Foot rotation indicator (FRI) point: A new gait planning tool to evaluate postural stability of biped robots

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Abstract

The focus of this paper is the problem of foot rotation in biped robots during the single support phase. Foot rotation is an indication of postural instability and should be carefully treated in a dynamically stable walk and avoided altogether in a statically stable walk.

We introduce the foot rotation indicator (FRI) point which is a point on the foot/ground contact surface where the net ground reaction force *would have to act* to keep the foot stationary. To ensure no foot rotation, the FRI point must remain within the convex hull of the foot support area.

In contrast with the ground projection of the center of mass (GCoM), which is a static criterion, the FRI point incorporates the robot dynamics. As opposed to the center of pressure (CoP) – better known as the zero moment point (ZMP) in the robotics literature – which may not leave the support area, the FRI point may. Due to these important properties the FRI point helps not only to monitor the state of static stability of a biped robot during the entire gait cycle, but indicates the stability robustness or the severity of instability of the gait as well. In response to a recent need the paper also resolves the misconceptions surrounding the CoP/ZMP equivalence.

1 Motivation

The problem of gait planning for biped robots is fundamentally different from the path planning for traditional fixed-base manipulator arms as succinctly pointed out in [12]. A biped robot may be viewed as a ballistic mechanism which intermittently interacts with its environment – the ground – through its feet. The foot/ground "joint" is *unilateral* since attractive forces are not present, and *underactuated* since control inputs are absent. Formally speaking, unilaterality and underactuation are the inherent characteristics of the locomotion mechanics and, at the same time, the root causes behind their postural instability and fall. A loss of postural stability may have potentially serious consequences and this calls for its thorough analysis in order to better predict and eliminate the possibility of fall.

Postural balance and stance foot equilibrium are profoundly inter-twined. A biped robot gait is said to be statically stable [9] and a human posture is said to be balanced[8] if the ground projection of its center of mass (GCoM)[1] falls within the convex hull of the foot support area (henceforth called the support polygon). The exit of the GCoM from the support polygon is equivalent to the presence of an uncompensated moment on the foot which causes it to rotate about a point on the polygon boundary.

Rotational equilibrium of the foot is therefore an important criterion for the evaluation and control of gait, and postural stability in legged robots. Indeed, foot rotation has been noted to reflect a loss of balance and an eventual fall in monopods[6] and bipeds[1] – two classes of legged robots most prone to instabilities. The exit of the GCoM from the support polygon is considered to be the determining factor of stability in the study of human posture as well[8].

Although the position of the GCoM is sufficient to determine the occurrence of foot rotation in a stationary robot, it is not so for a robot in motion. Instead it is the location of the *foot rotation indicator* (FRI) point, which we introduce in this paper, that indicates the existence of an unbalanced torque on the foot. The FRI point is a point on the foot/ground surface, within or outside the support polygon, where the net ground reaction force *would have to act* to keep the foot stationary. Farther away this point from the support boundary, larger is the unbalanced moment, and greater is the instability. To ensure no foot rotation, the FRI point must remain within the support polygon, regardless of the GCoM position. The FRI point is a dynamics-based criterion, and reduces to the GCoM position for a stationary robot.

We would like emphasize that the FRI point is distinctly different from the center of pressure CoP – better known as the zero moment point (ZMP)[1, 4, 5, 7, 9, 10, 11, 12] in the robotics literature – and frequently used in gait planning for biped robots. CoP is a point on the foot/ground surface where the net

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[1] GCoM: **G**round Projection of the **C**enter **o**f **M**ass.

ground reaction force *actually acts*. Regardless of the state of stability of the robot, the CoP may *never* leave the support polygon, whereas the FRI point does so whenever there is an unbalanced torque on the foot. In fact, the distance of the FRI point from the support polygon is an indication of the severity of this unbalanced torque and may be exploited during the planning stage.

