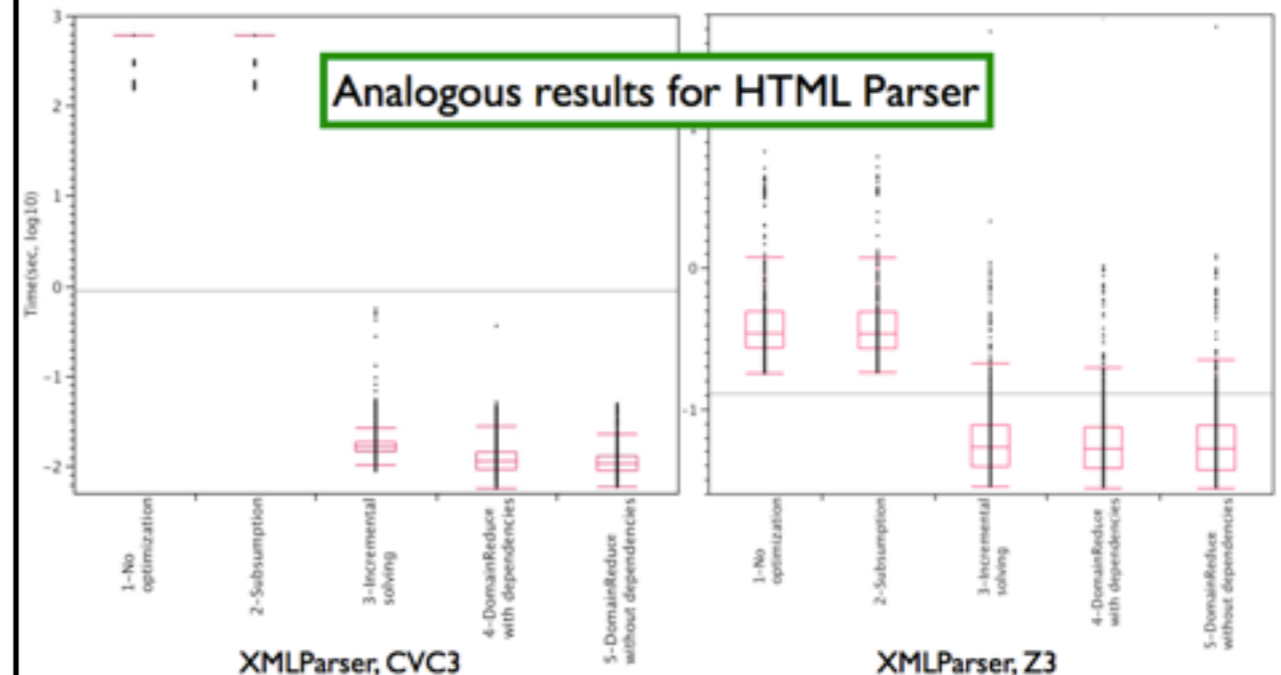
Tradeoff speed/likelihood of finding solution

Study Results (# constraints processed)

Subjects	PCs considered	PCs successfully processed									
		No optimization		Subsumption		Incremental solving		DomainReduce with dependencies		DomainReduce without dependencies	
		cvc3	z3	cvc3	z3	cvc3	z3	cvc3	z3	cvc3	z3
HTMLParser	1879	1879	1879	1879	1879	1879	1879	1879	1879	1879	1879
		43+1836	43+1836	43+1836	43+1836	43+1836	43+1836	43+1836	43+1836	43+1836	43+1836
XMLParser	1881	473	1881	473	1881	1881	1881	1881	1881	1881	1881
		49+424	49+1832	49+424	49+1832	49+1832	49+1832	49+1832	49+1832	49+1832	49+1832
K-NN	1930	261	936	261	936	271	937	262	878	111	0
		261+0	936+0	261+0	936+0	271+0	937+0	262+0	878+0	0+111	0+0

- Results for HTMLParser uninteresting
- Optimizations ineffective for Z3,
 - Useless or ineffective with one exception
 - DomainReduce even produce negative results for K-NN (worst case)
- Optimizations effective for CVC3
 - Small improvement for K-NN
 - Dramatic improvement for XMLParser (25% \Rightarrow 100%)

Study Results (time to process constraints)



Symbolic Execution and SMT Solving

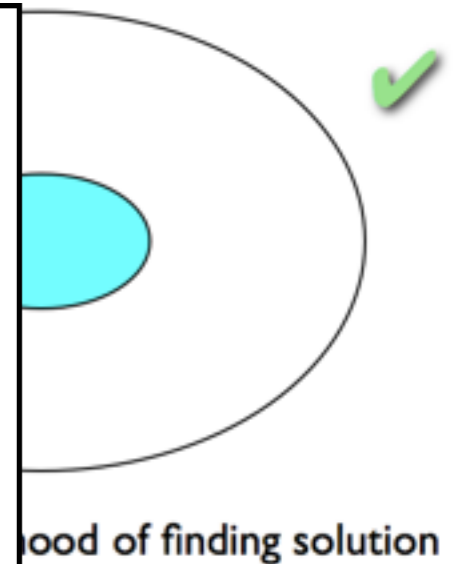
DomainReduce: Intuitive View

Restrict domain of constraints to be solved by leveraging solution of similar PC



Future work

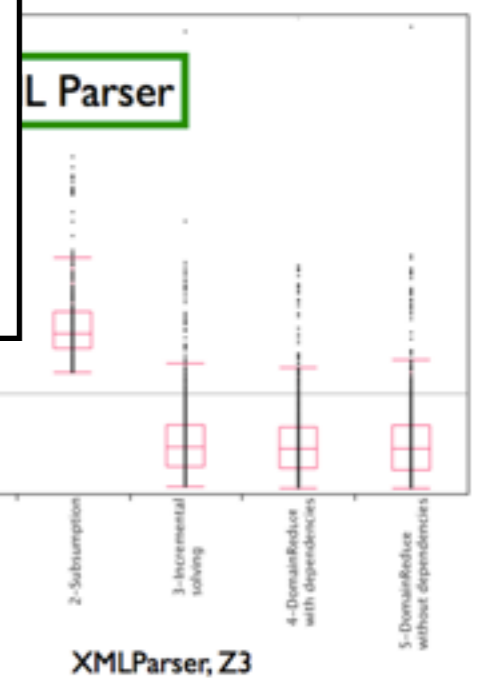
- More experiments (subjects, solvers, configurations)
- Investigate why optimizations work/don't work
- Apply optimizations in parallel
- More sophisticated optimizations (program structure or properties)
- Tighter integration



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(# cc

Subjects	PCs considered	No optimization	
		cvc3	z3
HTMLParser	1879	1879	1879
		43+1836	43+1836
XMLParser	1881	473	1881
		49+424	49+1832
K-NN	1930	261	936
		261+0	936+0

lts
(constraints)



- Results for HTMLParser
- Optimizations ineffective for Z3
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