This paper makes two main contributions. The first is the introduction of the FRI point which may be employed as a useful tool in gait planning in biped and other legged robots, as well as for the postural stability assessment in the human. The second contribution is in response to our discussion with other researchers regarding the misconceptions surrounding CoP and the CoP/ZMP equivalence. We review the physics behind both the concepts and show that CoP and ZMP are identical.

2 FRI point of a biped robot

In order to formally introduce the FRI point, we first treat the entire biped robot – a general n-segment extended rigid-body kinematic chain (sketch shown for example purpose in Fig. 1) – as a system and determine its response to the external force/torque. We may employ Newton or d'Alembert's principles for this purpose. The external forces acting on the robot are the resultant ground reaction force/torque, $\boldsymbol{R}$ and $\boldsymbol{M}$, acting at the CoP (denoted by point P, see Fig. 2), and the gravity. The equation for rotational dynamic equilibrium is obtained by noting that the sum of the external moments on the robot, computed either at its CoM or at *any* stationary reference point is equal to the sum of the rates of change of angular momentum of the individual segments about the same point. Taking moments at the origin O, we have

$$\boldsymbol{M} + \boldsymbol{OP} \times \boldsymbol{R} + \sum \boldsymbol{OG}_i \times m_i \boldsymbol{g} = \sum \dot{\boldsymbol{H}}_{Gi} + \sum \boldsymbol{OG}_i \times m_i \boldsymbol{a}_i \quad (1)$$

where m_i is the mass, G_i is the CoM location, $\boldsymbol{a}_i$ is the CoM linear acceleration, and $\boldsymbol{H}_{Gi}$ is the angular momentum about CoM, of the i^{th} segment. $\boldsymbol{M}$ is the frictional ground reaction moment (tangential).

An important aspect of our approach is to treat the stance foot as the focus of attention. Indeed, as the only robot segment interacting with the ground, the stance foot is a "special" segment subjected to joint forces, gravity forces and the ground reaction forces. Viewing from the stance foot, the dynamics of the rest of the robot may be completely represented by the ankle force/torque $-\boldsymbol{R}_1$ and $-\boldsymbol{\tau}_1$ (negative signs for convention). Fig. 2 artificially disconnects the ankle joint to clearly show the forces in action at that joint. The dynamic equilibrium equation of the foot (segment#1) is:

$$\boldsymbol{M} + \boldsymbol{OP} \times \boldsymbol{R} + \boldsymbol{OG}_1 \times m_1 \boldsymbol{g} - \boldsymbol{\tau}_1 - \boldsymbol{OO}_1 \times \boldsymbol{R}_1 = \dot{\boldsymbol{H}}_{G1} + \boldsymbol{OG}_1 \times m_1 \boldsymbol{a}_1. \quad (2)$$

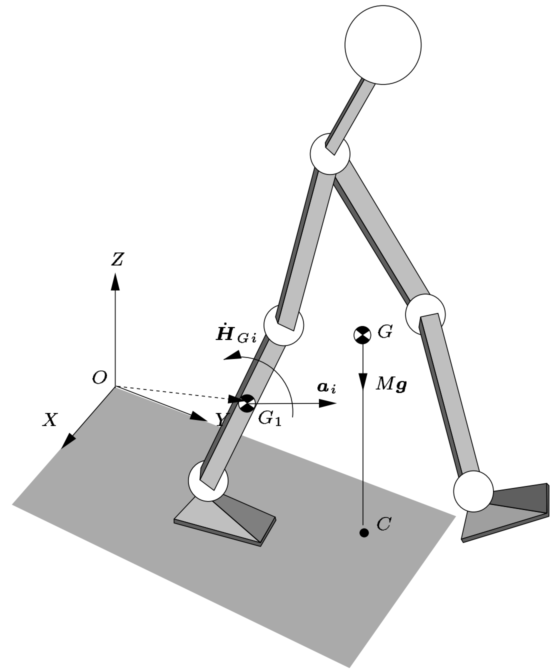

Figure 1: The sketch of a biped robot with the dynamic forces on the i-th segment. The GCoM is denoted by C.

The equations for *static* equilibrium of the foot are obtained by setting the dynamic terms (RHS) in Eq. 2 to zero:

$$\boldsymbol{M} + \boldsymbol{OP} \times \boldsymbol{R} + \boldsymbol{OG}_1 \times m_1 \boldsymbol{g} - \boldsymbol{\tau}_1 - \boldsymbol{OO}_1 \times \boldsymbol{R}_1 = \boldsymbol{0} \quad (3)$$

Recall that to derive Eq. 3 we could compute the moments at any other stationary reference point. Out of these the CoP represents a special point where Eq. 3 reduces to a simpler form

$$\boldsymbol{M} + \boldsymbol{PG}_1 \times m_1 \boldsymbol{g} - \boldsymbol{\tau}_1 - \boldsymbol{PO}_1 \times \boldsymbol{R}_1 = \boldsymbol{0}. \quad (4)$$

Considering only the tangential (XY) vector components of Eq. 4, we may write

$$\left(\boldsymbol{\tau}_1 + \boldsymbol{PO}_1 \times \boldsymbol{R}_1 - \boldsymbol{PG}_1 \times m_1 \boldsymbol{g}\right)_t = \boldsymbol{0} \quad (5)$$

where the subscript t implies the tangential components. Since $\boldsymbol{M}$ is tangential to the foot/ground surface its vector direction is normal to that surface[2]. In the presence of an unbalanced torque on the foot Eq. 5 is not satisfied for any point within the support polygon. One may, however, still find a point F outside the support boundary which satisfies Eq. 4, i.e.,

$$\left(\boldsymbol{\tau}_1 + \boldsymbol{FO}_1 \times \boldsymbol{R}_1 - \boldsymbol{FG}_1 \times m_1 \boldsymbol{g}\right)_t = \boldsymbol{0}. \quad (6)$$

The point F is called the FRI point and defined as,

[2] We ignore foot rotation about the ground normal as it does not contribute to a balance loss. Also we assume that the foot/ground friction is sufficiently large and there is no sliding.

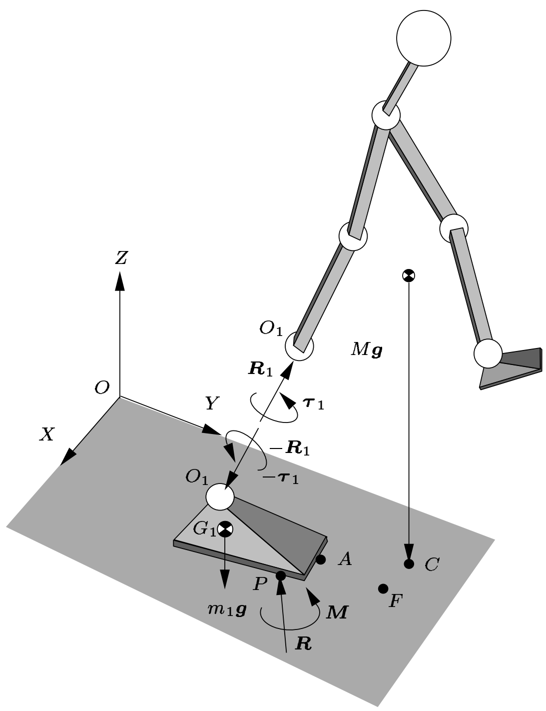

Figure 2: Biped robot foot artificially disconnected to show the intervening forces. The CoP and the FRI point are denoted by P and F, respectively.

> *The foot rotation indicator (FRI) point is a point on the foot/ground contact surface, within or outside the convex hull of the foot support area, at which the resultant moment of the force/torque impressed on the foot is normal to the surface.*

By impressed force/torque, we mean the force and torque at the ankle joint plus the weight of the foot, and not the ground reaction forces. An intuitive understanding is obtained by setting $\boldsymbol{\tau}_1 = \boldsymbol{0}$, $m_1 = 0$ in Eq. 6. In this case F is the point on the ground where the line of action of $\boldsymbol{R}_1$ penetrates, as was considered in [6] to analyze the hoof rotation in a monopod.

Finally, explicit expressions for the coordinates of F, $\boldsymbol{OF} : (OF_x, OF_y, OF_z = 0)$ are obtained by computing the dynamics of the robot *minus the foot* at F,

$$\boldsymbol{\tau}_1 + \boldsymbol{FO}_1 \times \boldsymbol{R}_1 + \sum_{i=2}^{n} \boldsymbol{FG}_i \times m_i \boldsymbol{g} = \sum_{i=2}^{n} \dot{\boldsymbol{H}}_{Gi} + \sum_{i=2}^{n} \boldsymbol{FG}_i \times m_i \boldsymbol{a}_i \quad (7)$$

Using Eq. 6 and considering only the tangential components,

$$\left(\boldsymbol{FG}_1 \times m_1 \boldsymbol{g} + \sum_{i=2}^{n} \boldsymbol{FG}_i \times m_i (\boldsymbol{g} - \boldsymbol{a}_i) \right)_t = \left(\sum_{i=2}^{n} \dot{\boldsymbol{H}}_{Gi} \right)_t . \quad (8)$$

Noting $\boldsymbol{FG}_i = \boldsymbol{FO} + \boldsymbol{OG}_i$ and $\boldsymbol{OF} = -\boldsymbol{FO}$, Eq. 8 may be rewritten as

$$\left(\sum_{i=2}^{n} \boldsymbol{OF} \times m_i (\boldsymbol{a}_i - \boldsymbol{g}) - \boldsymbol{OF} \times m_1 \boldsymbol{g} \right)_t = \left(- \boldsymbol{OG}_1 \times m_1 \boldsymbol{g} + \sum_{i=2}^{n} \dot{\boldsymbol{H}}_{Gi} + \sum_{i=2}^{n} \boldsymbol{OG}_i \times m_i (\boldsymbol{a}_i - \boldsymbol{g}) \right)_t . \quad (9)$$

Carrying out the operation, we may finally obtain:

$$OF_x = \frac{NUM_1}{DEN} \text{ and } OF_y = \frac{NUM_2}{DEN}, \quad (10)$$

where $DEN = m_1 g + \sum_{i=2}^{n} m_i (a_{iz} + g)$, $NUM_1 = m_1 OG_{1y} g + \sum_{i=2}^{n} m_i OG_{iy} (a_{iz} + g) - \sum_{i=2}^{n} m_i OG_{iz} a_{iy} + \sum_{i=2}^{n} \dot{H}_{Gix}$ and $NUM_2 = m_1 OG_{1x} g + \sum_{i=2}^{n} m_i OG_{ix} (a_{iz} + g) - \sum_{i=2}^{n} m_i OG_{iz} a_{ix} - \sum_{i=2}^{n} \dot{H}_{Giy}$.

2.1 Properties of FRI point

Some useful properties of the FRI point which may be exploited in gait planning are listed below:

1. *The FRI point indicates the* **occurrence** *of foot rotation* as already described.

2. *The location of the FRI point indicates the* **magnitude** *of the unbalanced moment on the foot.* The total moment M_A^I due to the impressed forces about a point A on the support polygon boundary (Fig. 2) is:

   $$M_A^I = \boldsymbol{AF} \times (m_1 \boldsymbol{g} - \boldsymbol{R}_1) \quad (11)$$

   which is proportional to the distance between A and F. If F is situated inside the support polygon M_A^I is counter-acted by the moment due to $\boldsymbol{R}$ and is precisely compensated, see Fig. 3, top, for a planar example. Otherwise, M_A^I is the uncompensated moment which causes the foot to rotate (Fig. 3, bottom).

3. *The FRI point indicates the* **direction** *of foot rotation.* This we derive from Eq.11 assuming that $m_1 \boldsymbol{g} - \boldsymbol{R}_1$ is directed downwards.

4. *The FRI point indicates the* **stability margin** *of the robot.* The *stability margin* of a robot against foot rotation may be quantified by the minimum distance of the support polygon boundary from the current location of the FRI point within the footprint. Conversely, when the FRI point is outside the footprint, this minimum distance is a measure of instability of the robot. An imminent foot rotation will be indicated by a motion of the FRI point towards the support polygon boundary.

2.2 Examples

The difference between the CoP, GCoM, and the FRI point will be analytically explored in Section 3. Here we provide two examples to aid our intuitive understanding. Fig. 4 depicts a planar inverted pendulum

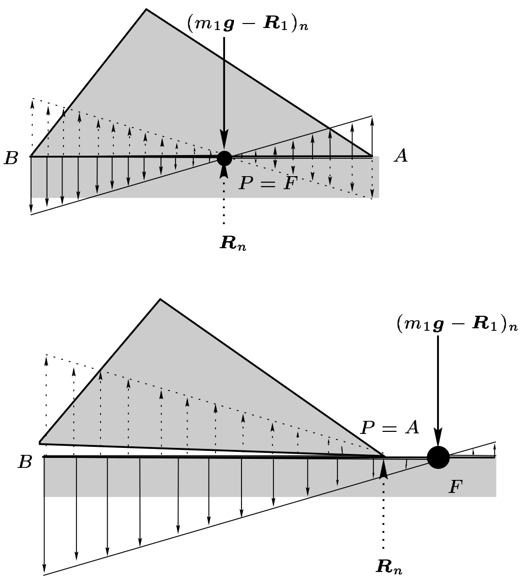

Figure 3: The magnitude of the moment experienced by a point on the support boundary is linearly proportional to the distance of this point from the FRI point (F). The magnitudes of the moments are shown by the arrow lengths. CW (i.e., negative) moments are shown by upward pointed arrows and the CCW (i.e., positive) moments are shown by downward pointed arrows. Moments are precisely compensated at top, so $P = F$, and the foot is stable. At bottom, $P \neq F$ and the foot is unstable. n stands for the normal component.

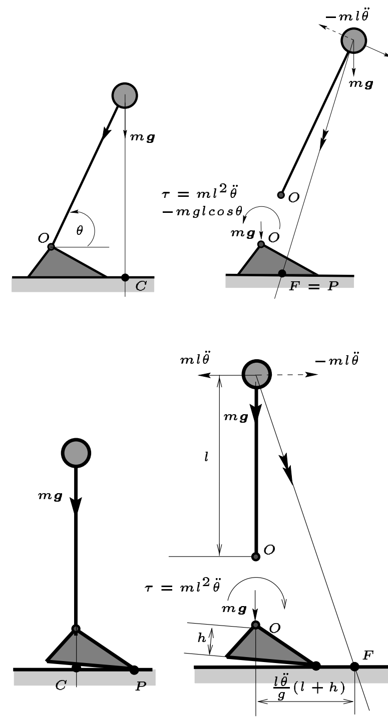

Figure 4: Two simple examples to compare and contrast the CoP (P), GCoM (C), and FRI point (F). In the top figure the foot is in static equilibrium since F is within the support line (although C is outside). P is coincident with F. At bottom, the foot is starting to rotate since F is outside the support line (although C is inside). P is at the tip about which the foot rotates.

connected by an "ankle" joint to a massless[3] triangular foot. In the first example (top) the pendulum configuration corresponds to a GCoM position denoted by C, outside the support polygon. The foot is however prevented from rotating by the ankle torque ($ml^2\ddot{\theta} - mlgcos\theta$) and the FRI point F is situated within the support line. Note that in order to stop the robot from tipping over [5] used a scheme to accelerate forward the heavy robot body. This generates a supplementary backward inertia force – similar to this example – which shifts the FRI point backward.

In the second example (bottom), the pendulum is vertically upright with its GCoM well within the support line. Despite, the foot starts to rotate due to the ankle torque $ml^2\ddot{\theta}$. The FRI point F is situated outside the support line at a horizontal distance $OF_y = \frac{l\ddot{\theta}}{g}(l+h)$ from O.

3 FRI point, CoP (ZMP) and GCoM compared

3.1 CoP reviewed

Although the term CoP most likely originated in the field of fluid mechanics, it is frequently used in the study of gait and postural balance. In Fig. 5, two types of interaction forces are shown to act at the foot/ground interface. They are the normal forces $\boldsymbol{f}_{ni}$, always directed upwards (Fig. 5, top left) and the frictional tangential forces $\boldsymbol{f}_{ti}$ (Fig. 5, top right). CoP may be defined as the point P where the resultant $\boldsymbol{R}_n = \sum \boldsymbol{f}_{ni}$ acts. With respect to a coordinate origin O, $\boldsymbol{OP} = \frac{\sum \boldsymbol{q}_i f_{ni}}{\sum f_{ni}}$, where $\boldsymbol{q}_i$ is the vector to the point of action of force $\boldsymbol{f}_i$. P must lie within the support polygon. The resultant of the tangential forces may be represented at P by a force $\boldsymbol{R}_t = \sum \boldsymbol{f}_{ti}$ and a moment $\boldsymbol{M} = \sum \boldsymbol{r}_i \times \boldsymbol{f}_{ti}$ where $\boldsymbol{r}_i$ is the vector from P to the point of application of $\sum \boldsymbol{f}_{ti}$.

The complete picture is shown in Fig. 5, bottom. The stance foot of the biped robot is subjected to a resultant ground reaction force $\boldsymbol{R} = \boldsymbol{R}_n + \boldsymbol{R}_t$ and a ground reaction moment $\boldsymbol{M}$. An analysis with a continuous distribution of ground reaction force was performed earlier[2, 3]. We point out that contrary to what appeared in [9] $\boldsymbol{R}$, and not $\boldsymbol{R}_n$, is the total ground reaction force. Note that CoP is identical to what has been termed as the "center of the actual ground reaction force" (C-ATGRF) in a recent paper[5].

[3]This simplifies our description without compromising the main issue.

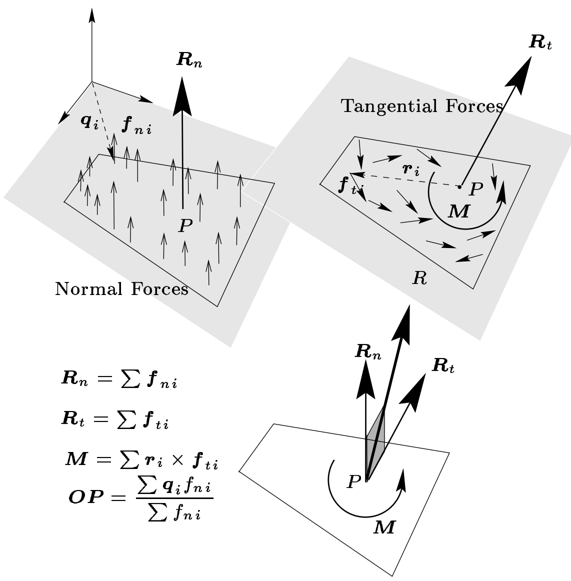

Figure 5: Analysis of CoP (Section 3.1). In the foot/ground interface we have the normal forces (top left) and the frictional tangential forces (top right). CoP is the point (P) where the resultant $\boldsymbol{R}_n$ of the normal forces act. At CoP, the tangential forces may be represented by a resultant force $\boldsymbol{R}_t$ and a moment $\boldsymbol{M}$ (bottom). Ground reaction force is $\boldsymbol{R} = \boldsymbol{R}_n + \boldsymbol{R}_t$.

3.2 Zero moment point (ZMP)

The concept of ZMP which we demonstrate to be identical to the CoP is known to have originally been introduced in 1969[13]. Since then it has been frequently used in biped robot control [1, 4, 5, 7, 9, 10, 11, 12] as a criterion of postural stability. Reference is often made to the *ZMP condition*[1], or the *ZMP stability criterion*[7], which states that the ZMP of a biped robot must be constrained within the convex hull of the foot support area to ensure the stability of the foot/ground contact[1], walk stability without falling down[1], dynamic stability of locomotion[10, 9], physical admissibility and realizability of gait[9]. Unfortunately, these justifications are not all equivalent, and the physical implications of some of these descriptions are not entirely clear.

A similar problem is encountered with the different definitions of ZMP, which, perhaps due to lack of rigor, are not always clearly understandable and has created confusion in the research community. Among the references reviewed we find the following definitions to be correct and consistent:

Def 1 **Hemami and Golliday 1977:** ZMP is the point where the vertical reaction force intersects the ground[4].

Def 2 **Arakawa and Fukuda 1997:** ZMP is the point on the floor at which the moment $\boldsymbol{T} : (T_x, T_y, T_z)$ generated by the reaction force and the reaction torque satisfies $T_x = 0$, and $T_y = 0$[1].

The term zero moment point is a misnomer since in general only two of the three moment components are zero. This raises question about the necessity of introducing a new name for an already well-known concept.

3.3 CoP = ZMP

Defs. 1, 2 of ZMP immediately correspond to the definition of CoP as we described in Section 3.1. It is also possible to show that CoP is the same point where the resultant moment generated by the inertia and gravity forces is tangential to the surface. To prove this let us first assume that this latter point, which we call D is distinct from the CoP. The dynamic equilibrium equation computed at D takes the form:

$$\boldsymbol{M} + \boldsymbol{DP} \times \boldsymbol{R} + \sum \boldsymbol{DG}_i \times m_i \boldsymbol{g} = \sum \dot{\boldsymbol{H}}_{Gi} + \sum \boldsymbol{DG}_i \times m_i \boldsymbol{a}_i \quad (12)$$

whereas, by definition D satisfies:

$$\left(\sum \dot{\boldsymbol{H}}_{Gi} + \sum \boldsymbol{DG}_i \times m_i(\boldsymbol{a}_i - \boldsymbol{g})\right)_t = \boldsymbol{0} \quad (13)$$

Comparing Eqs. 12 and 13, $(\boldsymbol{DP} \times \boldsymbol{R})_t = \boldsymbol{0}$. However, since $\boldsymbol{R} \neq \boldsymbol{0}$ and $\boldsymbol{DP} \nparallel \boldsymbol{R}$ in general, this is possible only if $\boldsymbol{DP} = \boldsymbol{0}$ or the points D and P are coincident. Other approaches have led to identical conclusion[2, 3]. CoP is defined in terms of the point of action of the ground reaction force and ZMP is traditionally defined in terms of the dynamic forces.

Another way to see this is to rewrite Eq. 12 as

$$(\boldsymbol{DP} \times \boldsymbol{R})_t = \left(\sum \dot{\boldsymbol{H}}_{Gi} + \sum \boldsymbol{DG}_i \times m_i(\boldsymbol{a}_i - \boldsymbol{g})\right)_t \quad (14)$$

in which the LHS corresponds to the traditional definition of CoP and the RHS corresponds to the definition used to compute the ZMP.

Since CoP=ZMP, ZMP may never leave the support polygon, contrary to what was incorrectly suggested in [7, 9]. Also, ZMP has no inherent relationship with a dynamically stable gait as has been previously implied[7, 10].

3.4 FRI point and CoP

In order to relate the FRI point and CoP let us rewrite Eq. 2, this time computing the moments at F:

$$\boldsymbol{M} + \boldsymbol{FP} \times \boldsymbol{R} + \boldsymbol{FG}_1 \times m_1 \boldsymbol{g} - \boldsymbol{\tau}_1 - \boldsymbol{FO}_1 \times \boldsymbol{R}_1 = \dot{\boldsymbol{H}}_{G1} + \boldsymbol{FG}_1 \times m_1 \boldsymbol{a}_1 \quad (15)$$

By substituting Eq. 6 in Eq. 15 we obtain:

$$(\boldsymbol{FP} \times \boldsymbol{R})_t = (\dot{\boldsymbol{H}}_{G1} + \boldsymbol{FG}_1 \times m_1 \boldsymbol{a}_1)_t \quad (16)$$

The FRI point and CoP are coincident if $\boldsymbol{FP} = \boldsymbol{0}$, i.e., if $(\dot{\boldsymbol{H}}_{G1} + \boldsymbol{FG}_1 \times m_1 \boldsymbol{a}_1)_t = \boldsymbol{0}$. This is possible if any one of the following conditions is satisfied: 1) $\boldsymbol{a}_1 = \boldsymbol{0}$ and angular acceleration is zero i.e., the foot is at rest or has uniform linear and angular motions, 2) $\boldsymbol{I}_1 = \boldsymbol{0}$ and $m_1 = 0$, i.e., the foot has zero mass and inertia, 3) $\boldsymbol{FG}_1 \parallel m_1 \boldsymbol{a}_1$ and $\boldsymbol{I}_1 = \boldsymbol{0}$.

It may be shown that for an idealized rigid foot the CoP is situated at a boundary point unless the foot is in stable equilibrium. Since the position of CoP cannot distinguish between the marginal state of static equilibrium and a complete loss of equilibrium of the foot (in both cases it is situated at the support boundary), its utility in gait planning is limited. FRI point, on the other hand, may exit the physical boundary of the support polygon and it does so whenever the foot is subjected to a net rotational moment.

3.5 CoP and GCoM

Referring to Fig. 1, GCoM, C satisfies

$$\boldsymbol{CG} \times \sum m_i \boldsymbol{g} = \boldsymbol{0} \tag{17}$$

where G is the center of mass of the entire robot and $\sum m_i = M$ is the total robot mass. Noting that $\boldsymbol{CG} \sum m_i = \sum \boldsymbol{CG}_i m_i$, and $\boldsymbol{CG}_i = \boldsymbol{CP} + \boldsymbol{PG}_i$ we can rewrite Eq. 17 as:

$$\boldsymbol{CP} \times \sum m_i \boldsymbol{g} + \sum \boldsymbol{PG}_i \times m_i \boldsymbol{g} = \boldsymbol{0} \tag{18}$$

Substituting in Eq. 1 we get

$$\boldsymbol{M} - \boldsymbol{CP} \times \sum m_i \boldsymbol{g} = \sum \dot{\boldsymbol{H}}_{Gi} + \sum \boldsymbol{PG}_i \times m_i \boldsymbol{a}_i \tag{19}$$

From above, P and C coincide if $\left(\sum \dot{\boldsymbol{H}}_{Gi} + \sum \boldsymbol{PG}_i \times m_i \boldsymbol{a}_i\right)_t = \boldsymbol{0}$ which is possible if the robot is stationary.

4 Conclusions

We introduced a new criterion called the FRI point that indicates the state of postural stability of a biped robot. The FRI point is a point on the foot/ground surface, within or outside the support polygon, where the net ground reaction force *would have to act* to keep the foot stationary. When the entire robot is stationary and stable, the FRI point is situated within the support polygon, and is coincident with GCoM and CoP. For stationary and unstable configurations, both GCoM and FRI point, which are coincident, are outside the support polygon. The CoP is at the polygon boundary.

In the presence of dynamics the GCoM and FRI point are non-coincident. When the foot is stable (implying that the robot possesses postural balance) the FRI point is situated within the support polygon and is coincident with the CoP. The CoP may never leave the support polygon, whereas the FRI point may. An exit of the FRI point from the support polygon signals postural instability. Farther away is the FRI point from the support boundary, larger is the unbalanced moment on the foot and greater is the instability. The distance between the FRI point and the nearest point on the polygon boundary is an useful indicator of the static *stability margin* of the foot.

Although postural stability of a biped robot (or a human being) is closely related to the static stability of its foot, the relationship between foot stability and natural anthropomorphic bipedalism is not at all clear. Even a simple observation of human locomotion will convince us that a significant part of the gait cycle involves foot rotation. One of our future goals is to measure the FRI point trajectory for natural human locomotion.

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